ECE606: Solid State Devices
Lecture 12 (from17)
High Field, Mobility
Hall Effect, Diffusion

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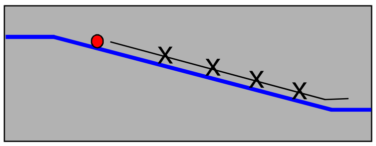
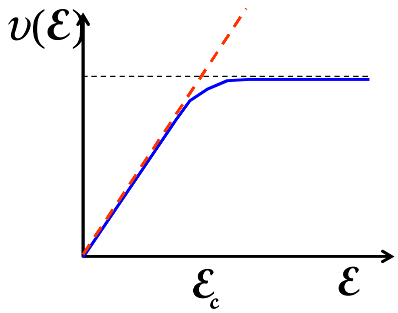
1) **High Field Mobility effects**
2) Measurement of mobility
3) Hall Effect for determining carrier concentration
4) Physics of diffusion
5) Conclusions

REF: ADF, Chapter 5, pp. 190-202

Mobility at High Fields?



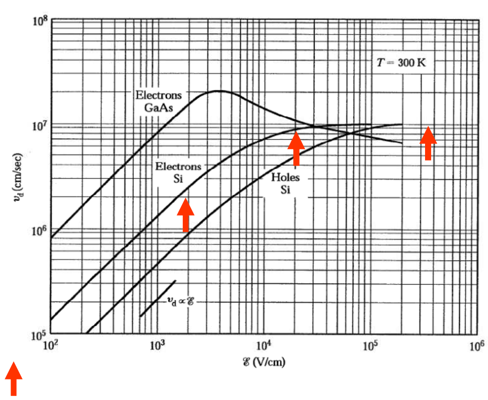
$$\upsilon = \frac{q\tau_N}{m_N^*}\mathcal{E}$$

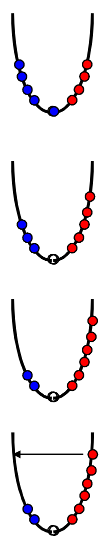


What causes velocity saturation at high fields?

Where does all the mobility formula in device simulator come from?

Velocity Saturation in Si/Ge





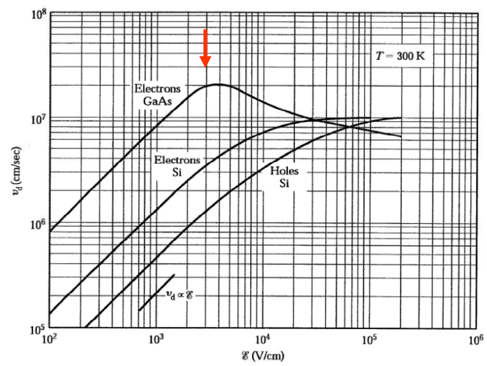
$\mathcal{E} = 0 \quad J_1 = J^+ - J^- = 0$

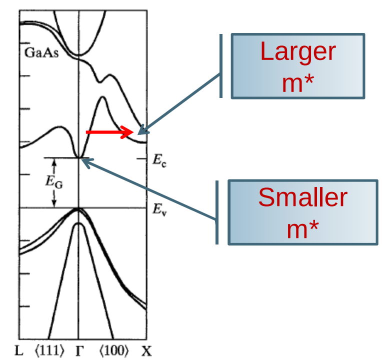
$\mathcal{E} \ll \mathcal{E}_c \quad J_2 = J^+ - J^- > J_1$

$\mathcal{E} \approx \mathcal{E}_c \quad J_3 = J^+ - J^- > J_2$

$\mathcal{E} \gg \mathcal{E}_c \quad J_4 = J^+ - J^- \approx J_3$

Velocity Overshoot & Inter-valley Transfer





What type of scattering would you need for inter-valley transfer?

Outline

1) High Field Mobility effects
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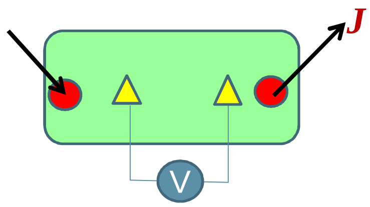
Problem of mobility measurement - unknown doping concentration

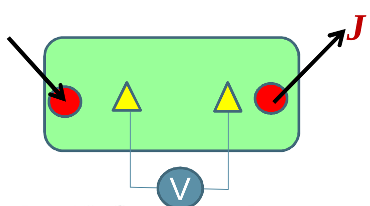


Four-Probe Measurement[1] measures voltage of device without measuring drop in current carrying wires

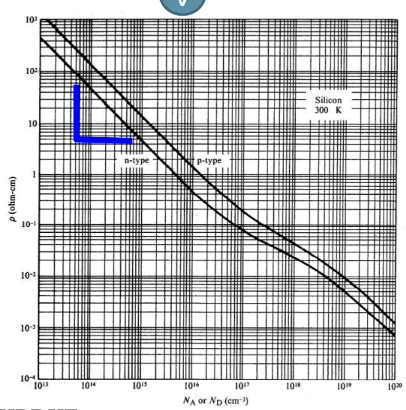
Can we find out the doping concentration and type by an *electrical measurement* without any knowledge of how the sample was prepared?

1 -http://en.wikipedia.org/wiki/Four-terminal_sensing

Problem of mobility measurement - unknown doping concentration



Four-Probe Measurement[1] measures voltage of device without measuring drop in current carrying wires



$$\mathcal{E} = \rho J$$

(in the low voltage limit where field-voltage curve is linear)

$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$= \frac{1}{q\mu_n N_D} \leftarrow \text{for n-type}$$

$$= \frac{1}{q\mu_p N_A} \leftarrow \text{for p-type}$$

1 -http://en.wikipedia.org/wiki/Four-terminal_sensing

Outline

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Trivia - FYI

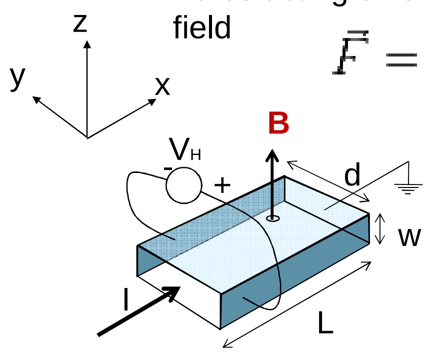
- Discovered in 1879 by Edwin Herbert Hall while he was working on his doctoral degree at Johns Hopkins University in Baltimore, Maryland.
- Done 18 years before the electron was discovered.
- 4 Nobel Prizes associated directly with it.
- Read the original article (*On a New Action of the Magnet on Electric Currents*) at http://www.stenomuseet.dk/skoletj/elmag/kilde9.html for a fascinating account of the discovery of the effect.

http://en.wikipedia.org/wiki/Hall_effect and http://en.wikipedia.org/wiki/Edwin_Hall

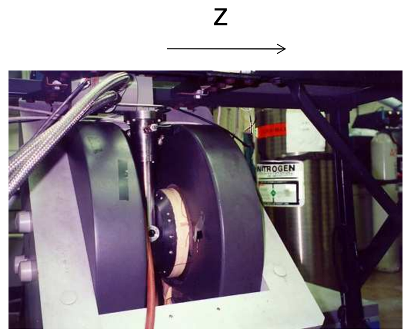
Set up for Hall Measurement

Force acting on a charged particle in a magnetic field

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$



- Applying a magnetic field one 'pushes' carriers towards one face
- This induces a voltage across the two faces perpendicular to current flow.
- We will relate this voltage to the current and magnetic field and deduce the density of carriers.



UIC system: 4-300K, 0-1.5 T

Drude Model for electrons *

Simple classical Newton's law expression
m – effective mass
v – drift velocity w/ scattering

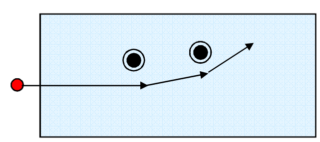
$$-q\mathcal{E} - q\boldsymbol{\upsilon}\times\mathbf{B} - \frac{m^*\boldsymbol{\upsilon}}{\tau} = 0$$

$$m^*\boldsymbol{\upsilon} = -q\tau\mathcal{E} - q\tau\boldsymbol{\upsilon}\times\mathbf{B}$$

$$\approx -q\tau\mathcal{E} - q\tau\left(-\frac{q\tau\mathcal{E}}{m*}\right)\times\mathbf{B}$$

$$= -q\tau\mathcal{E} + \frac{q^2\tau^2}{m^*}\mathcal{E}\times\mathbf{B}$$

$$\upsilon = -\frac{q\tau\mathcal{E}}{m^*} + \frac{q^2\tau^2}{m^{*2}}\mathcal{E}\times\mathbf{B}$$



Weak **B** field …

$$-q\mathcal{E} - \frac{m^*\boldsymbol{\upsilon}}{\tau} \approx 0$$

$$\upsilon = \frac{-q\tau\mathcal{E}}{m^*}$$

Perturbation works iff ….

$$\frac{q^2\tau^2 B}{m^* q\tau} = \frac{q\tau B}{m^*} \equiv \tau\omega_c << 1$$

* Same model works for holes, but with +q instead of -q

Drude Model & Hall Effect …

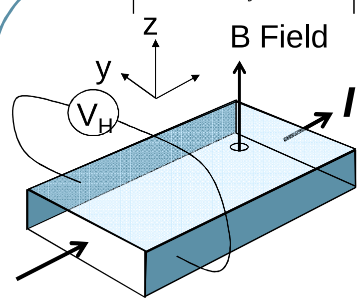
$$\mathbf{J}_n = -qn\,\boldsymbol{\upsilon}$$

$$= \frac{q^2 n\tau}{m^*}\mathcal{E} - \frac{q^2 n\tau}{m^*}\frac{q\tau}{m^*}\mathcal{E}\times\mathbf{B}$$

$$= \sigma_0\mathcal{E} - \sigma_0\mu\,\mathcal{E}\times\mathbf{B}$$

$$\begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ E_x & E_y & E_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0\mu B_z \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$



Hall Resistance

~0, since voltmeters have a very large internal resistance so very little current flows through

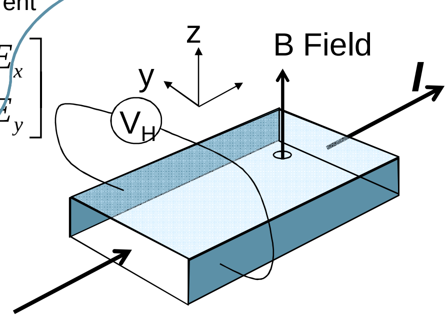
In the limit of small B

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0\mu B_z \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} \approx \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$R_H = \frac{E_y/B_z}{J_x} = -\frac{1}{qn}$$



J_x is measured, B_z is known, V_h is measured so E_y is known

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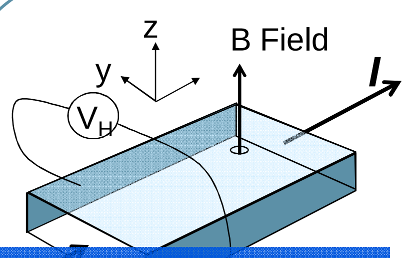
In the limit of small B

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0 \mu B_z \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$



$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} \approx \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

By a simple electrical measurement, we now know the concentration of electrons in the sample. From the intrinsic concentration of carriers in the semiconductor and temperature we can deduce the doping concentration in the sample

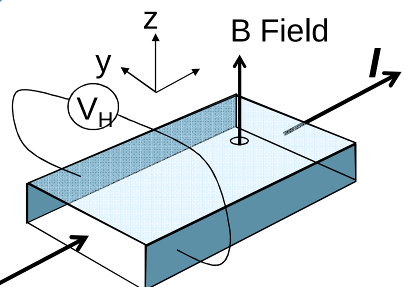
$$R_H = \frac{E_y / B_z}{J_x} = -\frac{1}{qn}$$

Hall Resistance

~0, since voltmeters have a very large internal resistance so very little current flows through

In the limit of small B

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0 \mu B_z \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

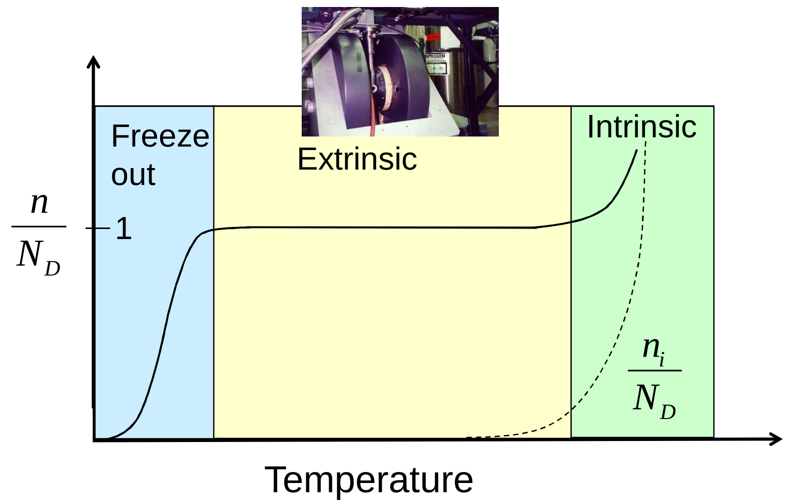


$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} \approx \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Why Bz set to zero in one eq and not the other?
=> analytical simplicity.
The system is also explicitly solvable in terms of Bz^2 and Ey.. Once Bz^z/Ey is small compared to 1/qn then one can neglect higher orders and come to the same result

$$R_H = \cdots = \frac{\ }{J_x} = \frac{\ }{qn}$$

Temperature-dependent Concentration



Outline

1) High Field Mobility effects
2) Measurement of mobility
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4) **Physics of diffusion**
5) Conclusions

Now the Diffusion term…

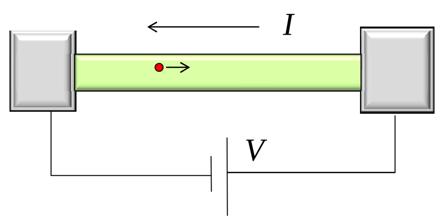
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

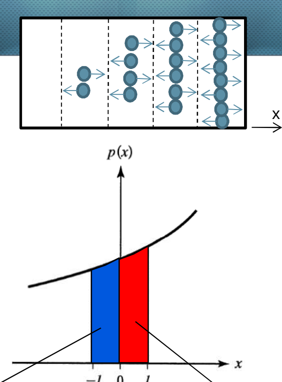
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$



Diffusion Flux …



Assuming independent random motion - on average, half of the electrons (at each x) will move to the left, half to the right ➔NET movement of electrons to left i.e. against the gradient. Opposite sign for holes.

$$J = \left[-\frac{q}{2}\left(\frac{p(0) + p(0) - \frac{dp}{dx} l}{2} \right) \times l + \frac{q}{2}\left(\frac{p(0) + p(0) + \frac{dp}{dx} l}{2} \right) \times l \right] \Bigg/ \frac{l}{\upsilon_{th}}$$

$$= q\frac{l\upsilon}{2}\frac{dp}{dx} \equiv qD\frac{dp}{dx}$$

Diffusion Flux …

This looks like a completely classical derivation.
Where is the Quantum Mechanics?

Quantum Mechanics is in the Diffusion coefficient and the Drift Velocity.
⇒Determines the available states
⇒Determines the capability to carry current

Scattering is built in here! Interactions of many, many, many electrons and he surrounding.

Without scattering one could not explain the equal partitioning into 2 directions!

$$= q\frac{l\upsilon}{2}\frac{dp}{dx} \equiv qD\frac{dp}{dx}$$

Einstein Relationship …

$$\frac{D}{\mu} = \frac{\frac{l\upsilon}{2}}{\frac{q\tau}{m_0^*}} = \frac{\frac{(\upsilon\tau)\times\upsilon}{2}}{\frac{q\tau}{m_0^*}} = \frac{\frac{1}{2}m_0^*\upsilon^2}{q} = \frac{k_B T}{q}$$

… because scattering dominates both phenomena

Einstein Relationship …

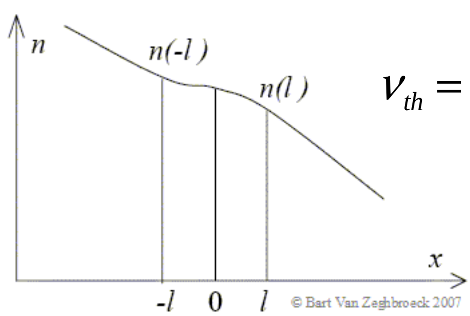
The derivation is based on the basic notion that carriers at non-zero temperature (Kelvin) have an additional thermal energy, which equals kT/2 per degree of freedom. It is the thermal energy, which drives the diffusion process. At T = 0 K there is no diffusion.

The reader should recognize that the random nature of the thermal energy would normally require a statistical treatment of the carriers. Instead we will use average values to describe the process. Such approach is justified on the basis that a more elaborate statistical approach yields the same results. To further simplify the derivation, we will derive the diffusion current for a one-dimensional semiconductor in which carriers can only move along one direction.

We now introduce the average values of the variables of interest, namely the thermal velocity, vth, the collision time, tc, and the mean free path, l. The thermal velocity is the average velocity of the carriers going in the positive or negative direction. The collision time is the time during which carriers will move with the same velocity before a collision occurs with an atom or with another carrier. The mean free path is the average length a carrier will travel between collisions.

http://ecee.colorado.edu/~bart/book/book/chapter2/ch2_7.htm

An alternate derivation



$$v_{th} = \frac{l}{\tau}$$

RMS velocity of carriers is decided by the average mean free path and average scattering time

Carriers that arrive at x=0 do so by travelling one mean free path from right to left or vice versa.

↓

Net current from left to right (at x=0) = charge on carrier*net flux of carriers from left to right

Figures and derivation from http://ecee.colorado.edu/~bart/book/book/chapter2/ch2_7.htm#fig2_7_8

Derivation for electron diffusion current

Electron flux at x=0 from left to right

$$\phi_{n,left\rightarrow right} = \frac{1}{2} v_{th} n(x=-l)$$

The factor half appears because the other half at $x=-l$ travels towards the left

Electron flux at x=0 from right to left

$$\phi_{n,right\rightarrow left} = \frac{1}{2} v_{th} n(x=l)$$

Net Flux

$$\phi_n = \phi_{n,left\rightarrow right} - \phi_{n,right\rightarrow left} = \frac{1}{2} \upsilon_{th} [n(x=-l) - n(x=l)]$$

Derivation continued

$$\phi_n = l \upsilon_{th} \frac{[n(x=-l) - n(x=l)]}{2l}$$

If the mean free path is small enough, then we can write this as

$$\phi_n = -l \upsilon_{th} \frac{dn}{dx}$$

The negative sign arises because we take the gradient is usually measured for increasing values of x. The current density is then given by.

$$J_n = -q\phi_n = ql \upsilon_{th} \frac{dn}{dx}$$

For holes, we repeat the derivation and find…

$$J_p = q\phi_p = -q l_{holes} \upsilon_{th,holes} \frac{dp}{dx}$$

Lump together the second and third terms to form a 'diffusion constant'

$$J_p = -qD_p \frac{dp}{dx} \qquad J_n = qD_n \frac{dn}{dx}$$

For thermally distributed carriers…

$$\frac{1}{2} m^* \upsilon_{th}^2 = \frac{k_B T}{2}$$

→ Equipartition theorem states that in equilibrium each carrier has thermal energy of kT/2 per degree of freedom.

↓

Our derivation is in one dimension➔ one degree of freedom

Use this to get a new insight into the relation between drift and diffusion

http://en.wikipedia.org/wiki/Equipartition_theorem#Derivations

Alternate way to obtain the diffusion constants

$$D = l\upsilon_{th} = \tau\upsilon_{th}^2 = \frac{q\tau}{m^*}\frac{m^*\upsilon_{th}^2}{q} = \mu\frac{k_BT}{q}$$

$$\frac{D}{\mu} = \frac{k_BT}{q}$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_BT}{q}$$

→ Einstein relations valid for electrons and holes AT EQULIBRIUM

The equation shows how thermal quantities governing diffusion can be related to those governing drift (mobility depends on the effective mass which describes motion of carriers in the presence of a field and scatterers)

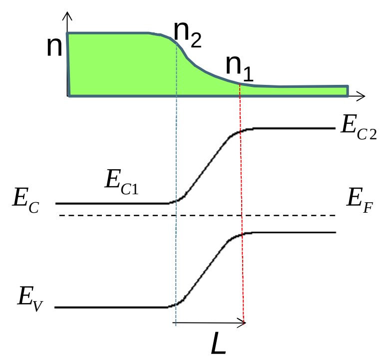
An alternate derivation of Einstein's relationship

No current in equilibrium

$$J_n = 0 = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$\Rightarrow \frac{1}{n}\frac{dn}{dx} = -\frac{\mu_n\mathcal{E}}{D_n}$$

$$n_2 = n_1 e^{-\int_0^L \frac{\mu_n\mathcal{E}}{D_n}} = n_1 e^{\frac{\mu_n V}{D_n}}$$



$$\frac{n_2}{n_1} = \frac{N_C e^{-(E_{C2}-E_F)/kT}}{N_C e^{-(E_{C1}-E_F)/kT}} = e^{-(E_{C2}-E_{C1})/kT} = e^{qV/kT}$$

$$\frac{qV}{kT} = \frac{\mu_n V}{D_n} \Rightarrow \frac{q}{kT} = \frac{\mu_n}{D_n}$$

Similar to relationship between c_n and e_n discussed in Chapter 5

Conclusion

1) Measurement of mobility and carrier concentration is particularly important for analysis of semiconductor devices.
2) Drift, diffusion, and recombination-generation constitute the elemental processes in semiconductor device physics.
3) We will put the pieces together in the next class.

ECE606: Solid State Devices
Lecture (from18)
Continuity Equations

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1) **Continuity Equation**
2) Example problems
3) Conclusion

Ref. Advanced Semiconductor Fundamentals , pp. 205-210

These equations have been state of the art in device modeling until 'recently' (10-15 years ago…)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

→ Continuity eqn. for electrons

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

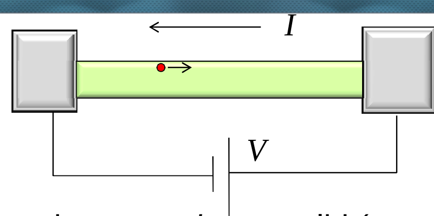
→ Continuity eqn. for holes

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

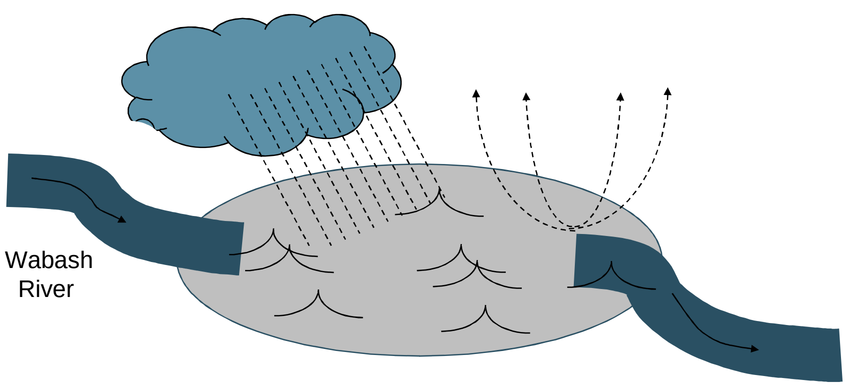
→ Poisson equation

Some critical remarks about these equations



- Continuity equations are *always* valid (regardless of the detailed physical model describing the device) because they describe a conservation law.
- Poisson equation as given does not account for explicit electron-electron repulsion. Might need to be modified for strongly correlated systems.
- Drift and Diffusion equations get modified when devices are so small that essentially no scattering takes place within the device.

Continuity Equation prequel: A Good Analogy

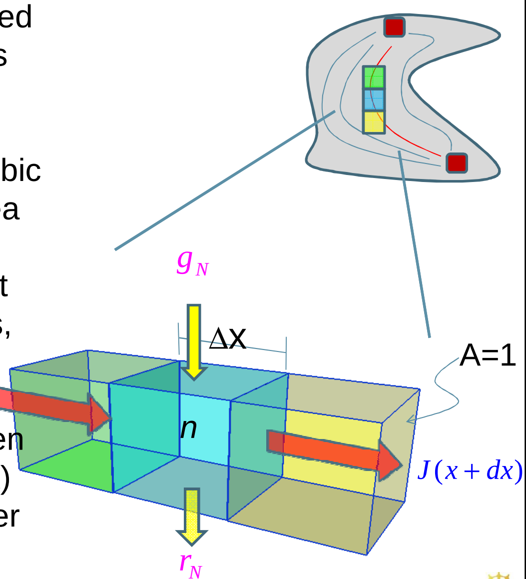


Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

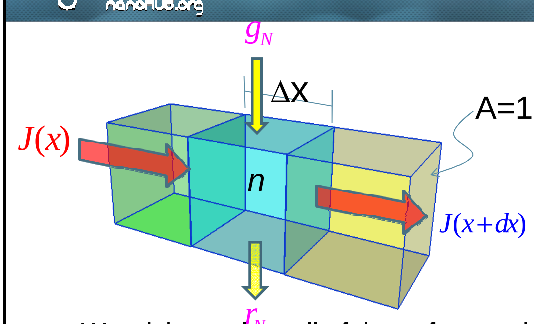
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet J_n + g_N - r_N$$

Now the Continuity Equations for electrons…

- Consider an arbitrarily shaped semiconductor with contacts that pump in current.
- Divide this arbitrary semiconductor into small cubic boxes of cross sectional area 'A' and length Δx.
- Boxes are large enough that concepts like effective mass, mobility etc. are valid inside these boxes.
- Electrons coming in are given by J(x), going out➔ J(x+Δx)
- Total number of electrons per cm^3 ➔n



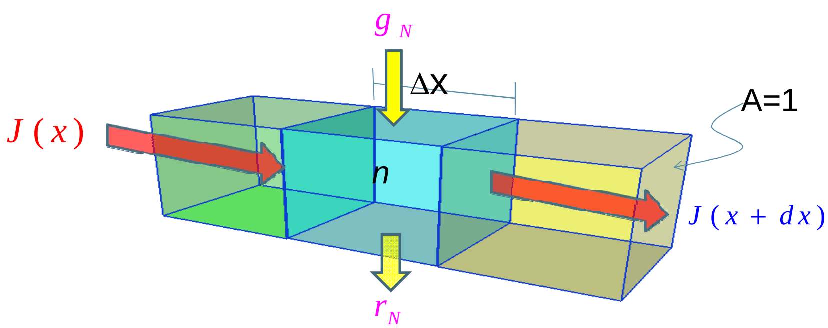
Now the Continuity Equations …



- g_N-> generation rate in (per cm^3 per sec) from external processes such as light.
- r_N-> recombination rate in (per cm^3 per sec) in the central box.

- We wish to relate all of these factors that affect the concentration of carriers in the central box.
- Our strategy (remember the analogy)– The *rate* of change of number of electrons INSIDE the central box should be equal to
  - No. of electrons coming in MINUS No. of electrons going out per sec (governed by current density J(x) and J(x+Δx)) PLUS
  - No. of electrons getting generated from external processes per sec (governed by generation rate g_N) MINUS
  - No. of electrons lost by recombination per sec (governed by recombination rate r_N)

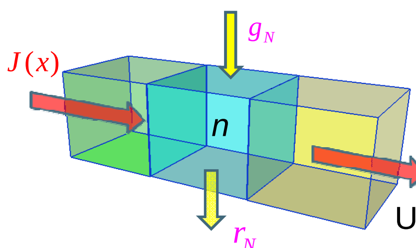
Let's just write the equations down…



$$\frac{\partial(A\times\Delta x\times n)}{\partial t}=\frac{A\times J_n(x)-A\times J_n(x+dx)}{-q}+A\times g_N\Delta x-A\times r_N\Delta x$$

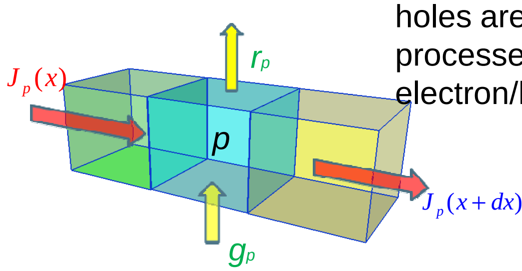
$$\frac{\partial n}{\partial t}=\frac{J_n(x)-J_n(x+dx)}{-q\Delta x}+g_N-r_N=\frac{1}{q}\nabla\bullet J_n+g_N-r_N$$

Continuity Equations for Electron/Holes



$$\frac{\partial n}{\partial t}=\frac{1}{q}\nabla\bullet J_n+g_N-r_N$$

Usually generation and recombination rates for electrons and holes are the same since the same processes create/destroy an electron/hole



$$\frac{\partial p}{\partial t}=-\frac{1}{q}\nabla\bullet J_P+g_P-r_P$$

Equations in place, learning to solve them…

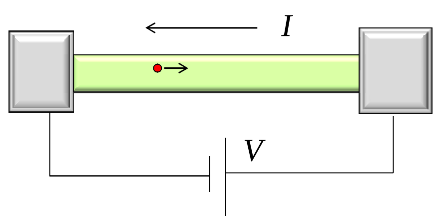
$$\frac{\partial n}{\partial t}=\frac{1}{q}\nabla\bullet\mathbf{J}_N-r_N+g_N$$

$$\mathbf{J}_N=qn\mu_N E+qD_N\nabla n$$

$$\frac{\partial p}{\partial t}=\frac{-1}{q}\nabla\bullet\mathbf{J}_P-r_P+g_P$$

$$\mathbf{J}_P=qp\mu_P E-qD_P\nabla p$$

$$\nabla\bullet E=q\left(p-n+N_D^+-N_A^-\right)$$



Two methods of solution:

Numerical and Analytical

Analytical Solutions

$$\nabla\bullet D=q\left(p-n+N_D^+-N_A^-\right)$$ ← Band-diagram

$$\frac{\partial n}{\partial t}=\frac{1}{q}\nabla\bullet\mathbf{J}_N-r_N+g_N$$

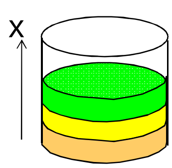
$$\mathbf{J}_N=qn\mu_N E+qD_N\nabla n$$

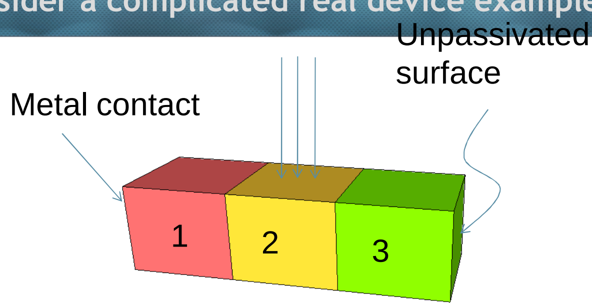
$$\frac{\partial p}{\partial t}=\frac{-1}{q}\nabla\bullet\mathbf{J}_P-r_P+g_P$$ ← Diffusion approximation, Minority carrier transport, Ambipolar transport

$$\mathbf{J}_P=qp\mu_P E-qD_P\nabla p$$

1) Continuity Equation
2) **Example problems**
3) Conclusion

Consider a complicated real device example





- Acceptor doped
- Light turned on in the middle section.
- The right region is full of mid-gap traps because of dangling bonds due to un-passivated surface.
- Interface traps at the end of the right region (That's where the dangling bonds are…)
- The left region is trap free.
- The left/right regions contacted by metal electrode.

Recall: Analytical Solution of Schrodinger Equation

1) $\frac{d^2\psi}{dx^2}+k^2\psi=0$ ⟶ 2N unknowns for N regions

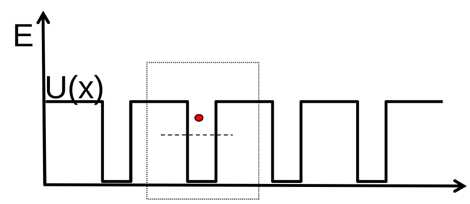
2) $\psi(x=-\infty)=0$, $\psi(x=+\infty)=0$ ⟶ Reduces 2 unknowns

3) $\psi\big|_{x=x_B^-}=\psi\big|_{x=x_B^+}$, $\frac{d\psi}{dx}\Big|_{x=x_B^-}=\frac{d\psi}{dx}\Big|_{x=x_B^+}$ ⟶ Set 2N-2 equations for 2N-2 unknowns (for continuous U)

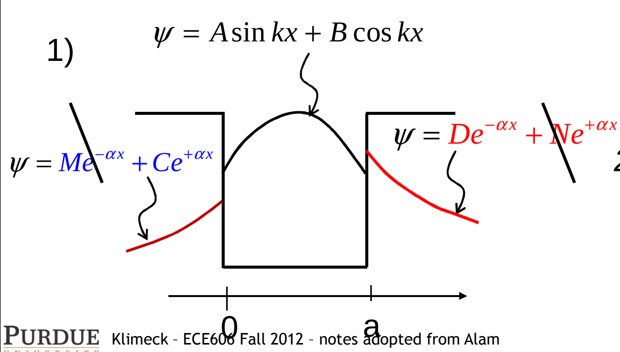
4) Det(coefficient matix)=0 And find E by graphical or numerical solution

5) $\int_{-\infty}^{\infty}|\psi(x,E)|^2\,dx=1$ for wave function

Recall: Bound-levels in Finite well



1)



2) Boundary Conditions

$\psi(x=-\infty)=0$

$\psi(x=+\infty)=0$

Analogously, we solve for our device

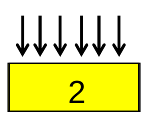
Solve the equations in different regions independently.

Bring them together by applying boundary conditions.

Region 2: Transient, Uniform Illumination, Uniform doping

(uniform)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\cancel{\nabla \bullet \mathbf{J}_N} - r_N + g_N \qquad \mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$



$$\frac{\partial(\cancel{n_0} + \Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$

Recall Shockley-Read-Hall

Acceptor doped

(uniform)

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\cancel{\nabla \bullet \mathbf{J}_p} - r_P + g_p \qquad \mathbf{J}_p = qp\mu_p E - qD_P \nabla p$$

$$\frac{\partial(\cancel{p_0} + \Delta p)}{\partial t} = -\frac{\Delta p}{\tau_p} + G$$

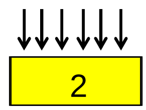
Majority carrier

Electric field still zero becaus new carriers balance

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right) = q\left(p_0 + \Delta n - n_0 - \Delta p + N_D^+ - N_A^-\right) = 0$$

Example: Transient, Uniform Illumination, Uniform doping, No applied electric field

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau_n} + G$$



Acceptor doped

$$\Delta n(x,t) = A + Be^{-t/\tau_n}$$

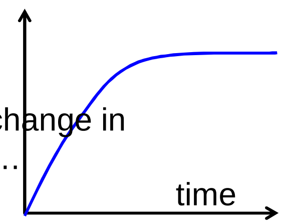
No carriers yet generated…

$$t = 0, \quad \Delta n(x,0) = 0 \Rightarrow A = -B$$

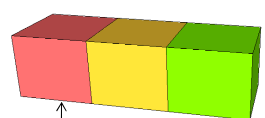
$$t \rightarrow \infty, \quad \Delta n(x,\infty) = G\tau_n = A$$

Steady state, no change in carriers with time…

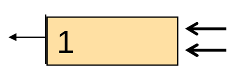
$$\Delta n(x,t) = G\tau_n\left(1 - e^{-t/\tau_n}\right)$$



Region 1: One sided Minority Diffusion at steady state



Steady state
Acceptor doped



$$\frac{\partial n}{\partial t} = 0 \text{(steady-state)}$$

$$r_N = 0 \text{(trap free)}$$

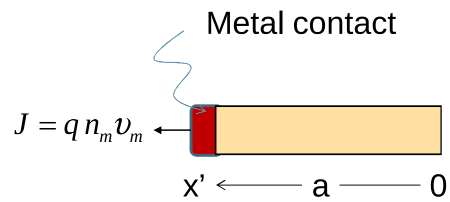
$$g_N = 0 \text{(nogeneration)}$$

$$E = 0$$

$$0 = D_N \frac{d^2 n}{dx^2}$$

$$D_N \frac{dn}{dx} \neq 0 \text{ (due to insertion of electrons from central region)}$$

Example: One sided Minority Diffusion



$$0 = D_N \frac{d^2 n}{dx^2}$$

$$\Delta n(x,t) = C + Dx'$$

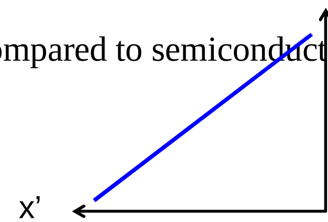
$x = a, \quad \Delta n(x' = a) = 0 \Rightarrow C = -Da$

(Metal has high electron density as compared to semiconductor)

$x = 0', \quad \Delta n(x' = 0') = C$

Just substitute x=0 in above eqn.

$$\Delta n(x,t) = \Delta n(x = 0')\left(1 - \frac{x'}{a}\right)$$

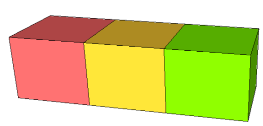


Region 3: Steady state Minority Diffusion with recombination

$$\frac{\partial n}{\partial t} = 0(\text{steady-state})$$

$$r_N \neq 0(\text{not trap free})$$

$$g_N = 0(\text{nogeneration})$$

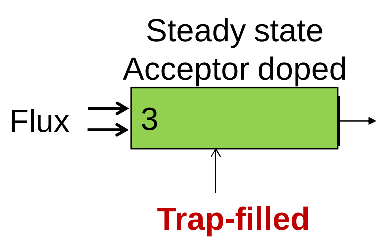


$$E = 0$$

$$D_N \frac{dn}{dx} \neq 0 \text{ (due to insertion of electrons from central region)}$$

$$0 = D_N \frac{d^2(n_0 + \Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

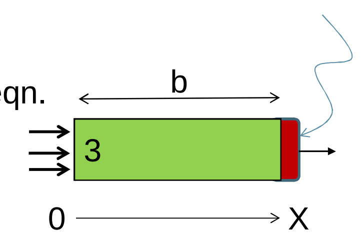
$$0 = D_N \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n}$$



Diffusion with Recombination …

$$D_N \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} = 0$$

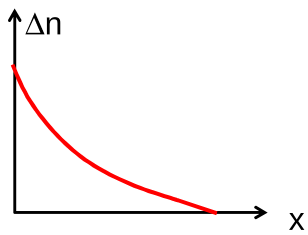
Functionally similar to Schrodinger eqn.



$$\Delta n(x,t) = E e^{x/L_n} + F e^{-x/L_n}$$

$$x = b, \quad \Delta n(x=b) = 0 \quad \Rightarrow F = -E e^{2b/L_n}$$

$$x = 0, \quad \Delta n(x=0) = E + F = \Delta n(x=0)$$

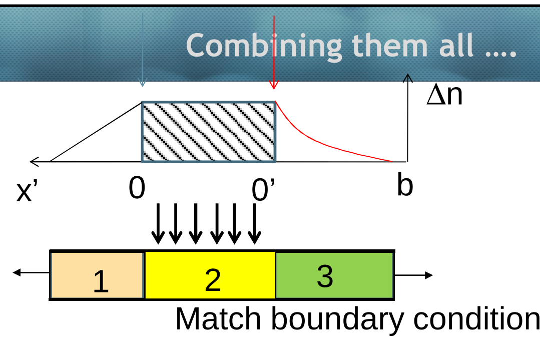


$$\Delta n(x,t) = \frac{\Delta n(0)}{(1 - e^{2b/L_n})}(e^{x/L_n} - e^{2b/L_n} e^{-x/L_n})$$

Combining them all ….

$$\Delta n_2(x) = G\tau_n =$$

$$\Delta n_2(0) = \Delta n_2(0')$$



$$\Delta n_1(x') = \Delta n(x=0)\left(1 - \frac{x'}{a}\right) = G\tau_n\left(1 - \frac{x'}{a}\right)$$

$$\Delta n(x) = \frac{\Delta n(0')}{(1 - e^{2b/L_n})}(e^{x/L_n} - e^{2b/L_n} e^{-x/L_n}) = \frac{G\tau_n (e^{x/L_n} - e^{2b/L_n} e^{-x/L_n})}{(1 - e^{2b/L_n})}$$

Calculating current $\quad \mathbf{J}_N = qn\mu_N E + qD_N \frac{dn}{dx}$

Conclusion

1) Continuity Equations form the basis of analysis of all the devices we will study in this course.

2) Full numerical solution of the equations are possible and many commercial software are available to do so.

3) Analytical solutions however provide a great deal of insight into the key physical mechanism involved in the operation of a device.