ECE606: Solid State Devices
Lecture 14
Electrostatics of p-n junctions

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Outline

1) **Introduction to p-n junctions**
2) Drawing band-diagrams
3) Accurate solution in equilibrium
4) Band-diagram with applied bias

Ref. Semiconductor Device Fundamentals, Chapter 5

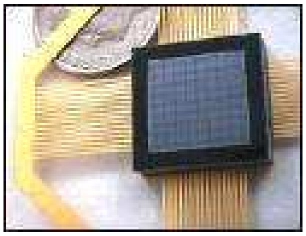
What is a Diode good for ….

solar cells

GaAs lasers

Organic LED

Avalanche Photodiode



GaN lasers

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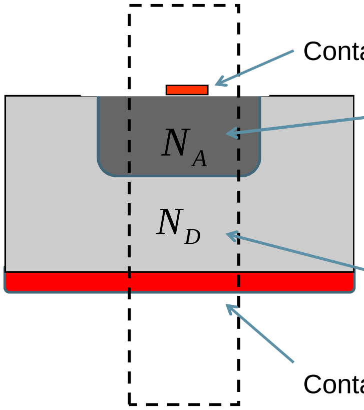
p-n Junction Devices …

Schematic of a p-n Diode



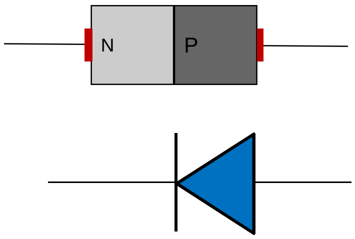
Contact

P-doped pocket

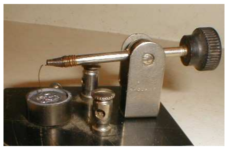
N-doped substrate

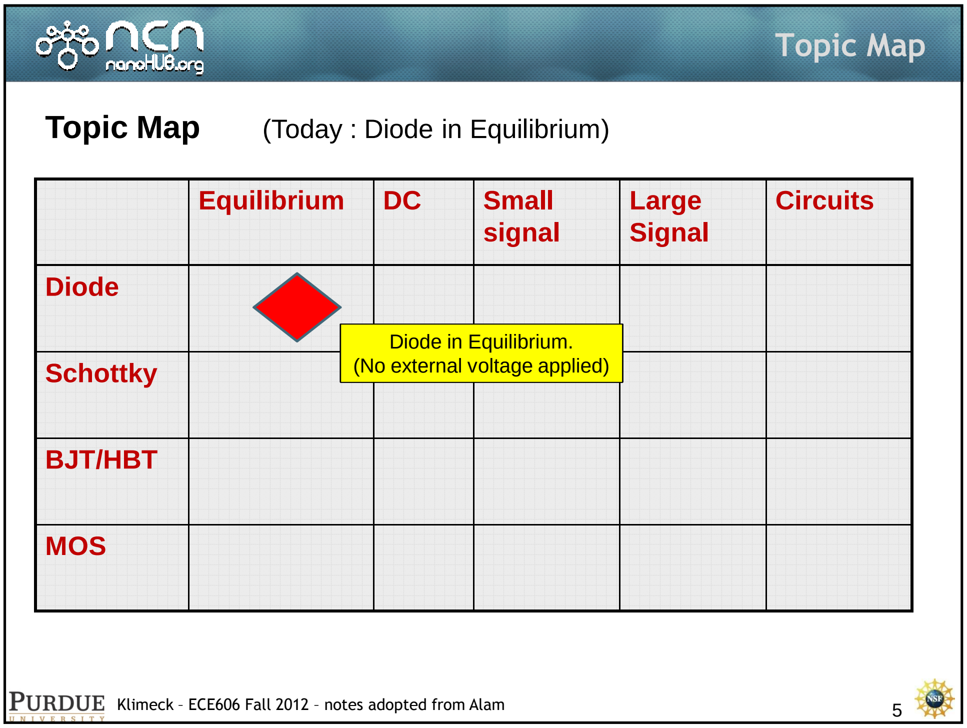
Contact

Symbol



Point-contact diode



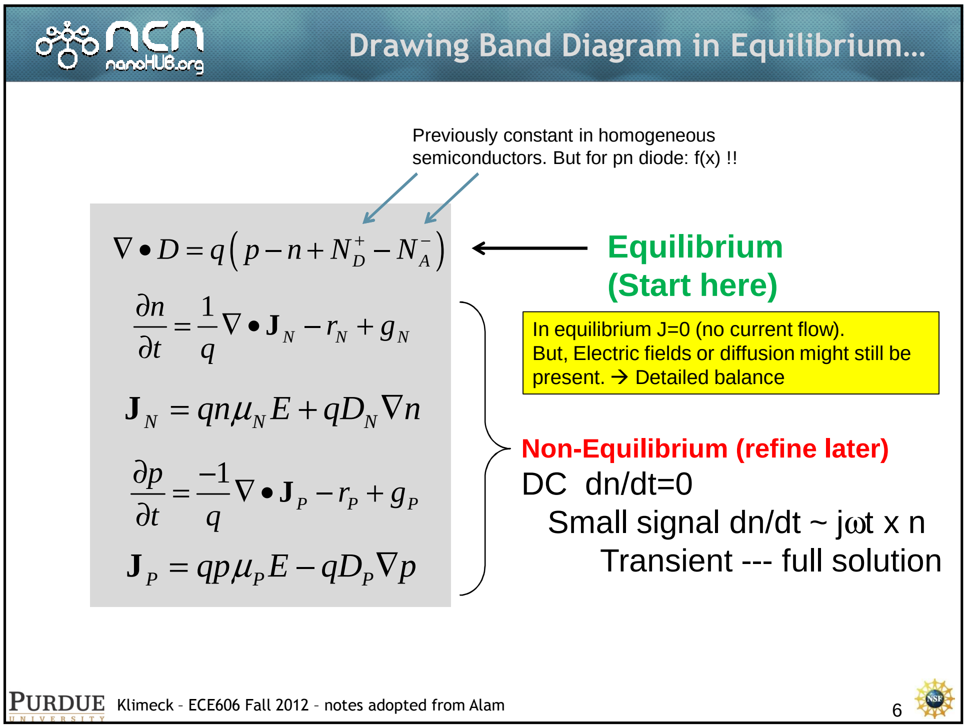


Topic Map

Topic Map (Today : Diode in Equilibrium)

	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode	◆				
Schottky					
BJT/HBT					
MOS					

Diode in Equilibrium.
(No external voltage applied)



Drawing Band Diagram in Equilibrium…

Previously constant in homogeneous semiconductors. But for pn diode: f(x) !!

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

Equilibrium (Start here)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

In equilibrium J=0 (no current flow).
But, Electric fields or diffusion might still be present. → Detailed balance

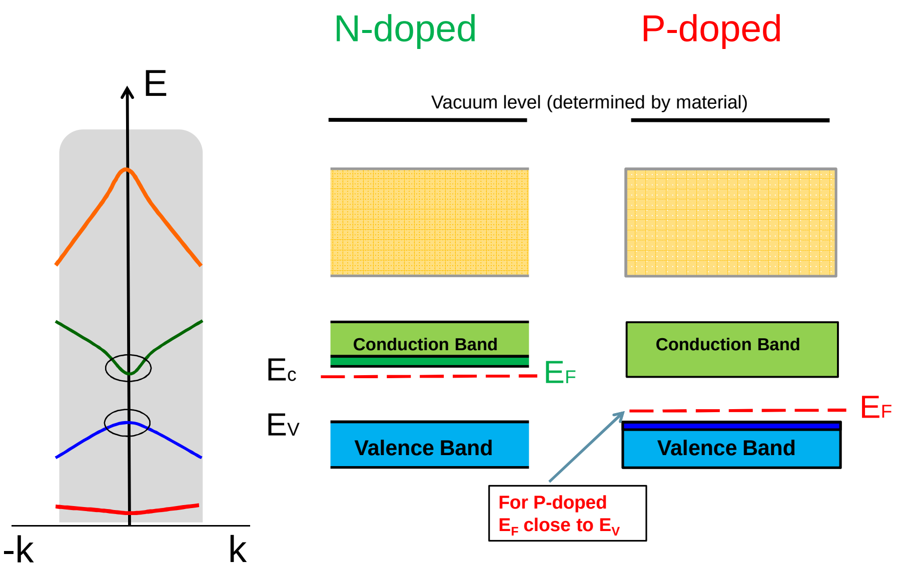
Non-Equilibrium (refine later)

DC dn/dt=0

Small signal dn/dt ~ jωt x n

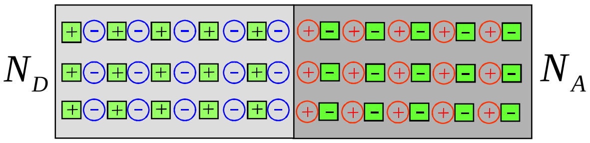
Transient --- full solution

P and N doped Material Side by Side …



For P-doped E_F close to E_V

Forming a Junction

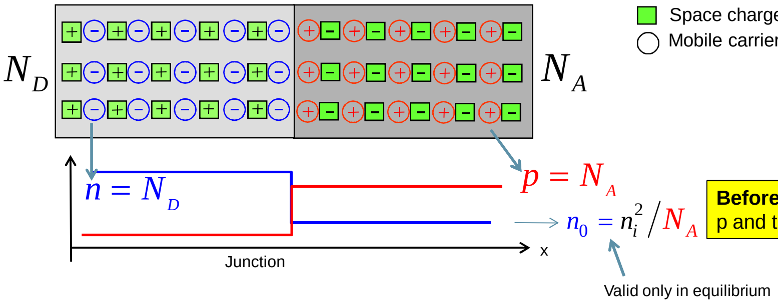


🟩 Space charge (fixed)
○ Mobile carriers

Donor-side (N-side)
Squares are fixed **donor atoms**.
Every donor atom has given away one **electron** (blue **circle**)

Acceptor-side (P-side)
Squares are fixed **acceptor atoms**.
Every acceptor atom has captured one **electron** (from the valence band).
Every acceptor atom leaves behind one **hole** in the valence band. (red **circle**)

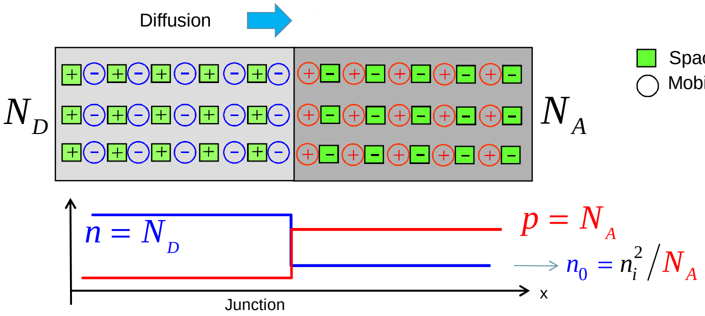
Forming a Junction



🟩 Space charge (fixed)
○ Mobile carriers

Before joining the p and the n side

Forming a Junction



🟩 Space charge (fixed)
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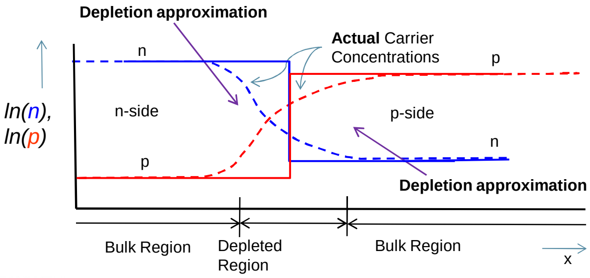
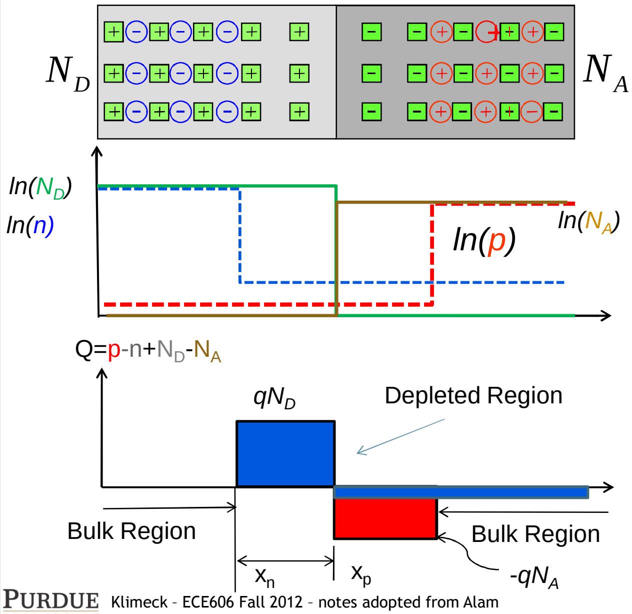
Before joining the p and the n side



After joining the p and the n side

Formation of a Junction



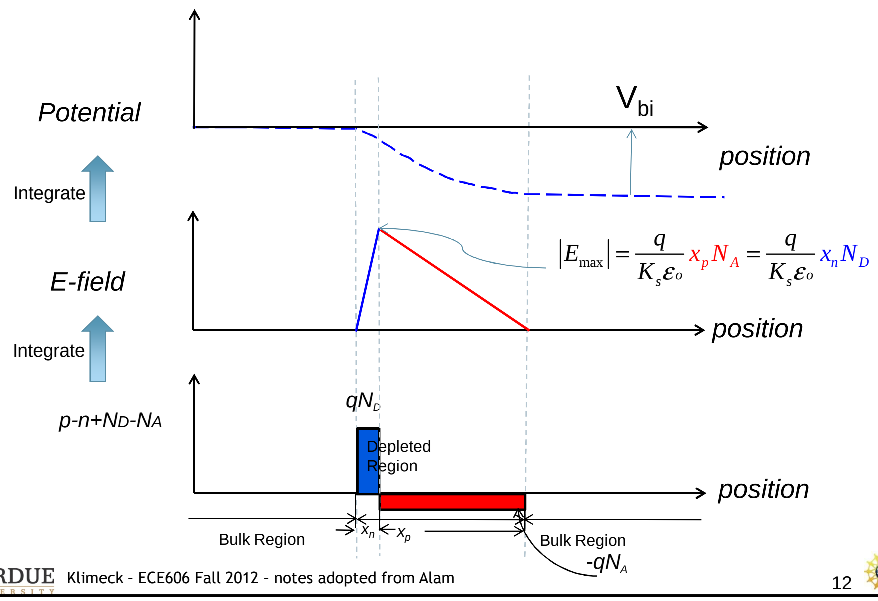
Net charge on **N-side**: qN_D (subtract green from blue line)
Space charge region: fixed donor atoms
Neglect electrons in p-side
X_n: depletion region in N-side

Net charge on **P-side**: $-qN_A$ (subtract brown from red line)
Space charge region: fixed acceptor atoms
X_p: depletion region in P-side

red and blue are of equal size
→ **charge balance**

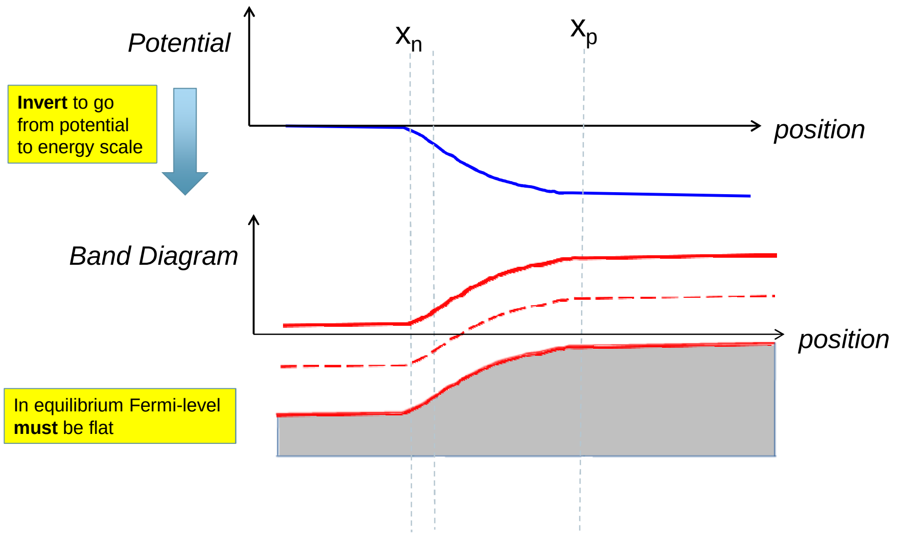
Depletion Region means depleted of “mobile” charges

Sketch of Electrostatics



$$\left|E_{\max}\right| = \frac{q}{K_s \varepsilon_o} x_p N_A = \frac{q}{K_s \varepsilon_o} x_n N_D$$

Sketch of Electrostatics



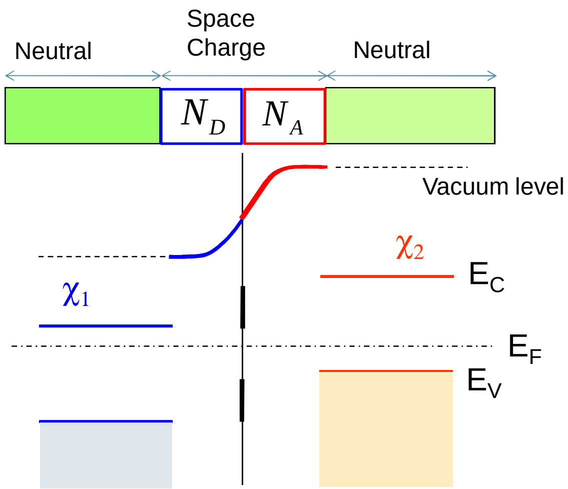
Invert to go from potential to energy scale

In equilibrium Fermi-level **must** be flat

Outline

1) Introduction to p-n junctions
2) **Drawing band-diagrams**
3) Analytical solution in equilibrium
4) Band-diagram with applied bias

Short-cut to Band-diagram

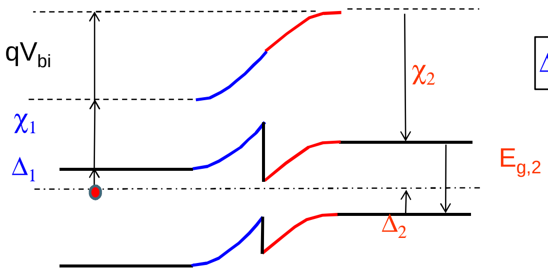


Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) "Transfer" vacuum slopes to join Ec/Ev

… is equivalent to solving the Poisson equation

Built-in Potential: boundary conditions @infinity



Always true in equilibrium

$$\Delta_1 + \chi_1 + qV_{bi} = \chi_2 + E_{g,2} - \Delta_2$$

delta$_{1,2}$ determined via doping concentrations

X$_{1,2}$ material parameters

Built-in potential V_{bi} unknown!

$$qV_{bi} = E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1$$

$$= \left(E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + (\chi_2 - \chi_1)$$

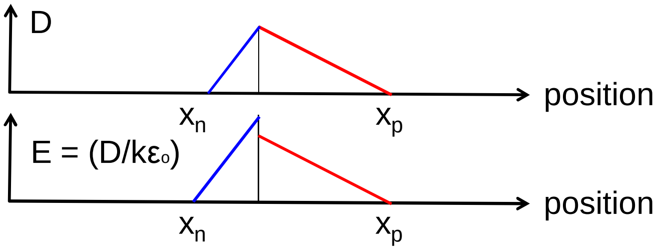
$$= k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1)$$

Interface Boundary Conditions

Homo - Junction

Hetero - Junction

Field not continuous across junction

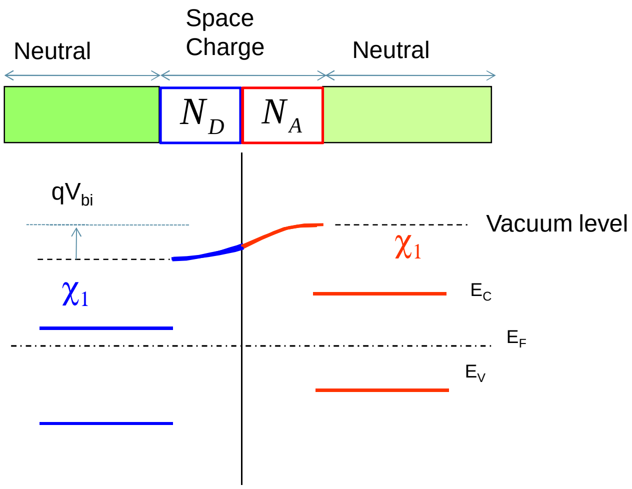


$$D_1 = K_1\varepsilon_0 E(0^-) = K_2\varepsilon_0 E(0^-) = D_2$$

$$E(0^-) = \frac{K_2}{K_1}E(0^+)$$

Displacement is continuous across the interface, field need not be ..

Built-in voltage for **Homo-junctions**

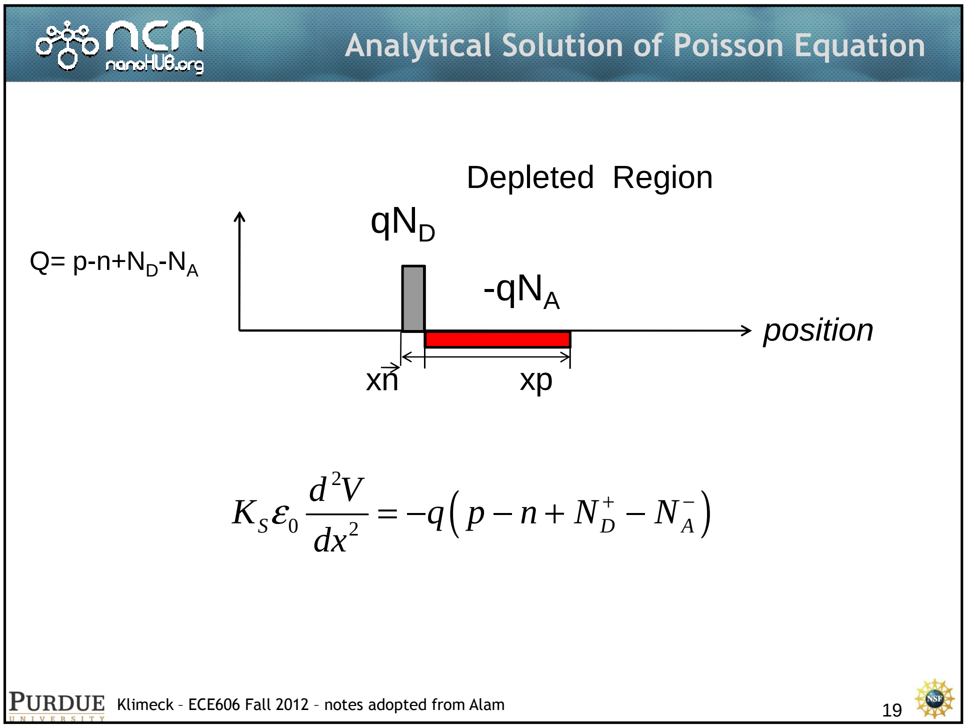


Drawing Recipe

1) Start with EF
2) Ec/Ev in bulk n-side
3) Ec/Ev in bulk p-side
4) Vacuum level in N
5) Vacuum level in P
6) Join vacuum levels
7) “Transfer” vacuum slopes to join Ec/Ev

Zero for homo-junctions

$$qV_{bi} = k_BT\ln\frac{N_AN_D}{N_{V,2}N_{C,1}e^{-E_{g,2}/k_BT}} + (\chi_2-\chi_1) = k_BT\ln\frac{N_AN_D}{N_VN_Ce^{-E_g/k_BT}} = k_BT\ln\frac{N_AN_D}{n_i^2}$$

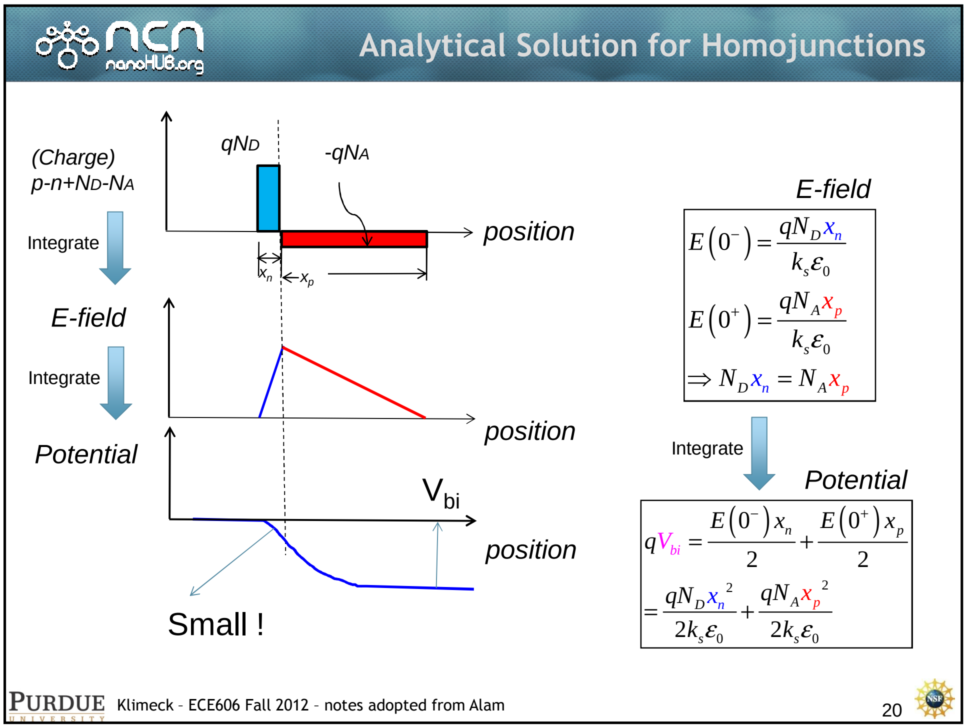


Analytical Solution of Poisson Equation

Depleted Region

Q= p-n+N_D-N_A

$$K_S\varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p-n+N_D^+-N_A^-\right)$$



Analytical Solution for Homojunctions

(Charge)
p-n+ND-NA

Integrate

E-field

Integrate

Potential

E-field

$$E\left(0^-\right) = \frac{qN_D x_n}{k_s\varepsilon_0}$$

$$E\left(0^+\right) = \frac{qN_A x_p}{k_s\varepsilon_0}$$

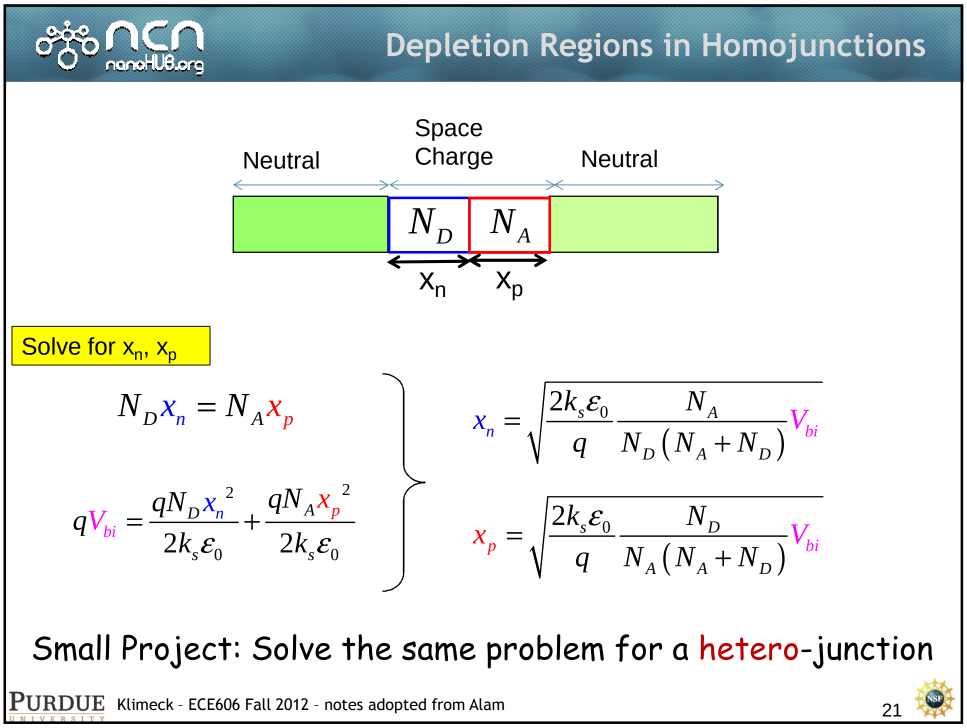
$$\Rightarrow N_D x_n = N_A x_p$$

Integrate

Potential

$$qV_{bi} = \frac{E\left(0^-\right)x_n}{2} + \frac{E\left(0^+\right)x_p}{2}$$

$$= \frac{qN_D x_n^{\ 2}}{2k_s\varepsilon_0} + \frac{qN_A x_p^{\ 2}}{2k_s\varepsilon_0}$$



Depletion Regions in Homojunctions

Solve for x_n, x_p

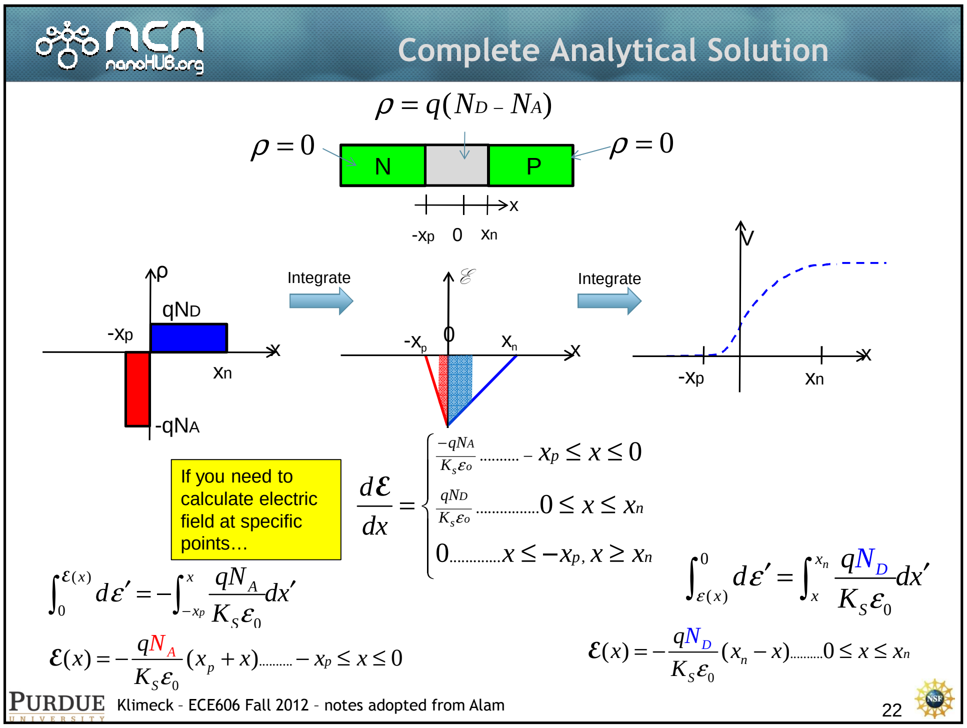
$$N_D x_n = N_A x_p$$

$$qV_{bi} = \frac{qN_D x_n^{\,2}}{2k_s\varepsilon_0} + \frac{qN_A x_p^{\,2}}{2k_s\varepsilon_0}$$

$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\,\frac{N_A}{N_D\left(N_A+N_D\right)}V_{bi}}$$

$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\,\frac{N_D}{N_A\left(N_A+N_D\right)}V_{bi}}$$

Small Project: Solve the same problem for a hetero-junction



Complete Analytical Solution

$$\rho = q(N_D - N_A)$$

If you need to calculate electric field at specific points…

$$\frac{d\mathcal{E}}{dx} = \begin{cases} \dfrac{-qN_A}{K_s\varepsilon_o} \text{..........} -x_p \le x \le 0 \\ \dfrac{qN_D}{K_s\varepsilon_o} \text{...............} 0 \le x \le x_n \\ 0 \text{............} x \le -x_p,\ x \ge x_n \end{cases}$$

$$\int_0^{\mathcal{E}(x)} d\varepsilon' = -\int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0}dx'$$

$$\int_{\varepsilon(x)}^{0} d\varepsilon' = \int_x^{x_n} \frac{qN_D}{K_S\varepsilon_0}dx'$$

$$\mathcal{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x) \text{..........} -x_p \le x \le 0$$

$$\mathcal{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \text{.........} 0 \le x \le x_n$$

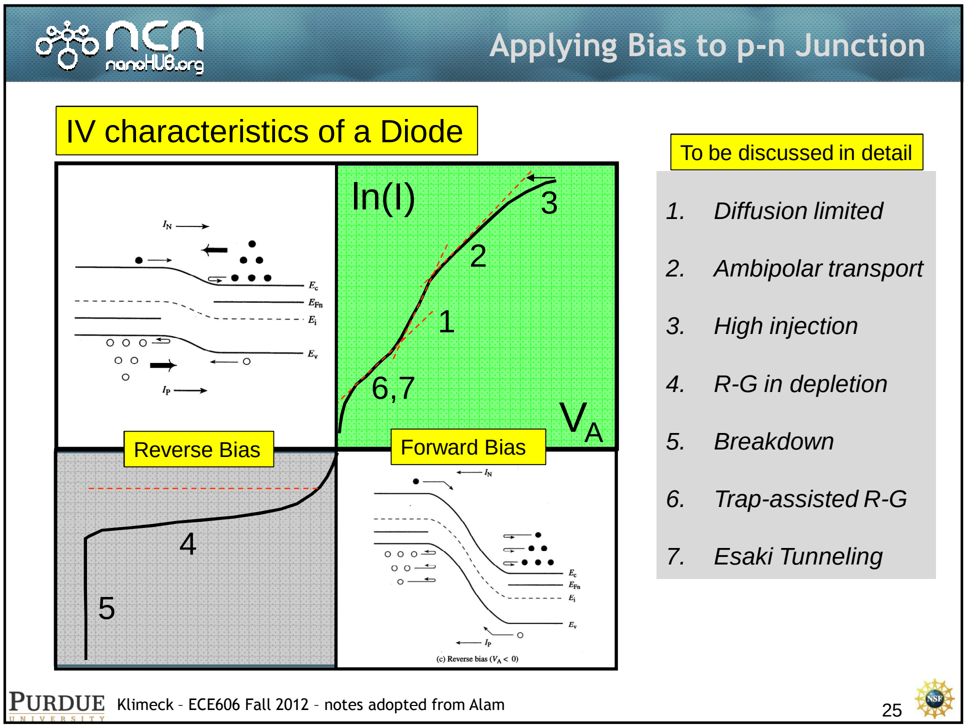
Outline

1) Introduction to p-n junction transistors
2) Drawing band-diagrams
3) Analytical solution in equilibrium
4) **Band-diagram with applied bias**

Topic Map

	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode		◆			
Schottky					
BJT/HBT					
MOS					

Diode in **Non-**Equilibrium
(External DC voltage applied)

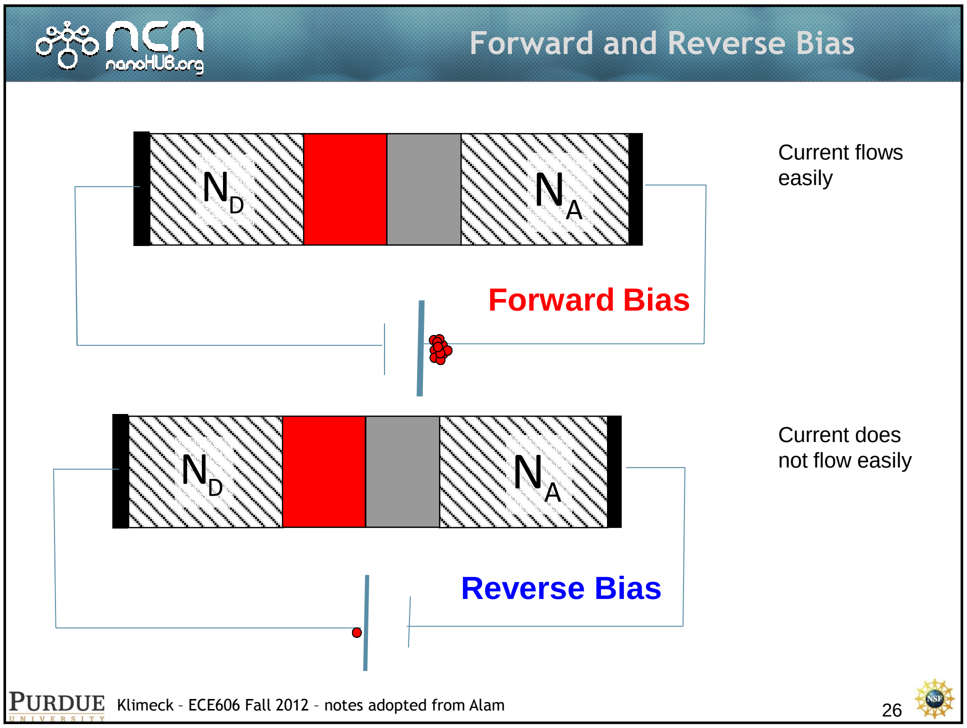


Applying Bias to p-n Junction

IV characteristics of a Diode

To be discussed in detail

1. *Diffusion limited*
2. *Ambipolar transport*
3. *High injection*
4. *R-G in depletion*
5. *Breakdown*
6. *Trap-assisted R-G*
7. *Esaki Tunneling*



Forward and Reverse Bias

Current flows easily

Forward Bias

Current does not flow easily

Reverse Bias

Band Diagram with Applied Bias…

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

← Band diagram (this segment)

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

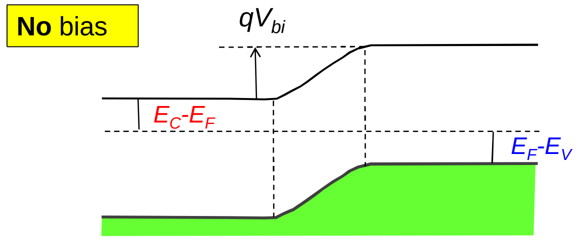
$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

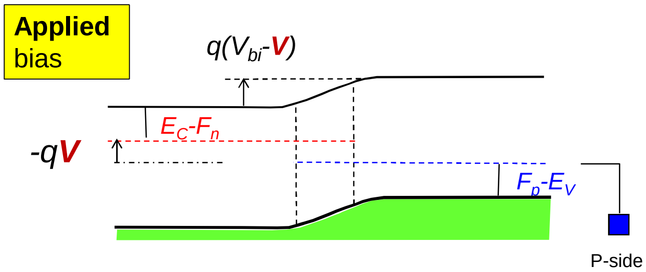
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

Next segment / lecture …

Applying a Bias: Poisson Equation



Question: Max value of $\mathbf{V_{bi}}$?
Answer: for degenerate s.c.,
if E_C-E_F=0, E_F-E_V=0 → $\mathbf{E_g}$

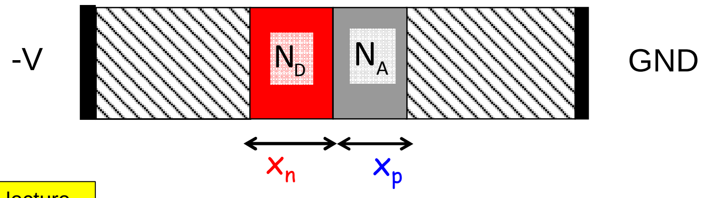


$$n(x) = n_i\, e^{(F_n - E_i)\beta}$$

$$p(x) = n_i\, e^{-(F_p - E_i)\beta}$$

$$n \times p = n_i^2\, e^{(F_n - F_p)\beta}$$

Depletion Widths



From previous lecture (homo-junction)

$$N_D x_n = N_A x_p$$

$$q\left(V_{bi}-V\right)=\frac{qN_D x_n^{\,2}}{2k_s\varepsilon_0}+\frac{qN_A x_p^{\,2}}{2k_s\varepsilon_0}$$

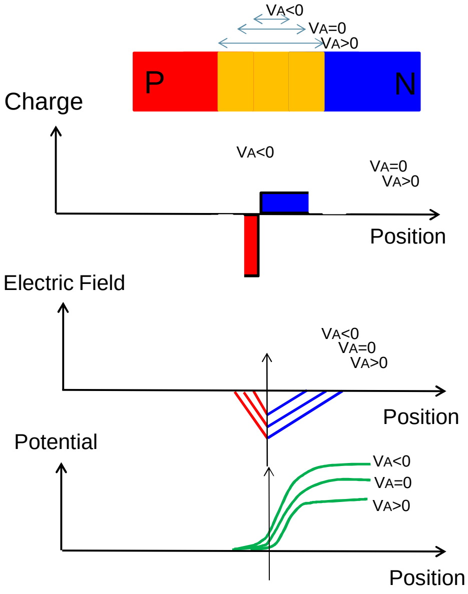
Applied bias

$$x_n=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}\left(V_{bi}-V\right)}$$

$$x_p=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}\left(V_{bi}-V\right)}$$

What about heterojunctions?

Fields and Depletion at Forward/Reverse Biases



Barrier height is **reduced** at **forward** biases

Significant **increase** of peak field at **reverse** bias

Summary
PN-Junction Electrostatics

1) Learning to draw **band-diagrams** is one of the most important topics you learn in this course. Band-diagrams are a graphical way of quickly solving the Poisson equation.

2) If you consistently follow the rules of drawing band-diagrams, you will always get correct results. Try to follow the rules, **not guess** the final result.

ECE606: Solid State Devices
p-n diode I-V characteristics

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Outline

1) **Derivation of the forward bias formula**
2) Solution in the nonlinear regime
3) I-V in the ambipolar regime
4) Conclusion

Ref. SDF, Chapter 6

Topic Map

	Equilibrium	**DC**	Small signal	Large Signal	Circuits
Diode		◆			
Schottky					
BJT/HBT					
MOSFET					

Diode in **Non-**Equilibrium
(External DC voltage applied)

Continuity Equations for p-n junction Diode

$$\nabla \bullet E = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

This section

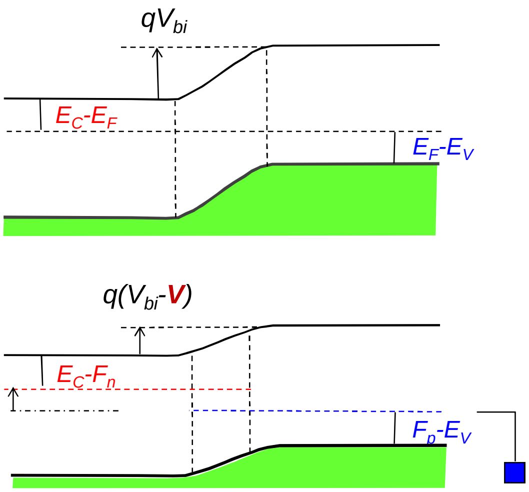
Applying a Bias: Poisson Equation

Review

No bias

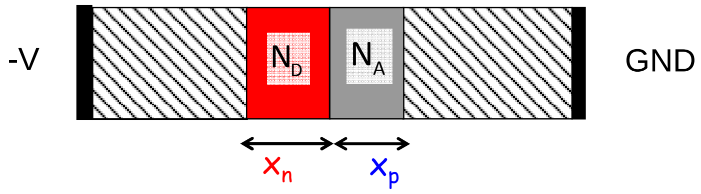
Applied bias

$-qV$



Depletion Widths

Review



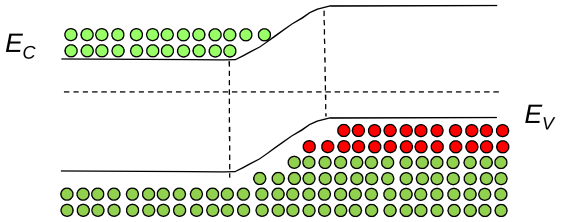
$$N_D x_n = N_A x_p$$

$$q(V_{bi}-V) = \frac{qN_D x_n^2}{2k_s\varepsilon_0} + \frac{qN_A x_p^2}{2k_s\varepsilon_0}$$

$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}(V_{bi}-V)}$$

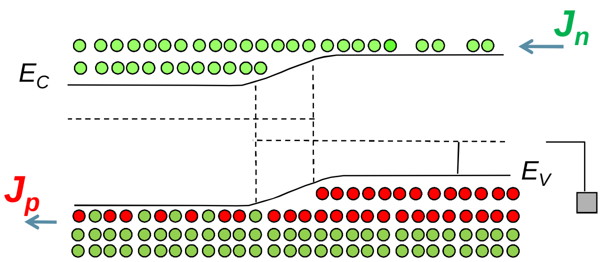
$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi}-V)}$$

Flat Quasi-Fermi Level up to Junction



Equilibrium: Many electrons N-side. Few electrons P-side.

Detailed balance of drift and diffusion forces. → **No net current**

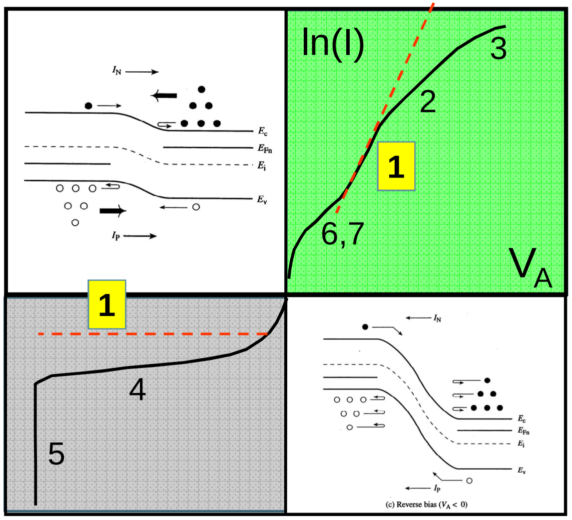


Forward Bias: Electron J_n and hole J_p currents across junction.

Diffusion force unchanged. (because doping did not change) But, drift force increased due to applied bias → **Net current**

Various Regions of I-V Characteristics

$$\ln(I) \sim \frac{q}{k_B T} V_A$$



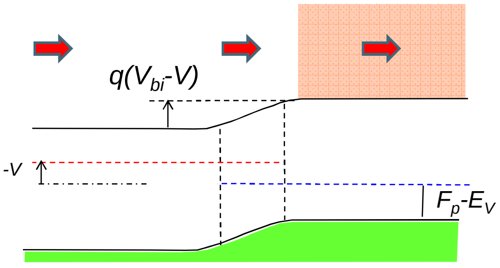
1. ***Diffusion limited***
2. *Ambipolar transport*
3. *High injection*
4. *R-G in depletion*
5. *Breakdown*
6. *Trap-assisted R-G*
7. *Esaki Tunneling*

Recall: One Sided Minority Diffusion

If current is continuous, one can calculate for the current anywhere. Position doesn't matter! Calculate at "easiest" position.

→Minority carrier current on P-side.
We know the solution from earlier
Assume steady state, no r_n, g_n

Steady state
Acceptor doped



$$\cancel{\frac{\partial n}{\partial t}} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{r_n} + \cancel{g_n}$$

$$J_n = qn\cancel{\mu_n}\mathcal{E} + qD_n\frac{dn}{dx}$$

$$0 = D_n\frac{d^2n}{dx^2}$$

Boundary Conditions

Boundary Conditions

$$n(x=0^+) = n_i\, e^{(F_n-E_i)\beta}$$

$$p(x=0^+) = n_i\, e^{-(F_p-E_i)\beta}$$

Difference of Quasi-Fermi-levels equals applied voltage

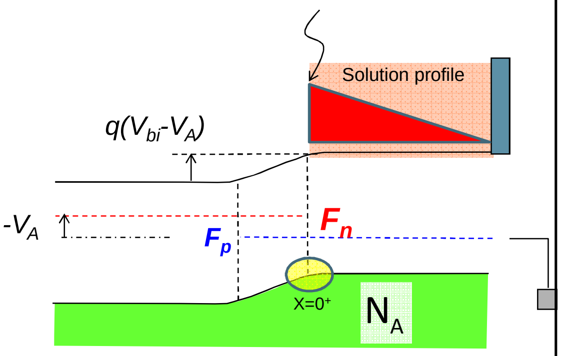
$$np = n_i^2 e^{(F_n-F_p)\beta} = n_i^2 e^{qV_A\beta}$$

$$p(0^+) = N_A$$

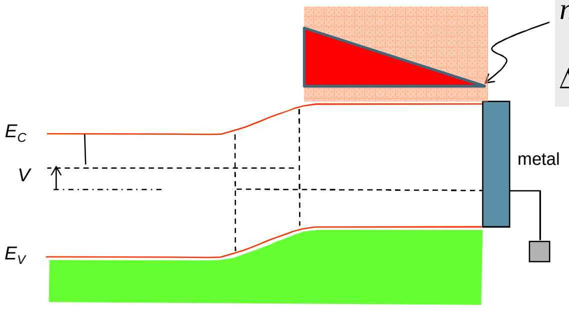
$$n(0^+) = \frac{n_i^2}{N_A} e^{qV_A\beta}$$

$$\Delta n(0^+) = n(0^+)_{V_G} - n(0^+)_{V_G=0}$$

$$= \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$



Right Boundary Condition



$$n(x=W_p) \approx \frac{n_i^2}{N_A}$$

$$\Delta n(x=W_p) = 0$$

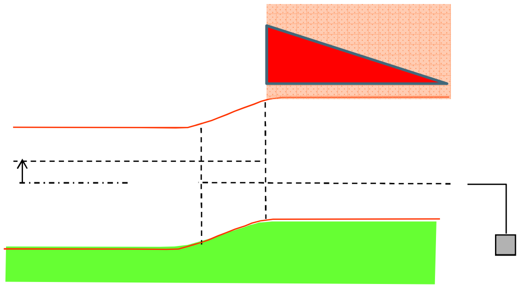
Infinite surface recombination velocity at metal → **excess** carrier concentration = 0

Example: One Sided Minority Diffusion

$$D_N \frac{d^2 n}{dx^2} = 0$$

Ansatz

$$\boxed{\Delta n(x,t) = C + Dx}$$



Plug in B.C.

$$x = W_p, \quad \Delta n(x = W_p) = 0 \Rightarrow C = -DW_p$$

$$x = 0', \quad \Delta n(x = 0) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right) = C$$

$$\boxed{\Delta n(x,t) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)}$$

Final result: Excess electron carrier concentration (P-side) as function of position

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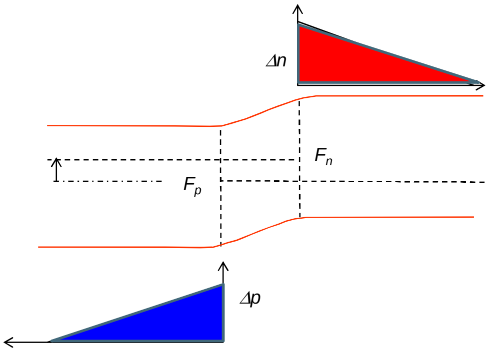
Electron & Hole Fluxes

$$\Delta n(x) = \frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)\left(1 - \frac{x}{W_p}\right)$$

$$\mathbf{J}_N = \cancel{qn\mu_N \mathcal{E}} + qD_N \nabla n$$

Current (electrons)

$$J_n = qD_n \left.\frac{dn}{dx}\right|_{x=0} = -\frac{qD_n}{W_p}\frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$



Current (holes)

$$J_p = -qD_p \left.\frac{dp}{dx}\right|_{x=0'} = -\frac{qD_p}{W_n}\frac{n_i^2}{N_D}\left(e^{qV_A\beta} - 1\right)$$

Total Current

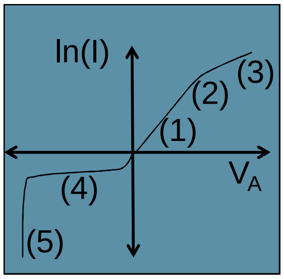
Forward Bias

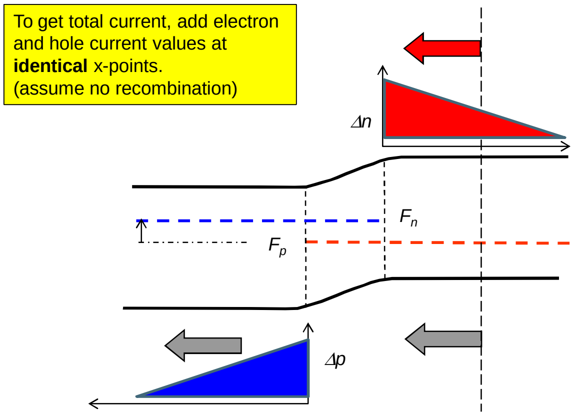
$\ln J_T \approx qV_A / k_B T + \ln(const.)$

Reverse Bias

$J_T \approx const.$ Diffusion current

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{qV_A\beta} - 1\right)$$




To get total current, add electron and hole current values at **identical** x-points.
(assume no recombination)

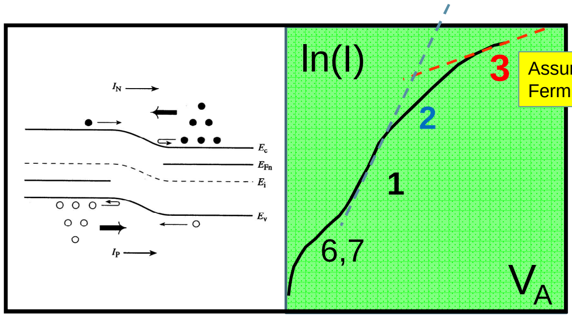
Outline

1) Derivation of the forward bias formula
2) **Solution in the nonlinear regime**
3) I-V in the ambipolar regime
4) Conclusion

Nonlinear Regime (3) …

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{\left(qV_A - \Delta F_n - \Delta F_p\right)\beta} - 1\right) = I_0\left(e^{q\left(V_A - aJ_n - bJ_p\right)\beta} - 1\right)$$

Today's lecture: Nonlinear Regime (2,3)



Assumption of **flat** Quasi-Fermi levels invalid here

Flat Quasi-Fermi Level up to Junction ?

$$\mathbf{J}_N = qn\mu_N\mathcal{E} + qD_N\frac{dn}{dx}$$

$$J_n = n\mu_n\frac{dF_n}{dx} \Rightarrow \Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

Drop of Quasi-Fermi level across the junction **proportional** to current!

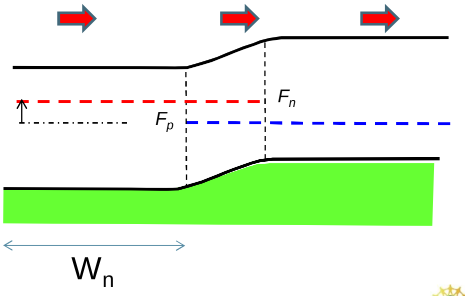
Rewrite ***n*** into non-equilibrium form, re-arrange $\mathbf{J}_n$ equation

$$n = n_i e^{\beta(F_n - E_i)} \qquad qD_N\frac{dn}{dx} = qD_N\beta\left[\frac{dF_n}{dx} - \mathcal{E}\right]\left[n_i e^{\beta(F_n - E_i)}\right]$$

New diffusion component: Plug this into original $\mathbf{J}_n$ equation

$$qD_N\frac{dn}{dx} = qD_N n\beta\left[\frac{dF_n}{dx} - \mathcal{E}\right]$$

$$= q\mu_N n\left[\frac{dF_n}{dx} - \mathcal{E}\right] \quad \because \frac{D_N}{\mu_n} = \frac{k_B T}{q}$$



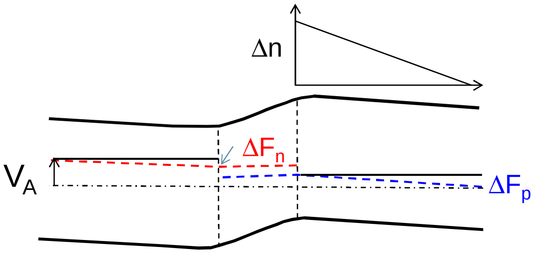
Forward Bias: Nonlinear Regime …

$$n(0^+) = \frac{n_i^2}{N_A} e^{(F_n - F_p)\beta}\Big|_{junction} = \frac{n_i^2}{N_A} e^{(qV_A - \Delta F_n - \Delta F_p)\beta} \Rightarrow \Delta n(0^+) = \frac{n_i^2}{N_A}\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$\Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

$$\Delta F_p = \frac{J_p W_n}{\mu_n N_D}$$



Still diffusion dominated transport? Since Quasi-Fermi levels are not flat in nonlinear regime (drift), this approximation becomes worse.

Outline

1) Derivation of the forward bias formula
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4) Tunneling and I-V characteristics
5) Conclusion

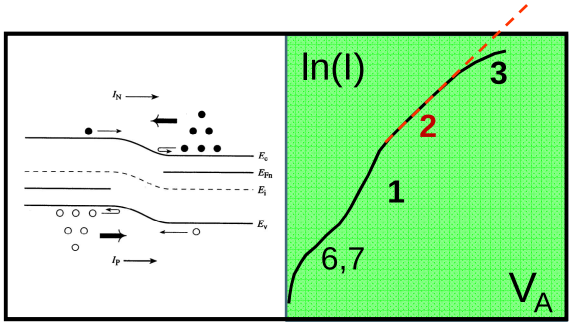
Region (2): Ambipolar Transport

$$J_T \approx -q\left[\frac{D_n}{W_p}+\frac{D_p}{W_n}\right] n_i e^{\left(qV_A-\Delta F_n-\Delta F_p\right)\beta/2}$$

$$\ln(J_T) \approx \frac{qV_A}{2k_BT}$$

Today's lecture: **Ambipolar** Transport regime (2)

Question: Where does the 2 come from?



Nonlinear Regime: Ambipolar Transport

$$np = n_i^2 e^{(F_n-F_p)\beta}$$

Here not negligibly small. Ambipolar transport !

$$\left(\frac{n_i^2}{N_A}+\Delta n\right)(N_A+\Delta p) = n_i^2\left(e^{q\left(V_A-\Delta F_n-\Delta F_p\right)\beta}-1\right)$$

Excess carrier concentrations >> N_A Thus…

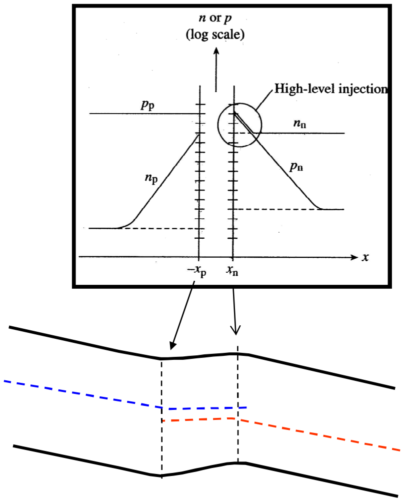
$$\Delta n \approx \Delta p = n_i\sqrt{\left(e^{q\left(V_A-\Delta F_n-\Delta F_p\right)\beta}-1\right)}$$

$$\approx n_i e^{q\left(V_A-\Delta F_n-\Delta F_p\right)\beta/2}$$

Currents

$$J_n = -qD_n\frac{\Delta n}{W_p} = \frac{qD_n n_i}{W_p} e^{\left(qV_A-\Delta F_n-\Delta F_p\right)\beta/2}$$

$$J_p = -qD_p\frac{\Delta n}{W_n} = \frac{qD_p n_i}{W_n} e^{\left(qV_A-\Delta F_n-\Delta F_p\right)\beta/2}$$



Note: junction never disappears, even for large forward bias!

Outline

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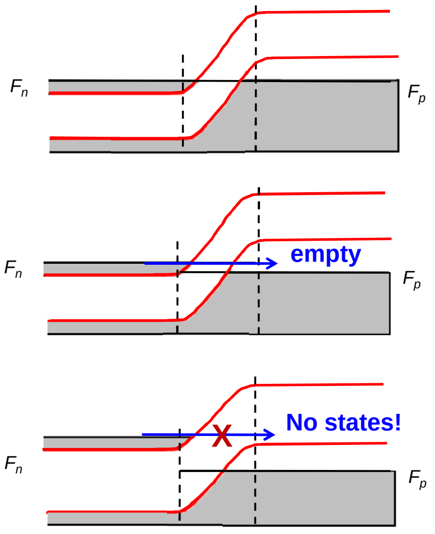
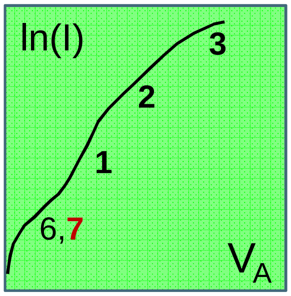
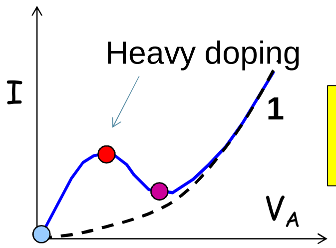
Forward Bias Nonlinearity (7): Esaki Diode

Esaki-Diode: **Heavily** doped diode

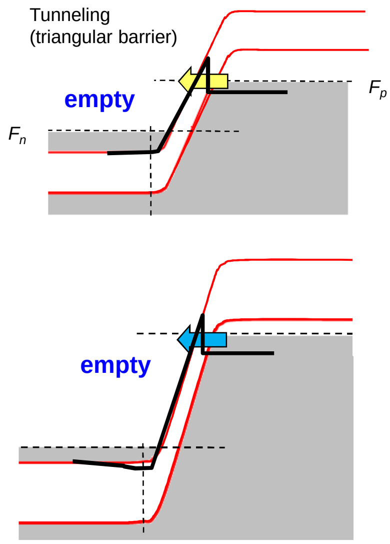


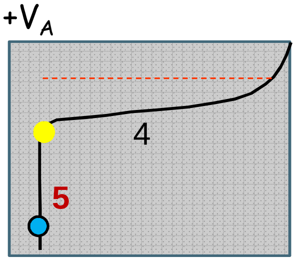




Tunneling in diodes.
Nobel Prize (Esaki)

Reverse Bias (5): Zener Tunneling





Zener tunneling occurs in every diode. (reverse bias)

Remember: Tunneling through a triangular barrier

$$I = qpT\upsilon$$

$$T = \frac{4}{4\cosh^2 \alpha d + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)\sinh^2 \alpha d}$$

(p.49 ADF)

Conclusion

1) I-V characteristics of a p-n junction is defined by many interesting phenomena including diffusion, ambipolar transport, tunneling etc.
2) The separate regions are identified by specific features. Once we learn to identify them, we can see if one or the other mechanism is dominated for a given technology.
3) In the next class, we will discuss a few more non-ideal effects.