ECE606: Solid State Devices
Lecture 15
p-n diode characteristics

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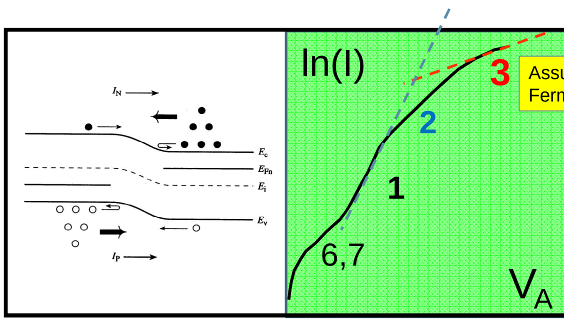
Outline

1) **Solution in the nonlinear regime**
2) I-V in the ambipolar regime
3) Tunneling and I-V characteristics
4) Non-ideal effects: Impact ionization
5) Non-ideal effects: Junction recombination
6) Conclusion

Nonlinear Regime (3) …

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{(qV_A-\Delta F_n-\Delta F_p)\beta}-1\right) = I_0\left(e^{q(V_A-aJ_n-bJ_p)\beta}-1\right)$$

Today's lecture: Nonlinear Regime (2,3)



Assumption of **flat** Quasi-Fermi levels invalid here

Flat Quasi-Fermi Level up to Junction ?

$$\mathbf{J}_N = qn\mu_N\mathcal{E} + qD_N\frac{dn}{dx}$$

$$J_n = n\mu_n\frac{dF_n}{dx} \quad \Rightarrow \Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

Drop of Quasi-Fermi level across the junction **proportional** to current!

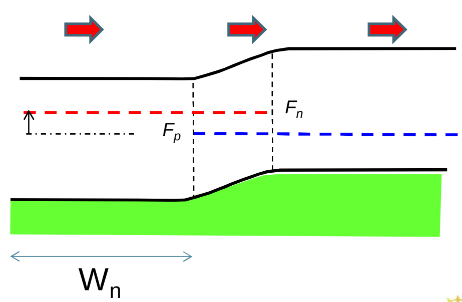
Rewrite ***n*** into non-equilibrium form, re-arrange $\mathbf{J}_n$ equation

$$n = n_i e^{\beta(F_n-E_i)} \qquad qD_N\frac{dn}{dx} = qD_N\beta\left[\frac{dF_n}{dx}-\mathcal{E}\right]\left[n_i e^{\beta(F_n-E_i)}\right]$$

New diffusion component: Plug this into original $\mathbf{J}_n$ equation

$$qD_N\frac{dn}{dx} = qD_N n\beta\left[\frac{dF_n}{dx}-\mathcal{E}\right]$$

$$= q\mu_N n\left[\frac{dF_n}{dx}-\mathcal{E}\right] \quad \because \frac{D_N}{\mu_n} = \frac{k_BT}{q}$$



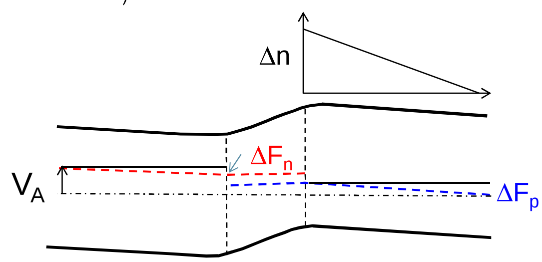
Forward Bias: Nonlinear Regime …

$$n(0^+) = \frac{n_i^2}{N_A} e^{(F_n - F_p)\beta} \Big|_{junction} = \frac{n_i^2}{N_A} e^{(qV_A - \Delta F_n - \Delta F_p)\beta} \Rightarrow \Delta n(0^+) = \frac{n_i^2}{N_A}\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$J_T = -q\left[\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right]\left(e^{(qV_A - \Delta F_n - \Delta F_p)\beta} - 1\right)$$

$$\Delta F_n = \frac{J_n W_n}{\mu_n N_D}$$

$$\Delta F_p = \frac{J_p W_n}{\mu_n N_D}$$



Still diffusion dominated transport? Since Quasi-Fermi levels are not flat in nonlinear regime (drift), this approximation becomes worse.

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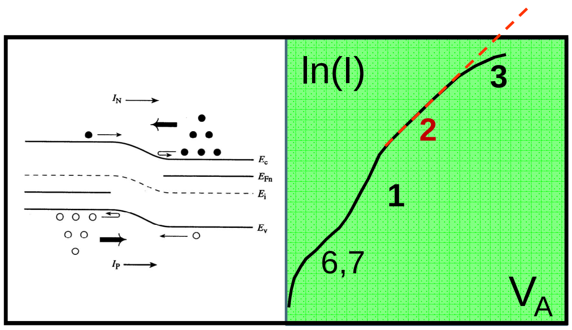
Region (2): Ambipolar Transport

$$J_T \approx -q\left[\frac{D_n}{W_p}+\frac{D_p}{W_n}\right] n_i e^{(qV_A-\Delta F_n-\Delta F_p)\beta/2}$$

$$\ln(J_T) \approx \frac{qV_A}{2k_BT}$$

Today's lecture: **Ambipolar** Transport regime (2)

Question: Where does the 2 come from?



Nonlinear Regime: Ambipolar Transport

$$np = n_i^2 e^{(F_n-F_p)\beta}$$

Here not negligibly small. Ambipolar transport !

$$\left(\frac{n_i^2}{N_A}+\Delta n\right)(N_A+\Delta p) = n_i^2\left(e^{q(V_A-\Delta F_n-\Delta F_p)\beta}-1\right)$$

Excess carrier concentrations >> N_A Thus…

$$\Delta n \approx \Delta p = n_i\sqrt{\left(e^{q(V_A-\Delta F_n-\Delta F_p)\beta}-1\right)}$$

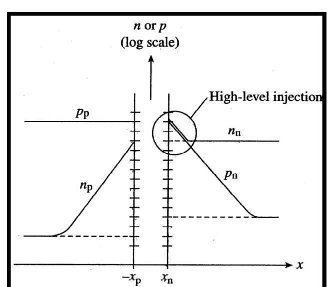
$$\approx n_i\, e^{q(V_A-\Delta F_n-\Delta F_p)\beta/2}$$

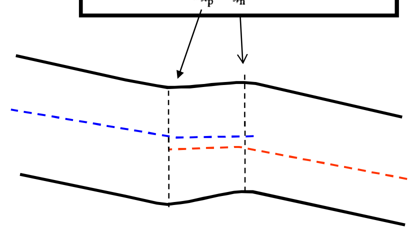
Currents

$$J_n = -qD_n\frac{\Delta n}{W_p} = \frac{qD_n n_i}{W_p} e^{(qV_A-\Delta F_n-\Delta F_p)\beta/2}$$

$$J_p = -qD_p\frac{\Delta n}{W_n} = \frac{qD_p n_i}{W_n} e^{(qV_A-\Delta F_n-\Delta F_p)\beta/2}$$

Note: junction never disappears, even for large forward bias!





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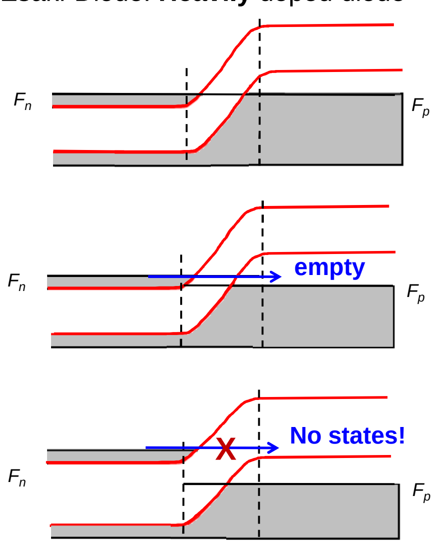
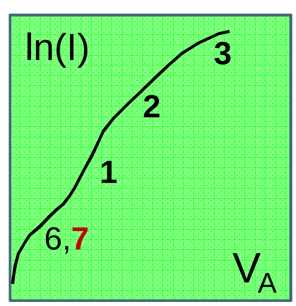
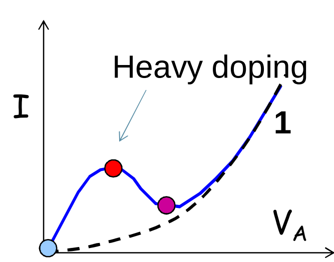
Forward Bias Nonlinearity (7): Esaki Diode

Esaki-Diode: **Heavily** doped diode

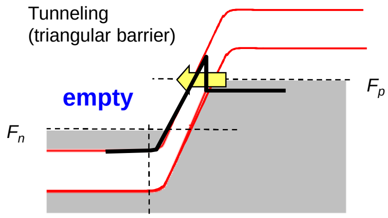


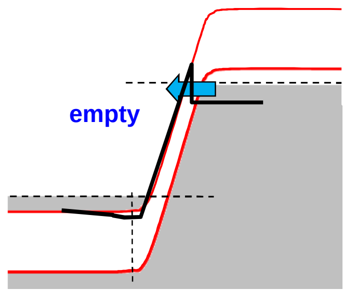


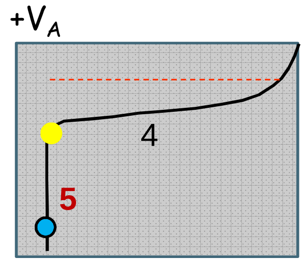


Tunneling in diodes. Nobel Prize (Esaki)

Reverse Bias (5): Zener Tunneling







Zener tunneling occurs in every diode. (reverse bias)

Remember: Tunneling through a triangular barrier

$$I = qpT\upsilon$$

$$T = \frac{4}{4\cosh^2 \alpha d + \left(\frac{\alpha}{k} - \frac{k}{\alpha}\right)\sinh^2 \alpha d}$$

(p.49 ADF)

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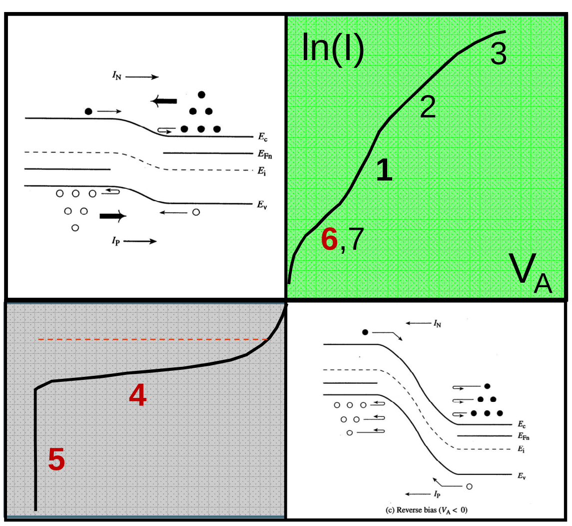
Various Regions of I-V Characteristics



1. *Diffusion limited*
2. *Ambipolar transport*
3. *High injection*
4. ***R-G in depletion***
5. ***Breakdown***
6. ***Trap-assisted R-G***
7. *Esaki Tunneling*

Slide 12

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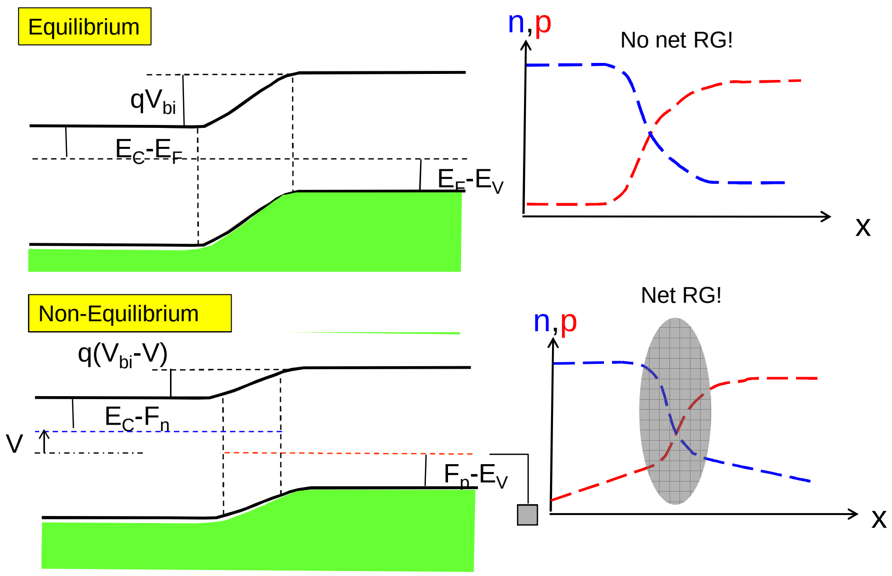
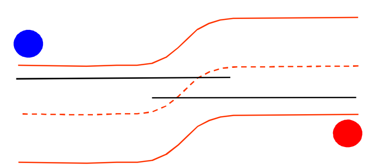
Applying Bias



(4,6) Junction Recombination

What is the recombination current?

$$I_R = -qA\int_0^W \frac{\partial n}{\partial t}\,dx$$



Shockley-Reed Hall

$$\frac{\partial n}{\partial t} = -\frac{[n(x)p(x) - n_i^2]}{\tau_p[n(x)+n_1] + \tau_n[p(x)+p_1]}$$

Follows from assuming midgap traps

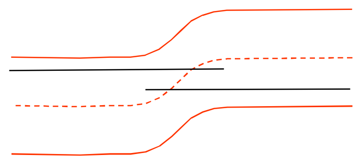
Assume $\tau_n = \tau_p$ $\quad E_i = E_T$ $\quad n_1 = p_1 = n_i$

$$\frac{\partial n}{\partial t} = -\frac{n_i^2 (e^{qV_A/kT} - 1)}{\tau[n(x) + p(x) + 2n_i]}$$

Note: Do you remember this HW ?

(np) Product within the Junction

Mass action in non-equilibrium

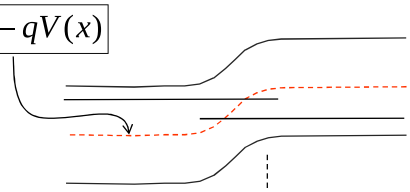


$$n(x)\,p(x) = n_i^{\,2} e^{(F_N - F_P)/kT}$$
$$= n_i^{\,2} e^{qV_A/kT}$$

For non-equilibrium at **low** current values.

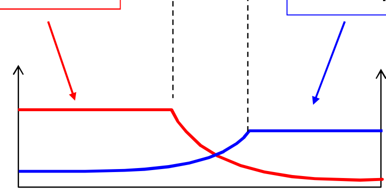
Electron/Hole Concentrations at Junction

$$E_i(x) = E_{iL} - qV(x)$$



$$n(x) = n_i e^{(F_N - E_i(x))/kT}$$
$$= n_i e^{[F_N - E_{iL} + qV(x)]/kT}$$

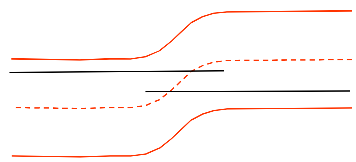
$$p(x) = \frac{n_i^{\,2} e^{qV_A/kT}}{n_i e^{[F_N - E_{iL} + qV(x)]/kT}}$$
$$= n_i e^{-[F_N - E_{iL} + qV(x)]/kT + qV_A/kT}$$



position

Junction Recombination

$$U_{FN} = \frac{F_N - E_{iL}}{kT} \qquad U_A = \frac{V_A}{kT/q}$$



$$\frac{\partial n}{\partial t} = -\frac{n_i (e^{U_A} - 1)}{\tau [e^{U_{FN}+U} + e^{-U_{FN}-U+U_A}]}$$

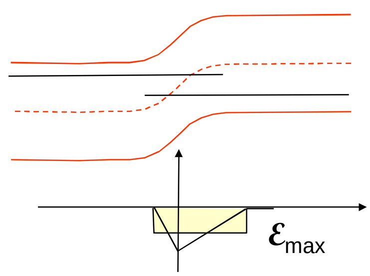
$$I_R = -qA\left(\frac{n_i}{\tau}\right) \times sinh\left(\frac{U_A}{2}\right) \times \int_0^W \frac{dx}{cosh[U_{FN} + U - U_A/2]}$$

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{e^{U_A/2}(e^{U_A/2} - e^{-U_A/2})}{e^{U_A/2}[e^{U_{FN}+U-U_A/2} + e^{-U_{FN}-U+U_A/2}]}$$

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{sinh(U_A/2)}{cosh[U_{FN} + U - U_A/2]}$$

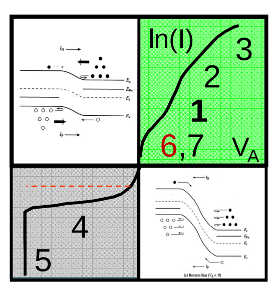
Junction Recombination in Forward Bias

$$\Rightarrow \frac{\partial n}{\partial t} = -\frac{n_i}{\tau} \frac{sinh(U_A/2)}{cosh[U_{FN} + U - U_A/2]}$$



$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau}) sinh(\frac{U_A}{2}) \int_0^W \frac{dx}{e^{(U_{FN}+U-U_A/2)}}$$

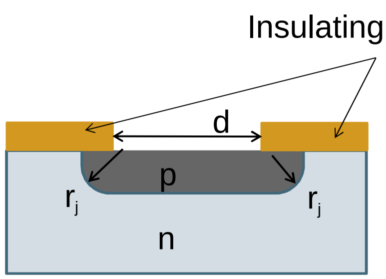
$$\Rightarrow I_R \approx -qA(\frac{n_i}{\tau}) \times sinh(\frac{U_A}{2}) \int_0^W \frac{dx}{e^{-(\mathcal{E}_{max}x)/(kT/2q)}}$$

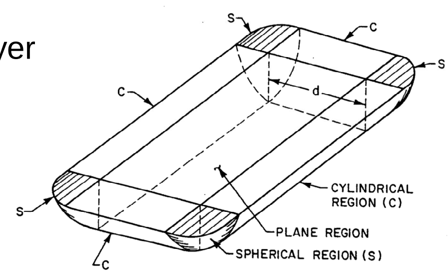


$$\boxed{\Rightarrow I_{Dep} = -qA\left[\frac{kT}{2q\mathcal{E}_{max}}\right]\left[\frac{n_i}{\tau} e^{qV_A/2kT}\right]}$$

Effective width — Excess Carrier at mid-junction

Junction Leakage in Practice





Junction Design Considerations
Electric field stronger at corners, sharp edges. → increased recombination!

Junction Recombination in Reverse Bias

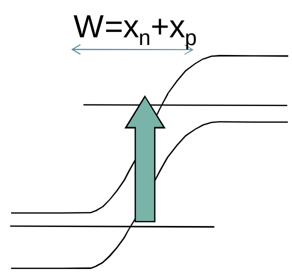
$$\frac{\partial n}{\partial t} = -\frac{n_i}{2\tau}$$

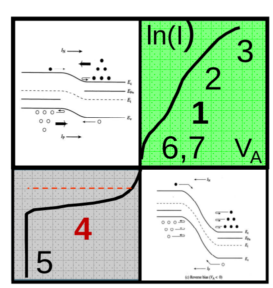
(Recombination in depletion region)

Integrate…

$$I_R \approx -qA\int_0^W \left(\frac{n_i}{2\tau}\right)dx$$

$$= -qA\frac{n_i W}{2\tau} \propto \sqrt{V_{bi} - V_A}$$





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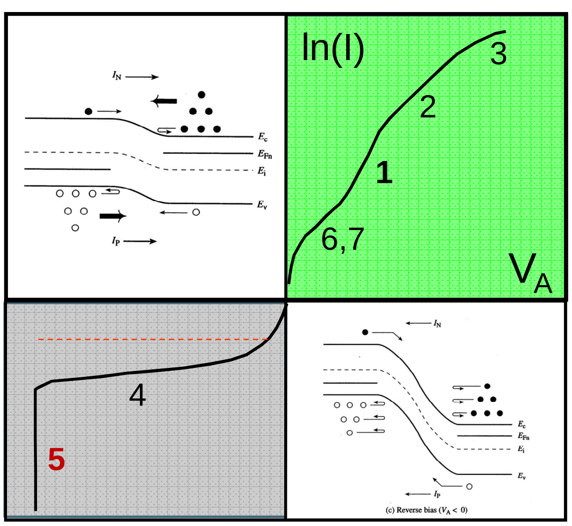
Avalanche Breakdown



1. *Diffusion limited*
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3. *High injection*
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5. ***Breakdown***
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Slide 22

MAA3 Asad: We should redraw the figure ...

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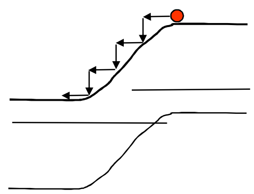
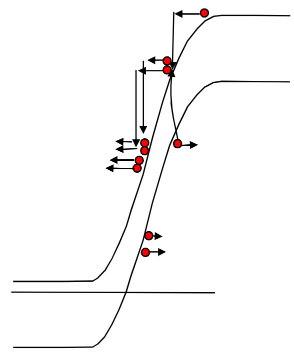
Reverse Bias



High Reverse Bias



Exponential current growth
(Impact Ionization or Inverse Auger process)

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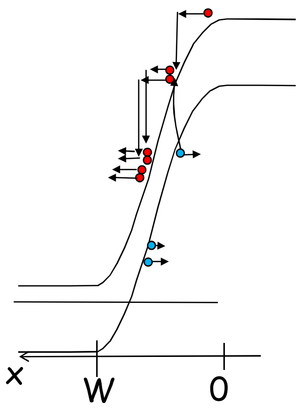
Impact-ionization and Flux Conservation



$$I_n(x+dx) = I_n(x) + \alpha_n I_n(x)dx + \alpha_p I_p(x)dx$$

Impact Ionization probabilities

$$\frac{I_n(x+dx) - I_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

$$\Rightarrow \frac{dI_n(x)}{dx} = \alpha_n I_n(x) + \alpha_p I_p(x)$$

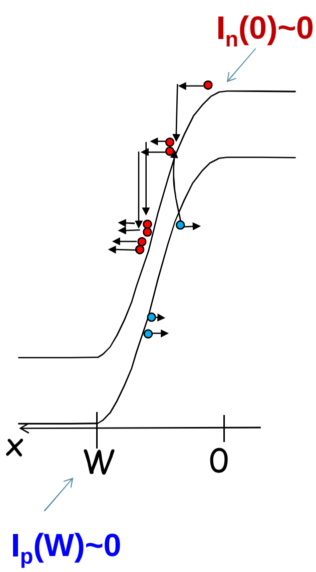
Steady state: Define $I_T = I_N + I_P$ (total current)

$$\frac{dI_n(x)}{dx} = \alpha_p \left[I_T - I_n(x) \right] + \alpha_n I_n(x)$$

$$\frac{dI_n(x)}{dx} - \left(\alpha_n - \alpha_p \right) I_n(x) = \alpha_p I_T$$

Differential equation

Impact-ionization



Solution form of differential equation

$$\frac{I_n(W)}{I_T} = \frac{\int_0^W \alpha_p e^{-\int_0^x (\alpha_n - \alpha_p) dx'} dx + \frac{I_n(0)}{I_T}}{1 + \int_0^W (\alpha_p - \alpha_n) e^{-\int_0^x (\alpha_n - \alpha_p) dx'} dx}$$

Reverse diffusion current

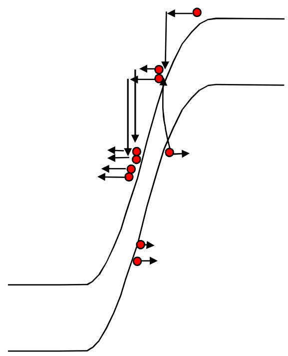
$$I_p(W) + I_n(W) = I_T \Rightarrow I_n(W) \approx I_T$$

$$\frac{I_n(0)}{I_T} \equiv \frac{1}{M_p}$$ Multiplication Factor

At x=W, I_N has grown exponentially, and I_P is now negligible.

$$\left(1 - \frac{1}{M_p}\right) \approx 1 = \int_0^W \alpha_p e^{-\int_0^x (\alpha_p - \alpha_n) dx'} dx$$

Impact-ionization



Simplify further...

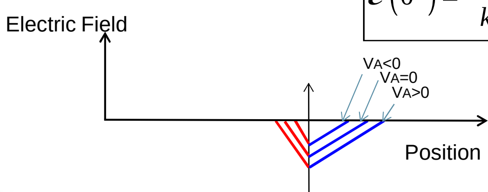
$$\int_0^W \alpha_p e^{-\int_0^x (\alpha_p - \alpha_n) dx'} dx \approx 1$$

$$\alpha_p = \alpha_n \Rightarrow \alpha_p W = 1$$ Assume: Significant impact ionization

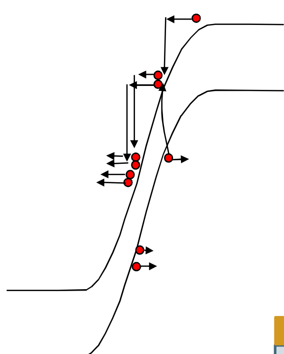
$$\alpha_p = A_0 e^{-B/\mathcal{E}}$$ from experiment and theory

Breakdown-Field

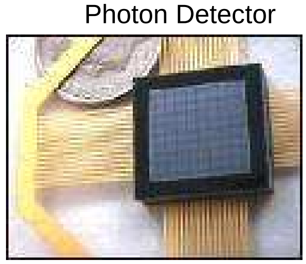
$$\mathcal{E}(0^-) = \frac{qN_D x_n}{k_s \varepsilon_0} = \left[\frac{2q}{k_s \varepsilon_0} \frac{N_D N_A}{N_D + N_A}(V_{bi} - V_A)\right]^{1/2}$$



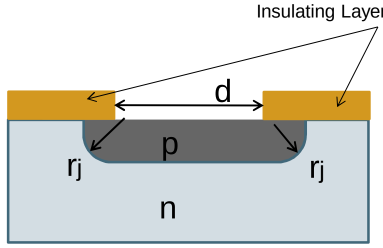
Impact-ionization: In Practice



Photon Detector



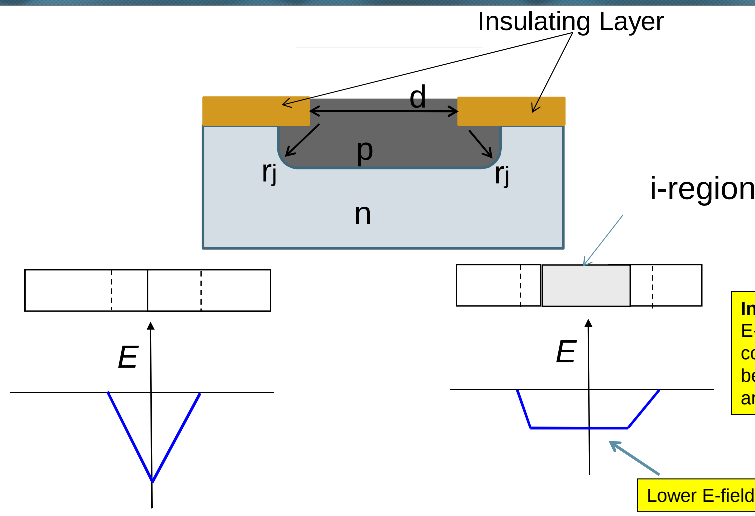
Good ….



High E-fields at junction corners → Breakdown

Bad….

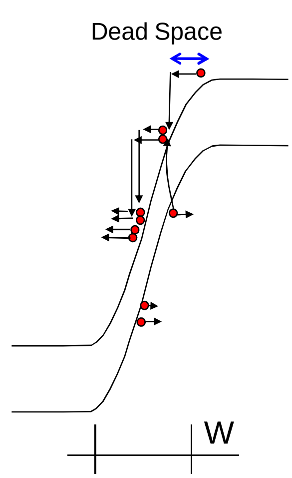
Junction Engineering



Intrinsic region: E-field has to be constant, because there are no charges!

Reduced field for p-i-n junction, because V_{bi} (area under the curve) must be the same.

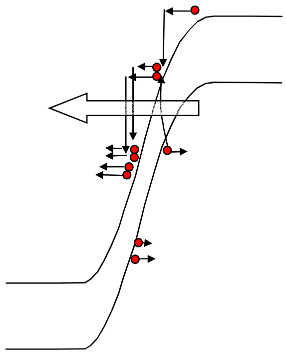
Modern Considerations: Dead Space

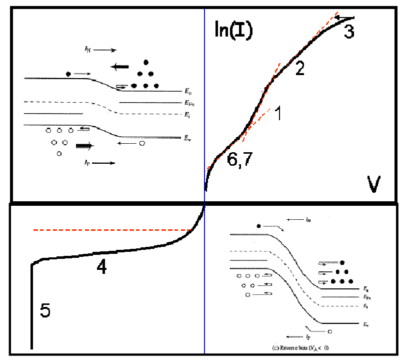


Dead Space: Space you need before an electron can impact ionize.

For very small (ballistic) junctions, electrons can cross the junction without inducing impact Ionization. (Dead space too small)

Zener Breakdown vs. Impact Ionization





How do you differentiate between Zener tunneling and impact-ionization?

Conclusion

1) Junction recombination is often used as a diagnostic tool for process maturity. Defects in junction arises from misplaced donor impurities, not necessary from deep-trap impurities.

2) Impact ionization plays an important role in wide variety of devices (e.g. avalanche photo-diodes).

3) In the next class, we will discuss AC response of p-n junction diodes.

ECE606: Solid State Devices
p-n diode AC Response

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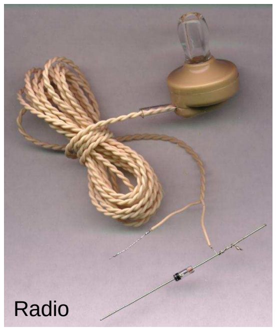
Topic Map

	Equilibrium	**DC**	**Small signal**	Large Signal	Circuits
Diode			◆		
Schottky					
BJT/HBT					
MOSFET					

Diode in **Non-**Equilibrium
(External DC+AC voltage applied)

Why should we study AC Response?

Motivation



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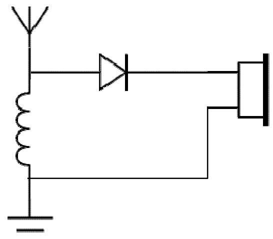
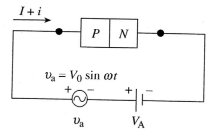
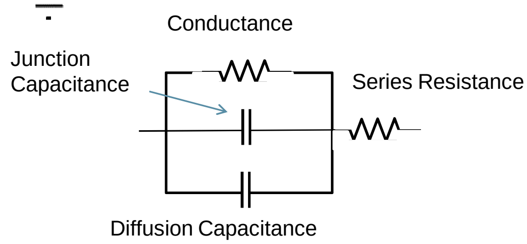




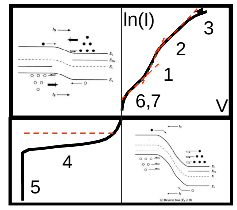


Outline

1) **Conductance and series resistance**
2) Majority carrier junction capacitance
3) Minority carrier diffusion capacitance
4) Conclusion

Ref. SDF, Chapter 7

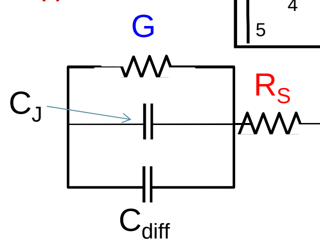
Forward Bias Conductance



$$I = I_o\left(e^{q(V_A - R_S I)\beta/m} - 1\right)$$

m = RG (2), diff (1), Ambipolar (2)

$$\ln\frac{I+I_o}{I_0} = q(V_A - R_S I)\frac{\beta}{m}$$

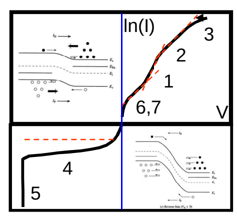


$$\frac{m}{q\beta(I+I_o)} = \frac{dV_A}{dI} - R_S$$

$$\boxed{\frac{1}{g_{FB}} = R_S + \frac{m}{q\beta(I+I_0)}}$$

Forward Bias Conductance

Reverse Bias Conductance

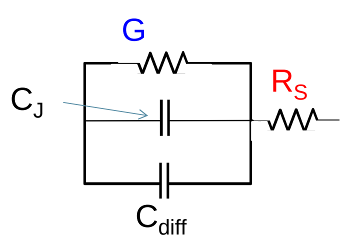


$$I = I_o\left(e^{q(V_A - R_S I)\beta/m} - 1\right) - \frac{qn_i}{2\tau}B_0\sqrt{V_{bi} - V_A}$$

$$\approx -I_0 - \frac{qn_i}{2\tau}B_0\sqrt{V_{bi} - V_A}$$

$$\boxed{\frac{1}{g_{RB}} = \frac{qn_i B_0}{2\tau\sqrt{V_{bi} - V_A}}}$$

Reverse Bias Conductance

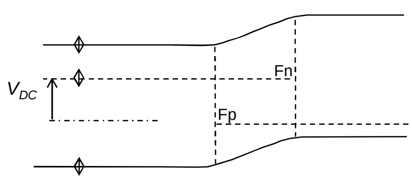


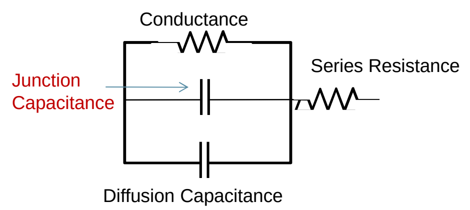
Outline

1) Conductance and series resistance
2) **Majority carrier junction capacitance**
3) Minority carrier diffusion capacitance
4) Conclusion

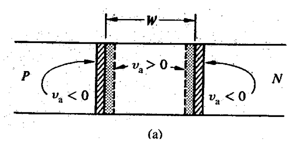
Junction Capacitance

Forward biased diode + AC signal

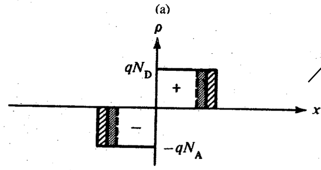


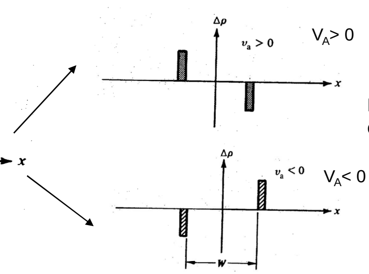


Depletion width modulation



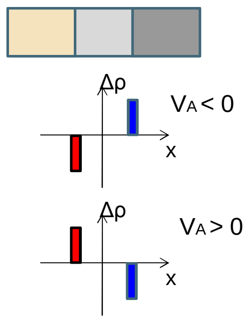
Charge modulation

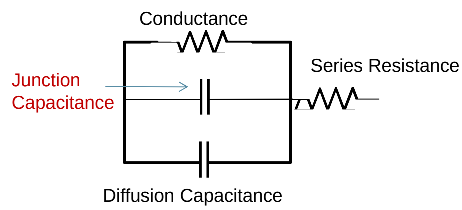




Majority carrier effect

Majority Carrier Junction Capacitance

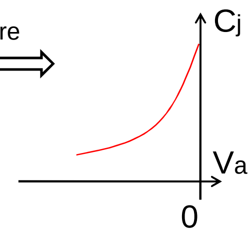




Majority Carrier Junction Capacitance

$$C_J = \frac{K_s \varepsilon_0 A}{W_n + W_p} = \frac{K_s \varepsilon_0 A}{\sqrt{\left(\frac{2K_s\varepsilon_0}{qN_D} + \frac{2K_s\varepsilon_0}{qN_A}\right)(V_{bi} - V_A)}}$$

Measure ⇒



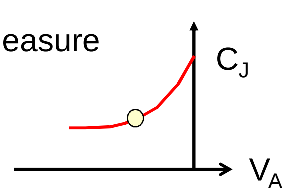
Measurement of Built-in Potential

$$\frac{1}{C_J^{\,2}} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}(V_{bi} - V_A)$$

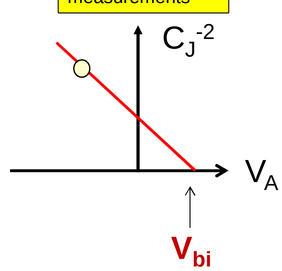
(Assume single sided p^+-n junction)

Derive V_{bi} from C_J measurements

measure



plot



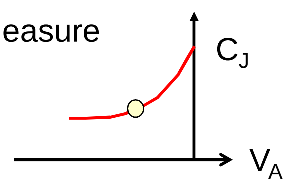
.. And Variable Doping

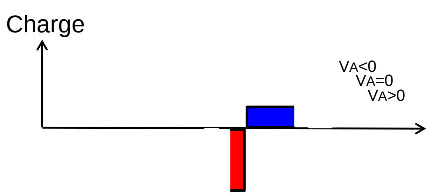
$$\frac{1}{C_J^{\,2}} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}(V_{bi}-V_A)$$

$$N_D(x) = \frac{2}{qK_s\varepsilon_0 A^2}\,\frac{1}{d\left(1/C_J^2\right)/dV_A}$$

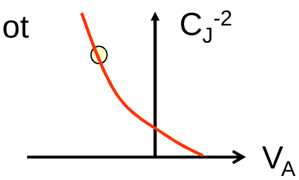
Measure doping concentration as a function of position

measure





plot



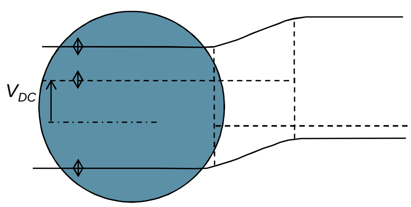
Dielectric Relaxation Time (majority side)

Majority side

$$J_n = qn\mu_N E + q\cancel{D_N}\nabla n$$

Neglect

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{R_n} + \cancel{G_n}$$



$$\frac{d(\Delta n)}{dt} = \frac{1}{q}\frac{d(qn\mu_N \mathcal{E})}{dx} = N_D\mu_N\frac{d\mathcal{E}}{dx}$$

N-side

$$\frac{d\mathcal{E}}{dx} = \frac{q}{k_S\varepsilon_0}(\cancel{p} - n_0 - \Delta n + N_D - \cancel{N_A})$$

$$\frac{d(\Delta n)}{dt} = -\frac{qN_D\mu_N}{k_S\varepsilon_0}\Delta n = -\frac{\sigma_0\Delta n}{k_S\varepsilon_0}$$

$$\Delta n(t) = n_0 e^{-\frac{\sigma_0 t}{k_S\varepsilon_0}} = n_0 e^{-\frac{t}{\tau_d}}$$

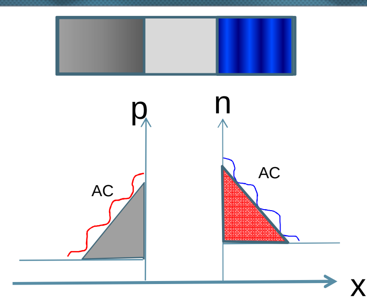
How long does it take for the signal to cross the junction?

$$\tau_d = \frac{K_s\varepsilon_0}{\sigma} \approx 0.1 \text{ ps}$$

Very fast

1) Conductance and series resistance
2) Majority carrier junction capacitance
3) **Minority carrier diffusion capacitance**
4) Conclusion

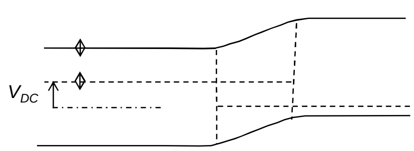
Diffusion Capacitance for Minority Carriers



Minority Carrier side

$$\mathbf{J}_N = qn\mu_N \mathcal{E} + qD_N \frac{dn}{dx}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + g_N$$



DC AC

$$\frac{\partial\left(n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right)}{\partial t} = D_N \frac{d^2\left(n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right)}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}}{\tau_n}$$

Diffusion Capacitance for Minority Carriers

$$\frac{\partial\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{\partial t}=D_N\frac{d^2\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{dx^2}-\frac{\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}}{\tau_n}$$

$$j\omega\Delta n_{ac}e^{j\omega t}=D_N\frac{d^2\Delta n_{dc}}{dx^2}+e^{j\omega t}\frac{d^2\Delta n_{ac}}{dx^2}-\frac{\Delta n_{dc}}{\tau_n}-e^{j\omega t}\frac{\Delta n_{ac}}{\tau_n}$$

$$\text{DC}:\quad 0=D_N\frac{d^2\Delta n_{dc}}{dx^2}-\frac{\Delta n_{dc}}{\tau_n}\Rightarrow\Delta n_{dc}=Ae^{-\frac{x}{L_n}}+Be^{+\frac{x}{L_n}}$$

$$\text{AC}:\quad 0=D_N\frac{d^2\Delta n_{ac}}{dx^2}-\left(j\omega\tau_n+1\right)\frac{\Delta n_{ac}}{\tau_n}\Rightarrow\Delta n_{ac}=Ce^{-\frac{x}{L_n*}}+De^{+\frac{x}{L_n*}}\rightarrow Ce^{-\frac{x}{L_n*}}$$

$$L_n^*=\sqrt{D_n\tau_n/\left(1+j\omega\tau_n\right)}\qquad\tau_n^*=\tau_n/\left(1+j\omega\tau_n\right)$$

AC Boundary Conditions

$$\Delta n_{dc}(x=0)=\frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}}-1\right)$$

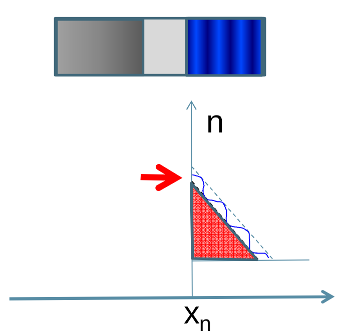
$$\left(\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)=\frac{n_i^2}{N_A+\Delta p_{ac}e^{j\omega t}}\left(e^{q\frac{V_{dc}+V_{ac}e^{j\omega t}}{kT}}-1\right)$$

$$\left(\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)\approx\frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}}e^{\frac{qV_{ac}e^{j\omega t}}{kT}}-1\right)$$

Taylor expansion

$$\approx\frac{n_i^2}{N_A}\left\{e^{\frac{qV_{dc}}{kT}}\left(1+\frac{qV_{ac}e^{j\omega t}}{kT}\right)-1\right\}$$

$$\Delta n_{ac}(x=0)=\frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}=C$$



AC Current and Impedance

$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} = C$$

$$\Delta n_{ac}(x) = Ce^{-\frac{x}{L_n^*}} + De^{+\frac{x}{L_n^*}} \rightarrow Ce^{-\frac{x}{L_n^*}}$$

Finally…

$$J_{ac} = -qD_n \left.\frac{d\Delta n_{ac}}{dx}\right|_{x=0} = \frac{qD_n}{L_n^*}\frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}$$

AC Current

$$Y_{ac} = \frac{J_{ac}}{V_{ac}} = \frac{q^2 D_n}{L_n^* kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} \equiv G_0\sqrt{1+j\omega\tau_n}$$

AC Impedance

Diffusion Conductance and Capacitance

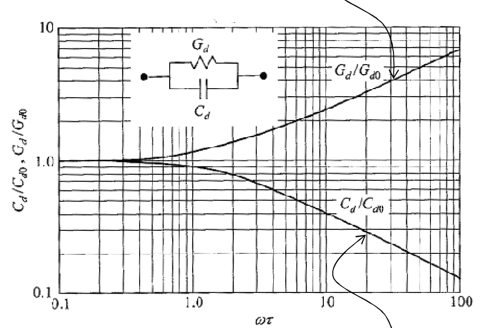
$$G_D \propto \sqrt{\omega}$$

$$Y_{ac} = G_D + j\omega C_D \equiv G_0\sqrt{1+j\omega\tau_n}$$

Separate in real & imaginary parts …

$$G_D = \frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}+1\right]^{1/2}$$

$$\omega C_D = \frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}-1\right]^{1/2}$$



Product of G_D and C_D frequency-independent

$$C_D \propto 1/\sqrt{\omega}$$

Conclusion

1) Small signal response relevant for many analog applications.
2) Small signal parameters always refer to the DC operating conditions, as such the parameter changes with bias condition.
3) Important to distinguish between majority and minority carrier capacitance. Their relative importance depends on specific applications.