ECE606: Solid State Devices
Lecture 16
p-n diode AC Response

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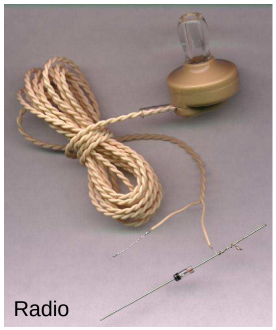
Topic Map

	Equilibrium	**DC**	**Small signal**	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSFET					

Diode in **Non-**Equilibrium
(External DC+AC voltage applied)

Why should we study AC Response?

Motivation



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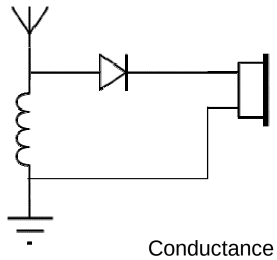
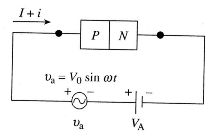
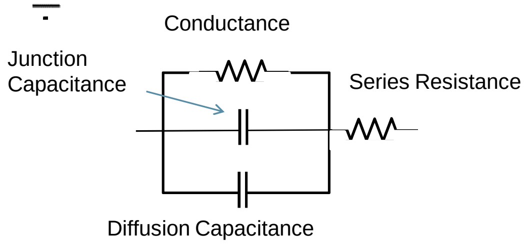




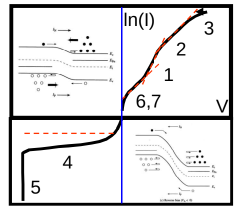


Outline

1) **Conductance and series resistance**
2) Majority carrier junction capacitance
3) Minority carrier diffusion capacitance
4) Conclusion

Ref. SDF, Chapter 7

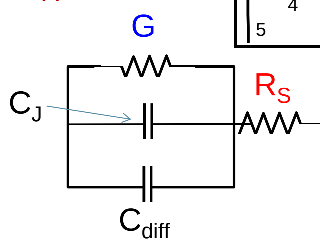
Forward Bias Conductance



$$I = I_o\left(e^{q(V_A - R_S I)\beta/m} - 1\right)$$

m = RG (2), diff (1), Ambipolar (2)

$$\ln\frac{I + I_o}{I_0} = q(V_A - R_S I)\frac{\beta}{m}$$



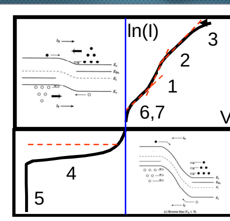
$$\frac{m}{q\beta(I + I_o)} = \frac{dV_A}{dI} - R_S$$

$$\boxed{\frac{1}{g_{FB}} = R_S + \frac{m}{q\beta(I + I_0)}}$$

Forward Bias Conductance

nanoHUB.org

Reverse Bias Conductance

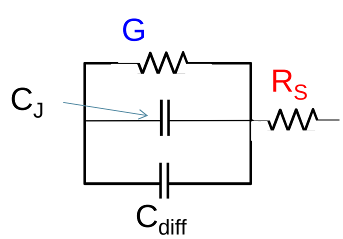


$$I = I_o\left(e^{q(V_A - R_S I)\beta/m} - 1\right) - \frac{qn_i}{2\tau}B_0\sqrt{V_{bi} - V_A}$$

$$\approx -I_0 - \frac{qn_i}{2\tau}B_0\sqrt{V_{bi} - V_A}$$

$$\boxed{\frac{1}{g_{RB}} = \frac{qn_i B_0}{2\tau\sqrt{V_{bi} - V_A}}}$$

Reverse Bias Conductance

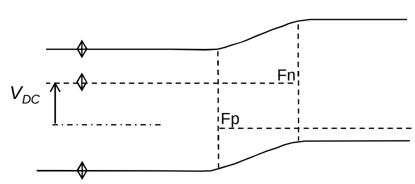


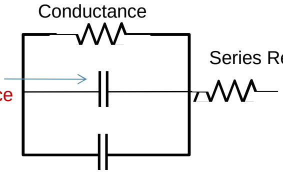
Outline

1) Conductance and series resistance
2) **Majority carrier junction capacitance**
3) Minority carrier diffusion capacitance
4) Conclusion

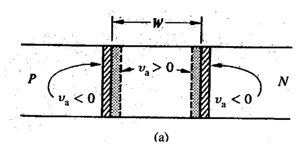
Junction Capacitance

Forward biased diode + AC signal

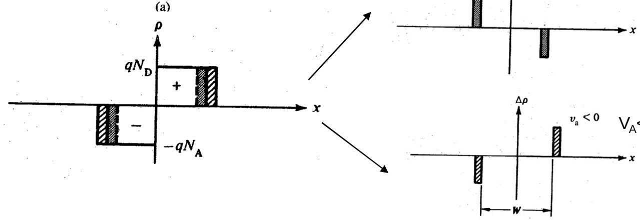




Depletion width modulation

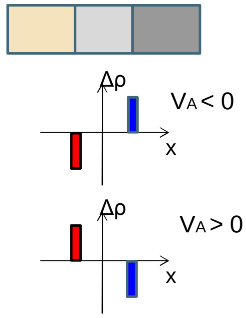


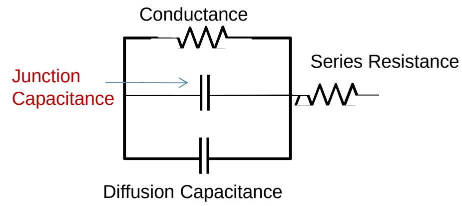
Charge modulation



Majority carrier effect

Majority Carrier Junction Capacitance

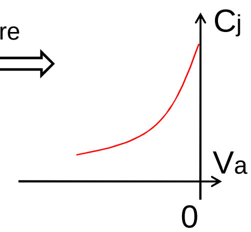




Majority Carrier Junction Capacitance

$$\boxed{C_J = \frac{K_s\varepsilon_0 A}{W_n + W_p} = \frac{K_s\varepsilon_0 A}{\sqrt{\left(\frac{2K_s\varepsilon_0}{qN_D} + \frac{2K_s\varepsilon_0}{qN_A}\right)(V_{bi} - V_A)}}}$$

Measure

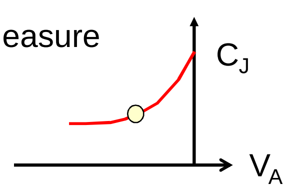


Measurement of Built-in Potential

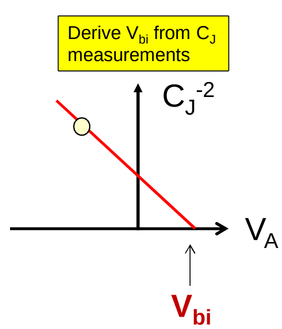
$$\boxed{\frac{1}{C_J^{\,2}} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}(V_{bi} - V_A)}$$

(Assume single sided p^+-n junction)

measure



plot

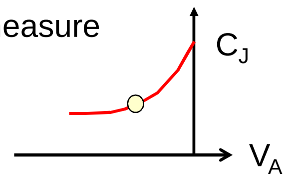


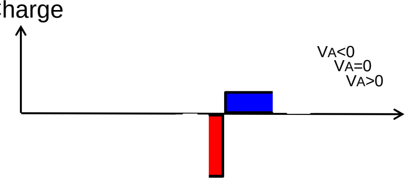
.. And Variable Doping

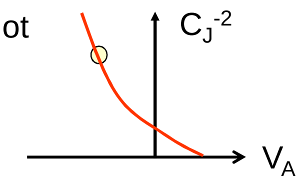
$$\frac{1}{C_J^{\,2}} \approx \frac{2}{qN_D(x)K_s\varepsilon_0 A^2}(V_{bi} - V_A)$$

$$\boxed{N_D(x) = \frac{2}{qK_s\varepsilon_0 A^2}\,\frac{1}{d\left(1/C_J^2\right)/dV_A}}$$

Measure doping concentration as a function of position







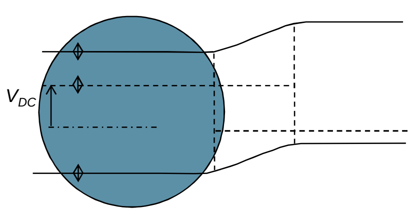
Dielectric Relaxation Time (majority side)

Majority side

$$J_n = qn\mu_N E + q\cancel{D_N}\nabla n$$

Neglect

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} - \cancel{R_n} + \cancel{G_n}$$



$$\frac{d(\Delta n)}{dt} = \frac{1}{q}\frac{d\left(qn\mu_N \mathcal{E}\right)}{dx} = N_D\mu_N\frac{d\mathcal{E}}{dx}$$

N-side

$$\frac{d\mathcal{E}}{dx} = \frac{q}{k_S\varepsilon_0}\left(\cancel{p} - n_0 - \Delta n + N_D - \cancel{N_A}\right)$$

$$\frac{d(\Delta n)}{dt} = -\frac{qN_D\mu_N}{k_S\varepsilon_0}\Delta n = -\frac{\sigma_0\Delta n}{k_S\varepsilon_0}$$

$$\boxed{\Delta n(t) = n_0 e^{-\frac{\sigma_0 t}{k_S\varepsilon_0}} = n_0 e^{-\frac{t}{\tau_d}}}$$

How long does it take for the signal to cross the junction?

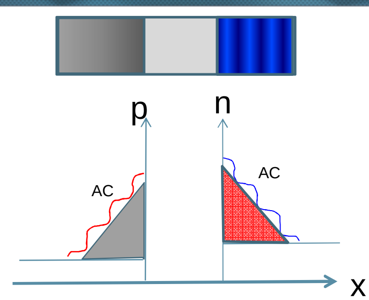
$$\boxed{\tau_d = \frac{K_s\varepsilon_0}{\sigma} \approx 0.1\ \text{ps}}$$

Very fast

Outline

1) Conductance and series resistance
2) Majority carrier junction capacitance
3) **Minority carrier diffusion capacitance**
4) Conclusion

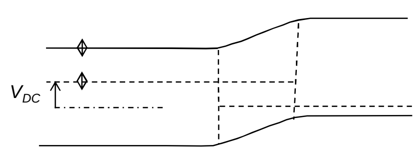
Diffusion Capacitance for Minority Carriers



Minority Carrier side

$$\mathbf{J}_N = qn\cancel{\mu_N}\mathcal{E} + qD_N\frac{dn}{dx}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + g_N$$



$$\frac{\partial\left(n_0 + \underbrace{\Delta n_{dc}}_{\text{DC}} + \underbrace{\Delta n_{ac} e^{j\omega t}}_{\text{AC}}\right)}{\partial t} = D_N\frac{d^2\left(n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t}\right)}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}}{\tau_n}$$

Diffusion Capacitance for Minority Carriers

$$\frac{\partial\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{\partial t} = D_N \frac{d^2\left(n_0+\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}\right)}{dx^2} - \frac{\Delta n_{dc}+\Delta n_{ac}e^{j\omega t}}{\tau_n}$$

$$\boxed{j\omega\Delta n_{ac}e^{j\omega t} = D_N \frac{d^2\Delta n_{dc}}{dx^2} + e^{j\omega t}\frac{d^2\Delta n_{ac}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} - e^{j\omega t}\frac{\Delta n_{ac}}{\tau_n}}$$

$$\boxed{\text{DC}: \quad 0 = D_N \frac{d^2\Delta n_{dc}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} \Rightarrow \Delta n_{dc} = Ae^{-\frac{x}{L_n}} + Be^{+\frac{x}{L_n}}}$$

$$\boxed{\text{AC}: \quad 0 = D_N \frac{d^2\Delta n_{ac}}{dx^2} - \left(j\omega\tau_n+1\right)\frac{\Delta n_{ac}}{\tau_n} \Rightarrow \Delta n_{ac} = Ce^{-\frac{x}{L_n^*}} + De^{+\frac{x}{L_n^*}} \rightarrow Ce^{-\frac{x}{L_n^*}}}$$

$$L_n^* = \sqrt{D_n\tau_n/\left(1+j\omega\tau_n\right)} \qquad \tau_n^* = \tau_n/\left(1+j\omega\tau_n\right)$$

AC Boundary Conditions

$$\Delta n_{dc}(x=0) = \frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}} - 1\right)$$

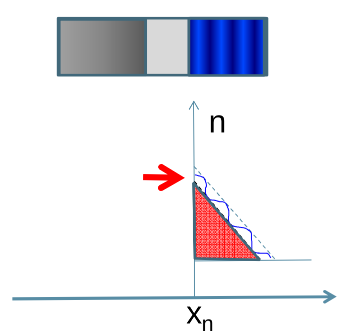
$$\left(\Delta n_{dc} + \Delta n_{ac}e^{j\omega t}\right) = \frac{n_i^2}{N_A + \Delta p_{ac}e^{j\omega t}}\left(e^{q\frac{V_{dc}+V_{ac}e^{j\omega t}}{kT}} - 1\right)$$

$$\left(\Delta n_{dc} + \Delta n_{ac}e^{j\omega t}\right) \approx \frac{n_i^2}{N_A}\left(e^{\frac{qV_{dc}}{kT}} e^{\frac{qV_{ac}e^{j\omega t}}{kT}} - 1\right)$$

Taylor expansion

$$\approx \frac{n_i^2}{N_A}\left\{e^{\frac{qV_{dc}}{kT}}\left(1 + \frac{qV_{ac}e^{j\omega t}}{kT}\right) - 1\right\}$$

$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}} = C$$



AC Current and Impedance

$$\Delta n_{ac}(x=0)=\frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}=C$$

$$\Delta n_{ac}(x)=Ce^{-\frac{x}{L_n*}}+De^{+\frac{x}{L_n*}}\rightarrow Ce^{-\frac{x}{L_n*}}$$

Finally…

$$\boxed{J_{ac}=-qD_n\left.\frac{d\Delta n_{ac}}{dx}\right|_{x=0}=\frac{qD_n}{L_n^*}\frac{qV_{ac}}{kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}}$$

AC Current

$$\boxed{Y_{ac}=\frac{J_{ac}}{V_{ac}}=\frac{q^2D_n}{L_n^*kT}\frac{n_i^2}{N_A}e^{\frac{qV_{dc}}{kT}}\equiv G_0\sqrt{1+j\omega\tau_n}}$$

AC Impedance

Diffusion Conductance and Capacitance

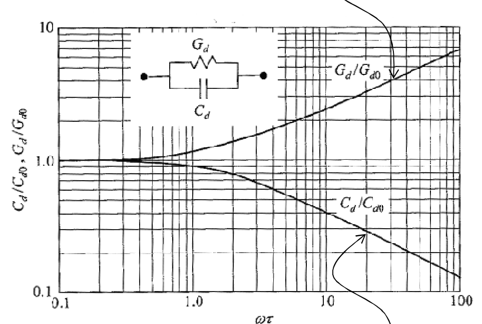
$$G_D\propto\sqrt{\omega}$$

$$Y_{ac}=G_D+j\omega C_D\equiv G_0\sqrt{1+j\omega\tau_n}$$

Separate in real & imaginary parts …

$$G_D=\frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}+1\right]^{1/2}$$

$$\omega C_D=\frac{G_0}{\sqrt{2}}\left[\sqrt{1+\omega^2\tau_n^{\ 2}}-1\right]^{1/2}$$



Product of G_D and C_D frequency-independent

$$C_D\propto 1/\sqrt{\omega}$$

1) Small signal response relevant for many analog applications.
2) Small signal parameters always refer to the DC operating conditions, as such the parameter changes with bias condition.
3) Important to distinguish between majority and minority carrier capacitance. Their relative importance depends on specific applications.

ECE606: Solid State Devices
p-n diode
Large Signal Response

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Topic Map

Digital Signals: switch on and off

	Equilibrium	**DC**	Small signal	**Large Signal**	Circuits
Diode					
Schottky					
BJT/HBT					
MOSFET					

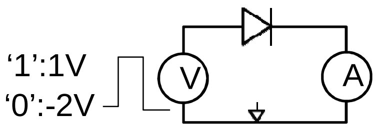
Outline

1) **Large signal response and charge control model**
2) Turn-off characteristics
3) Turn-on characteristics
4) Other applications
5) Conclusion

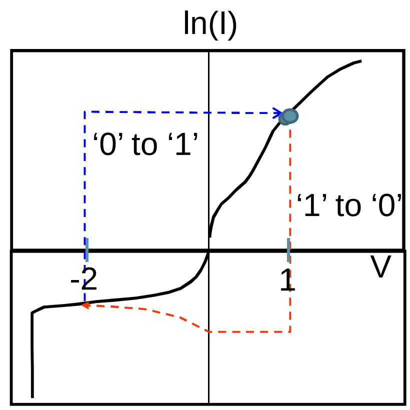
Ref. SDF, Chapter 8

Digital, Large Signal Applications

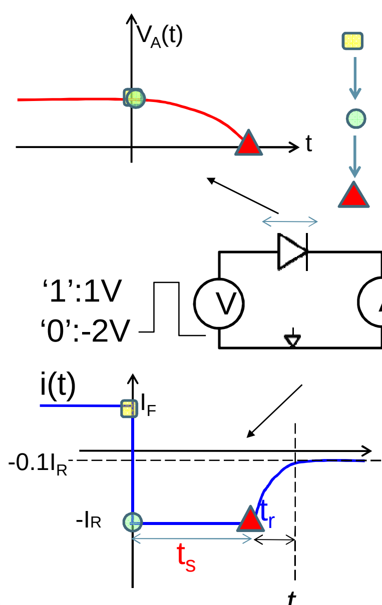
—— :If transition is slow, every point is in quasi-equlibrium
→treat them like DC



---:If transition is very fast



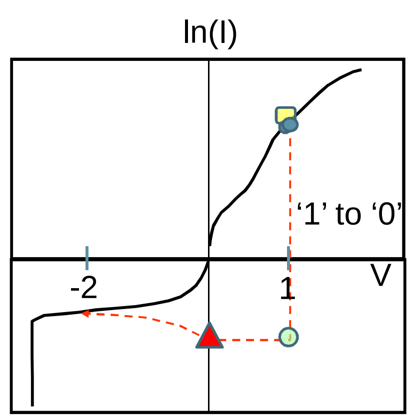
A Closer Look to Fast Transition



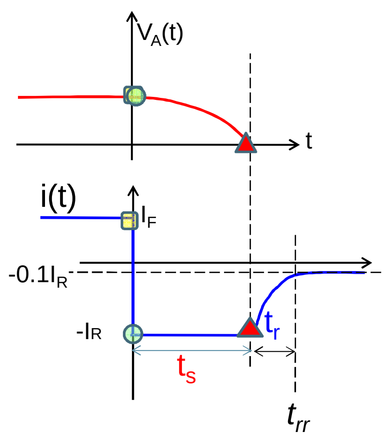
Before transition occur

constant voltage, current change from I_F to I_R

constant current, voltage change form 1V to 0V



Definitions



t_s … Charge storage time

t_r … Recovery time

t_{rr} .. Reverse recovery time

Continuity Equations

Full analytical solution impossible for large signal….

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N \mathcal{E} + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

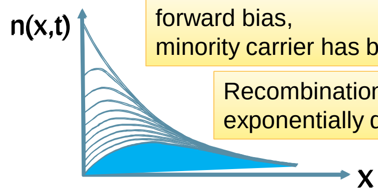
$$\mathbf{J}_P = qp\mu_P \mathcal{E} - qD_P \nabla p$$

$$\frac{\partial Q_n}{\partial t} = i_{n,diff} - \frac{Q_n}{\tau_n}$$

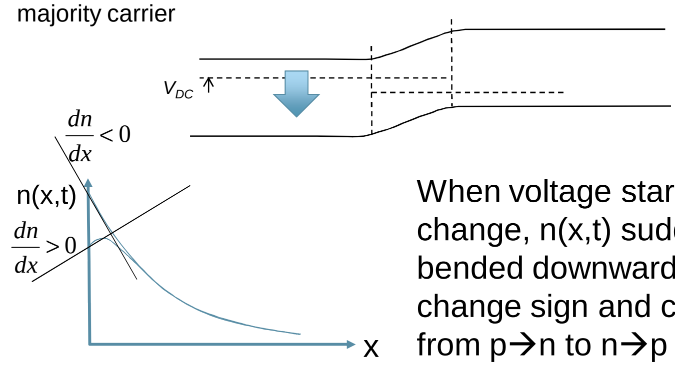
$$\frac{\partial Q_p}{\partial t} = i_{p,diff} - \frac{Q_p}{\tau_n}$$

Charge control equations:
Approximation when you have large transient response

How Does Current Flip Without Voltage Flipping?



Where did the charge go?
1. Back to the left-hand side
2. Recombine with the trap and the majority carrier



When voltage start to change, n(x,t) suddenly bended downwards, dn/dx change sign and current flip from p→n to n→p

Large Signal Charge Control Model

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{dJ_n}{dx} - r_N + \cancel{g_N}$$

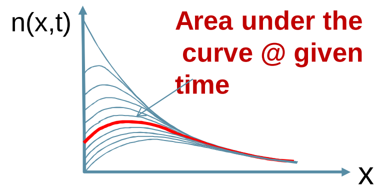
$$\downarrow$$

$$\frac{\partial(\Delta n)}{\partial t} = D_N \frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

minority carrier

$$\mathbf{J}_N = \cancel{qn\mu_N\mathcal{E}} + qD_N\frac{dn}{dx}$$

Large Signal Charge Control Model



$$\frac{\partial(\Delta n)}{\partial t} = D_N \frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

↓ x(qA), integrate

$$\int_0^{W_p} \frac{\partial(qA\Delta n)}{\partial t} dx = \int_0^{W_p} D_N \frac{d}{dx}\frac{d(qA\Delta n)}{dx} dx - \int_0^{W_p} \frac{qA\Delta n}{\tau_n} dx$$

Current going out | Current coming in

$$\frac{\partial Q}{\partial t} = D_N \left.\frac{d(qA\Delta n)}{dx}\right|_{x=W_p} - D_N \left.\frac{d(qA\Delta n)}{dx}\right|_{x=0} - \frac{Q}{\tau_n}$$

Recombination

$$Q \equiv \int_0^{W_p} (qA\Delta n)\, dx$$

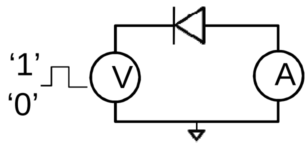
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

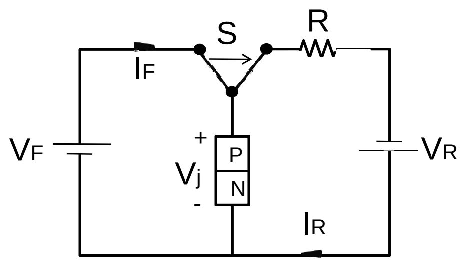
(Total charge that is building in)= (net electrons flowing in) – (recombination)

Outline

1) Large signal response and charge control model
2) **Turn-off characteristics**
3) Turn-on characteristics
4) Other applications
5) Conclusion

Turn-off Characteristics: Determine (t_s)

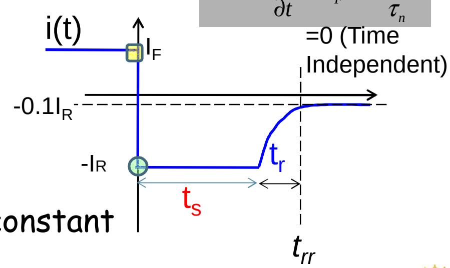




$$t<0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(0^-)}{\tau_n}$$

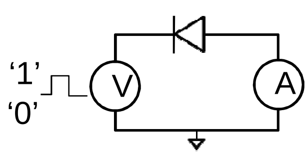
=0 (Time Independent)

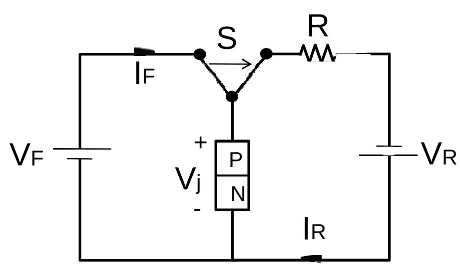
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$



Why does the current remain constant even with t>0?

Boundary Condition



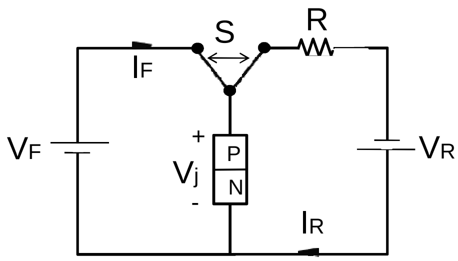


$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

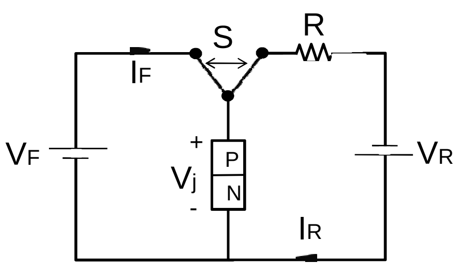
$$t<0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(0^-)}{\tau_n} = 0$$

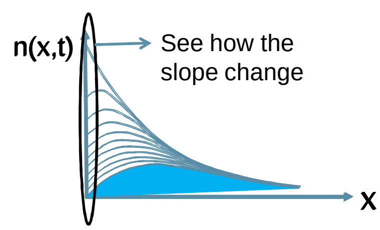
$$Q(0^-) = I_F \tau_n = Q(0^+)$$

Note. For a capacitor, voltage can not change instantly. So charge can not change instantly ....



Since V_j can't be larger than the band gap, which is smaller than V_R, the diode will be forced to supply the negative current I_R





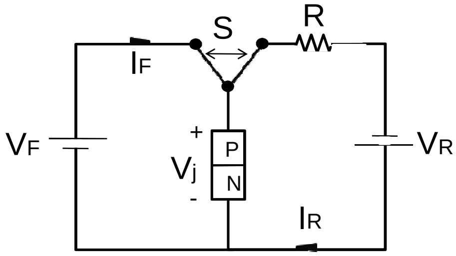
Approximately constant!

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n} \longrightarrow t > 0 \quad \frac{\partial Q}{\partial t} = -I_R - \frac{Q}{\tau_n}$$

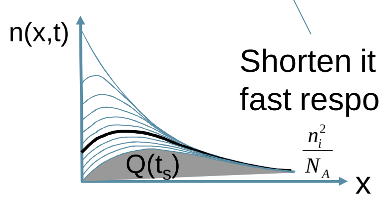
$$\int_{Q(0+)}^{Q(t_s)} \frac{dQ}{-\left(I_R + \frac{Q}{\tau_n}\right)} = \int_0^{t_s} dt$$

$$t_s = \tau_n \ln \frac{I_R + Q(0^+)/\tau_n}{I_R + Q(t_s)/\tau_n}$$

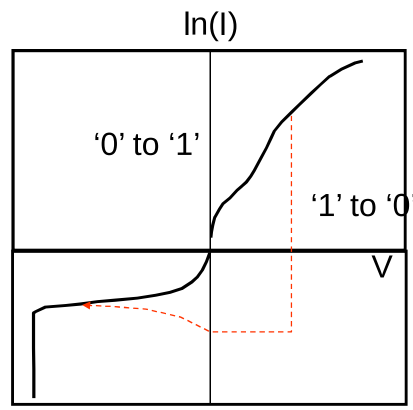
Storage Time



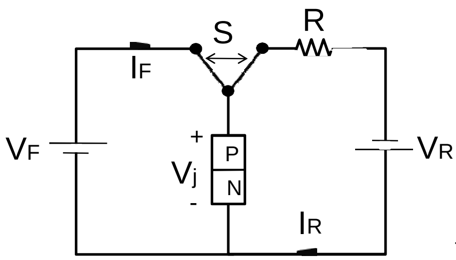
$$t_s = \tau_n \ln \frac{I_R + Q(0^+)/\tau_n}{I_R + \cancel{Q(t_s)}/\tau_n} = \tau_n \ln \frac{I_R + I_F}{I_R}$$

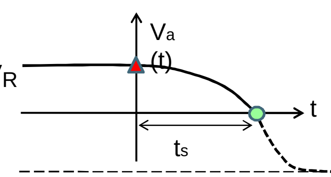


Shorten it for fast response!



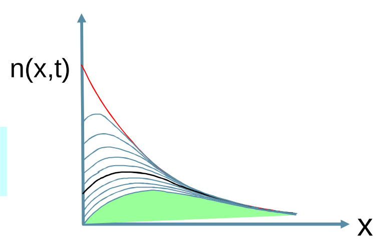
Turn-off *Voltage* Transient





$$\upsilon_A(t) = \frac{kT}{q} \ln \frac{n_p(0,t)}{n_{po}}$$

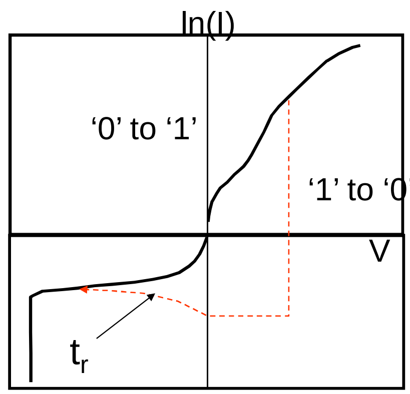
$$Q_n(t) = \tau_p \ln\left(-I_R + (I_R + I_F)e^{-t/\tau_p}\right)$$



Allows easy calculation of $n_p(0,t)$.

Recovery Time

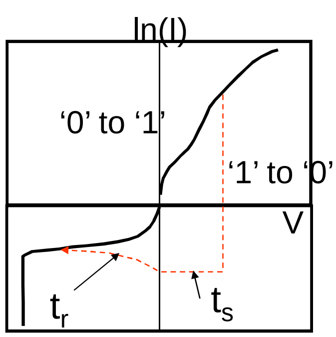
$$erf\sqrt{\frac{t_r}{\tau_p}}+\frac{e^{-\frac{t_r}{\tau_p}}}{\sqrt{\pi\frac{t_r}{\tau_p}}}=1+0.1\frac{I_F}{I_R}$$



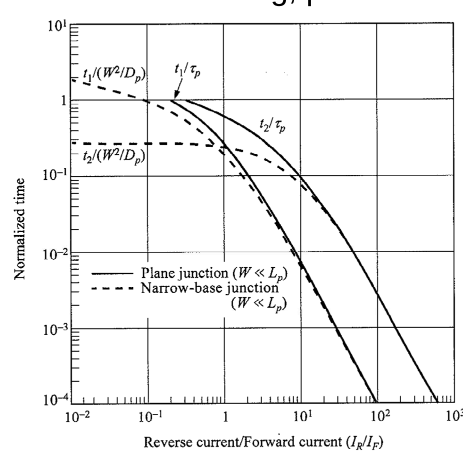
Useful formula …

$$erf\,(\sqrt{x})=\left[1-\mathrm{e}^{-x\frac{1.27+0.15x}{1+0.15x}}\right]^{0.5}$$

Recovery Time



Ref. Sze/Ng, p. 117

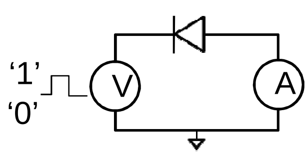


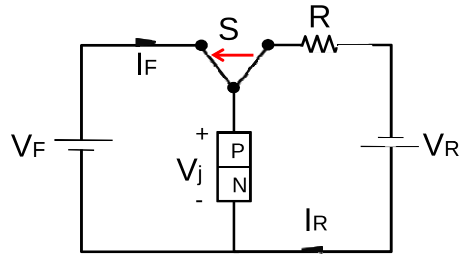
$$t_{rr}=t_r+t_f\approx\left[\begin{array}{ll}\dfrac{W_p^2}{2D_n}\left(\dfrac{I_R}{I_F}\right)^{-2} & (W_p\ll L_n)\\ \dfrac{\tau_p}{2}\left(\dfrac{I_R}{I_F}\right)^{-2} & (W_p\gg L_n)\end{array}\right.$$

Outline

1) Large signal response and charge control model
2) Turn-off characteristics
3) **Turn-on characteristics**
4) Other applications
5) Conclusion

Turn-on Characteristics: Boundary Condition



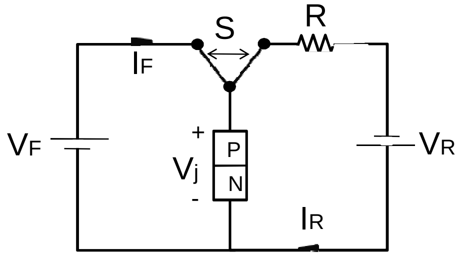


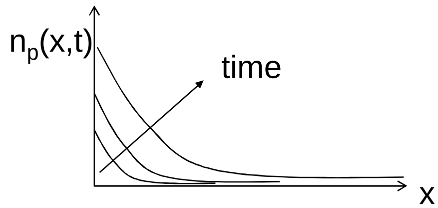
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t \to \infty \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(t=\infty)}{\tau_n}$$

$$Q(t=\infty) = I_F \tau_n$$

Turn-on Characteristics





$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t > 0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q}{\tau_n}$$

$$Q(t) = Q(t \to \infty)\left(1 - e^{-\frac{t}{\tau_n}}\right) = I_F \tau_n \left(1 - e^{-\frac{t}{\tau_n}}\right)$$

Check:
$Q(t=0)=0$
$Q(t \to \infty) = I_F \tau_n$

Outline

1) large signal response and charge control model
2) turn-off characteristics
3) turn-on characteristics
4) **other applications**
5) conclusion

Diffusion Time

Electrons get scattered to the other side, the average time is…

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$\downarrow$$

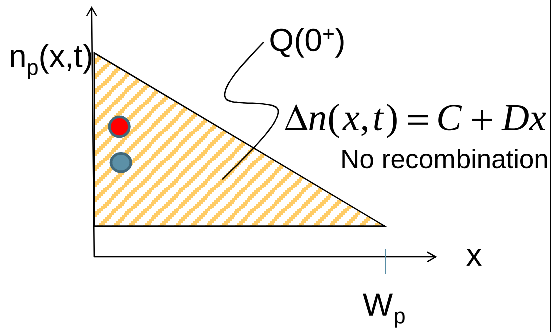
$$\frac{Q(0^+) - Q(t=\infty)}{\tau_{diff}} = i_{diff}$$

$$\downarrow$$

$$\tau_{diff} = \frac{Q(0^+)}{i_{diff}} = \frac{q\left[\frac{\Delta n_p(0)}{2}\right]W_p}{qD_n\frac{\Delta n_p(0)}{W_p}} = \frac{W_p^2}{2D_n} \sim \frac{1}{2} \times \frac{W_p}{(D_n/W_p)}$$

Only half of them goes to the right

Diffusion velocity



Two line derivation of Steady State Diode Current

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$\tau_n \rightarrow \tau_{diff}$$

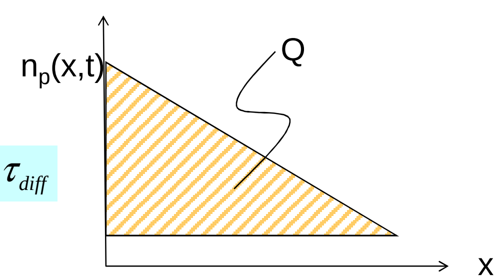
$$\downarrow$$

$$i_{diff} = \frac{Q}{\tau_{diff}}$$

$$\downarrow$$

$$i_{diff} = \frac{Q}{\tau_{diff}} = \frac{q \times \frac{1}{2}\frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right) \times W_p}{\frac{W_p^2}{2D_n}} = q\frac{D_n}{W_p}\frac{n_i^2}{N_A}\left(e^{qV_A\beta} - 1\right)$$

Exact Expression!



Large signal response of devices of great importance for digital applications.

Analytical solution of partial differential equation often difficult (if not impossible), therefore approximate methods like Charge-control approximation often help simplify the solution and still provide a great deal of insight into the dynamics of switching operation.

Be careful in using the boundary condition which is often dictated by external circuit conditions.