

# ECE606: Solid State Devices

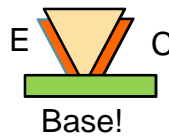
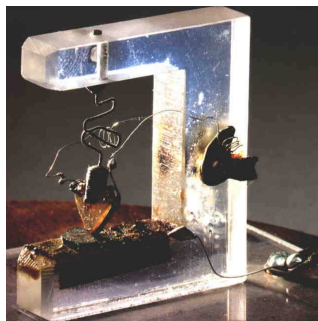
## Lecture 18

### Bipolar Transistors

#### a) Introduction

#### b) Design (I)

Gerhard Klimeck  
gekco@purdue.edu



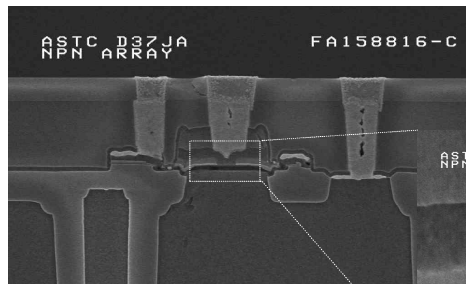
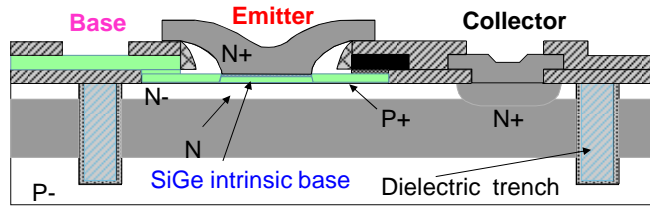
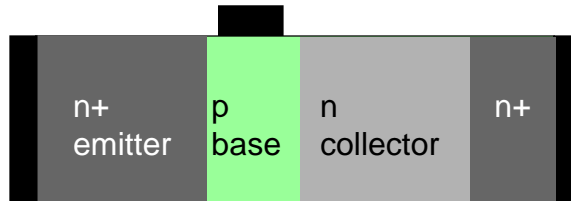
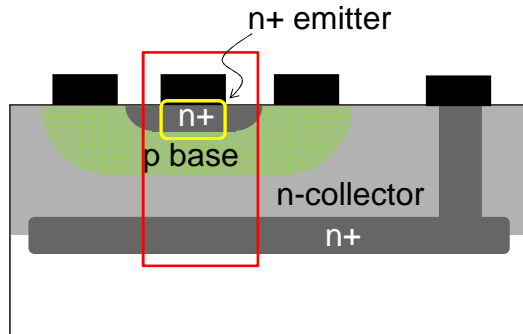
Point contact **Germanium** transistor

Ralph Bray from Purdue missed the invention of transistors.

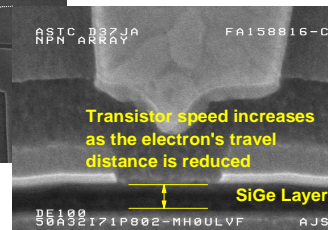
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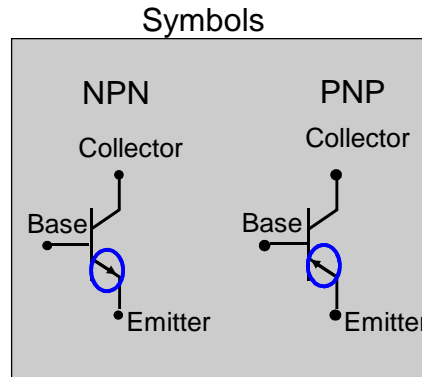
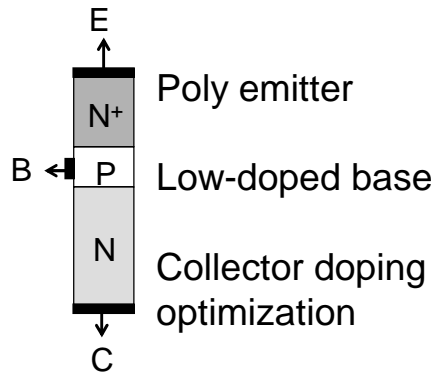
Transistor research was also in advanced stages in Europe (radar)

**Double  
Diffused BJT**



Why do we need all these design?






$$I_C + I_B + I_E = 0 \quad (\text{DC})$$

$$V_{EB} + V_{BC} + V_{CE} = 0$$

- 1) **Equilibrium and forward band-diagram**
- 2) Currents in bipolar junction transistors
- 3) Eber's Moll model
- 4) Intermediate Summary
- 5) Current gain in BJTs
- 6) Considerations for base doping
- 7) Considerations for collector doping
- 8) Conclusions

**REF:** SDF, Chapter 10

|          | Equilibrium   | DC | Small signal | Large Signal | Circuits |
|----------|---|----|--------------|--------------|----------|
| Diode    |   |    |              |              |          |
| Schottky |   |    |              |              |          |
| BJT/HBT  |  |    |              |              |          |
| MOS      |   |    |              |              |          |

$$\nabla \cdot D = q(p - n + N_D^+ - N_A^-)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

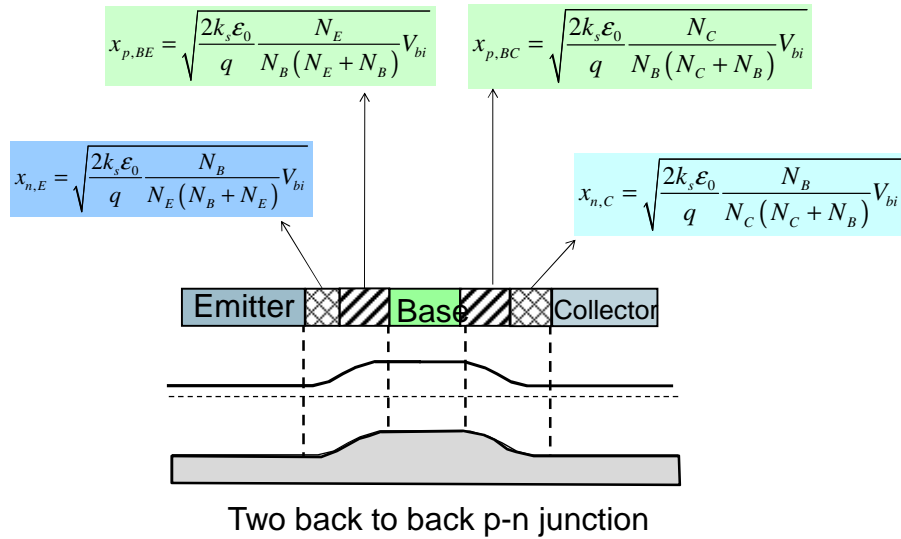
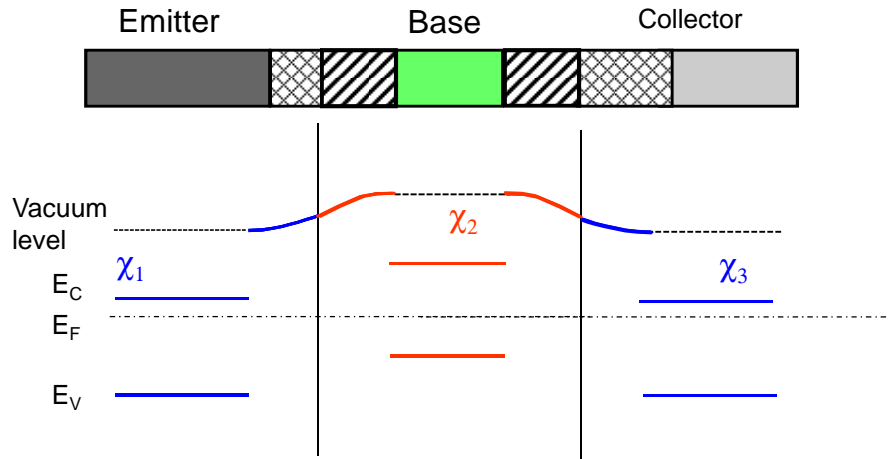
$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

← **Equilibrium**

DC  $dn/dt=0$   
 Small signal  $dn/dt \sim j\omega n$   
 Transient --- Charge control model


NPN homojunction BJT



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|                | Equilibrium | DC  | Small signal | Large Signal | Circuits |
|----------------|-------------|---|--------------|--------------|----------|
| Diode          |             |   |              |              |          |
| Schottky       |             |   |              |              |          |
| <b>BJT/HBT</b> |             |  |              |              |          |
| MOS            |             |   |              |              |          |



$$\nabla \cdot D = q(p - n + N_D^+ - N_A^-)$$

← **Non-equilibrium**

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

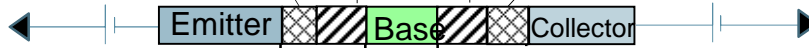
DC  $dn/dt=0$   
 Small signal  $dn/dt \sim j\omega n$   
 Transient --- Charge control model

$$x_{p, BE} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_E}{N_B(N_E + N_B)} (V_{bi} - V_{EB})}$$

$$x_{p, BC} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_C}{N_B(N_C + N_B)} (V_{bi} - V_{CB})}$$

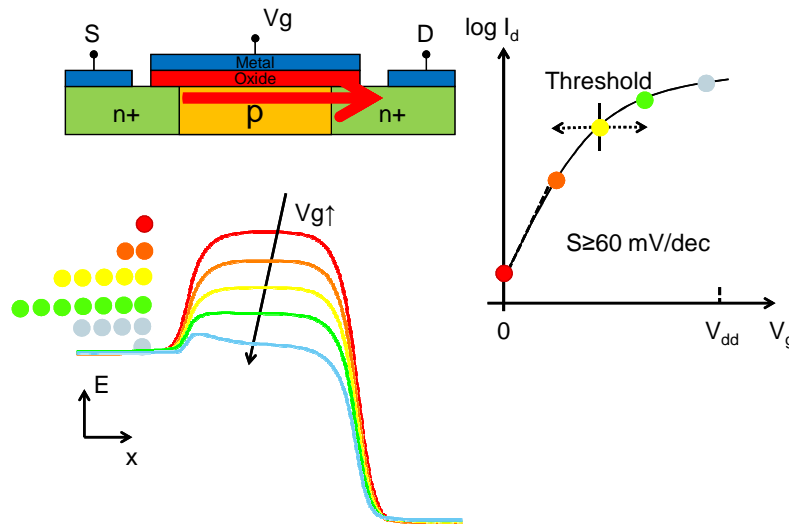
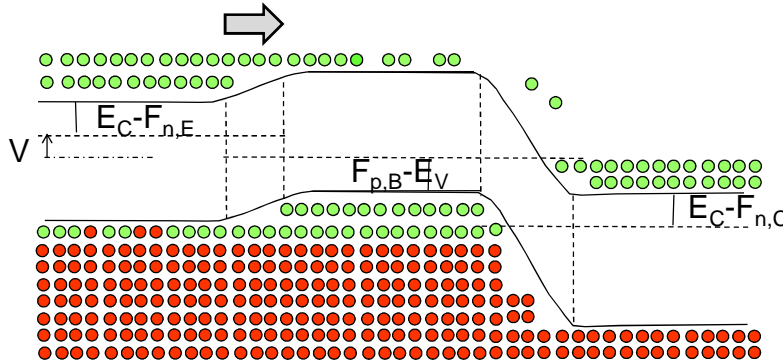
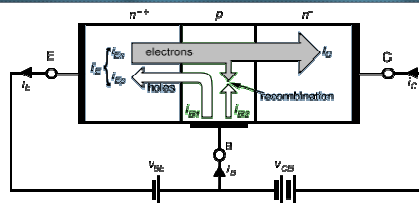
$$x_{n, E} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_B}{N_E(N_B + N_E)} (V_{bi} - V_{EB})}$$

$$x_{n, C} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_B}{N_C(N_C + N_B)} (V_{bi} - V_{CB})}$$

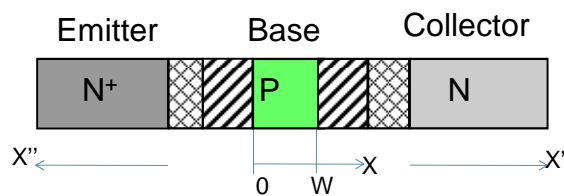
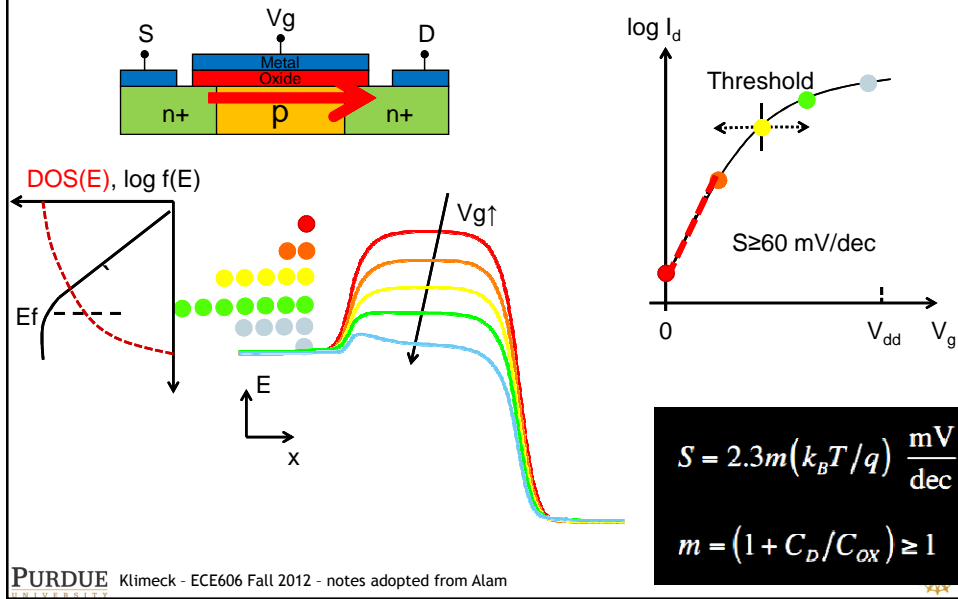


Assume current flow is small...  
 fermi level is flat

Input small amount of holes results in large amount of electron output







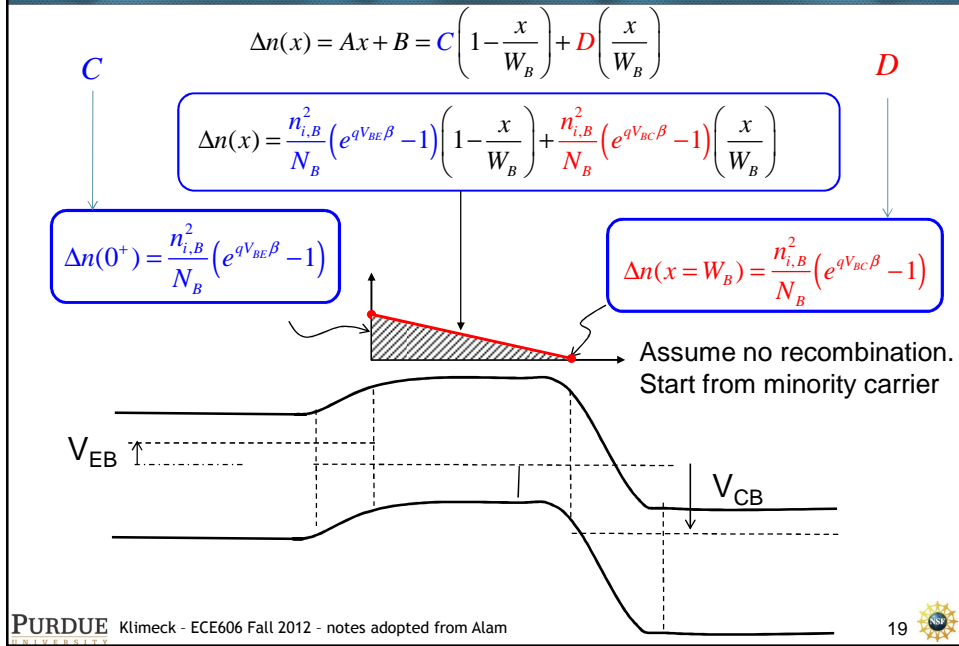
$$N_E = N_{D,E}, \dots, N_B = N_{A,B}, \dots, N_C = N_{D,C}$$

$$D_E = D_P, \dots, D_B = D_N, \dots, D_C = D_P$$

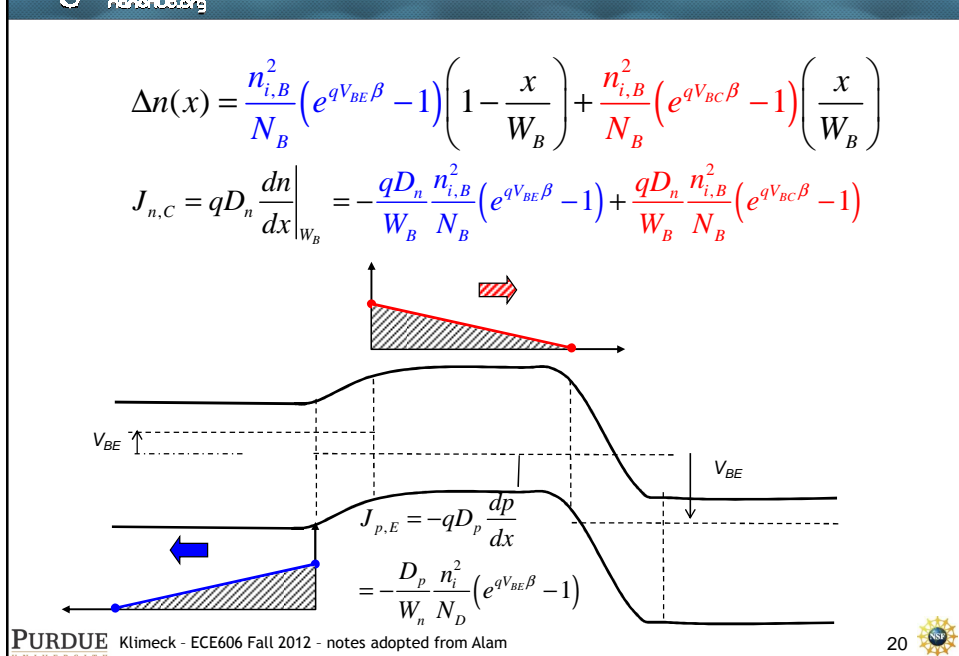
$$n_{E0} = n_{p0}, \dots, p_{B0} = p_{n0}, \dots, n_{C0} = n_{p0}$$

Doping  
Minority carrier diffusion  
Majority carriers

### Carrier Distribution in Base

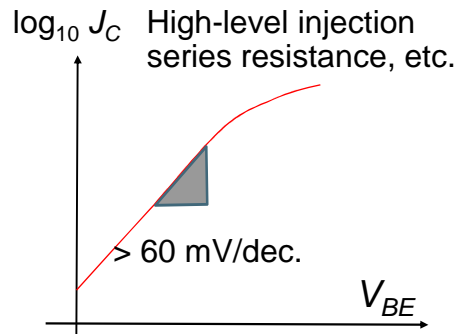
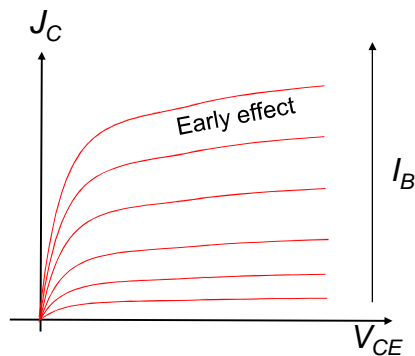


### Collector and Emitter Electron Current



Normal, Active Region  
 EB: Forward biased  
 BC: Reverse biased

$$J_{n,C} = -\frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BE}\beta} - 1) + \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BC}\beta} - 1)$$



$W_B$  is not independent of bias  
 $\Rightarrow$  Early Effect

same physics of diode, rollover



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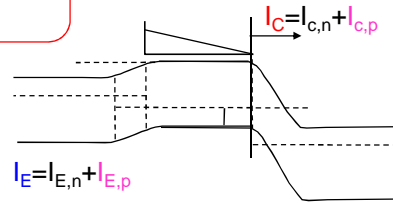
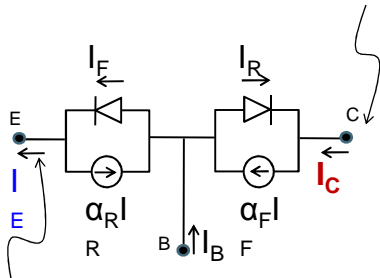
**REF:** SDF, Chapter 10



Hole diffusion in collector

$$I_C = -A \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BE}\beta} - 1) + A \left[ \frac{qD_n n_{i,B}^2}{W_B N_B} + \frac{qD_p n_{i,C}^2}{W_C N_C} \right] (e^{qV_{BC}\beta} - 1)$$

$$\equiv \alpha_F I_{F0} (e^{qV_{BE}\beta} - 1) - I_{R0} (e^{qV_{BC}\beta} - 1)$$



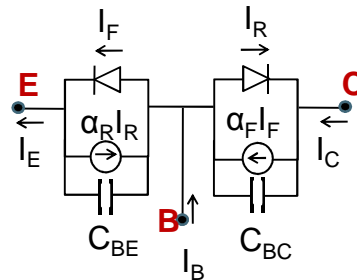
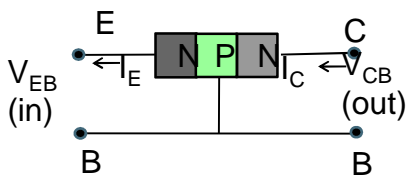
$$I_F = I_{F0} (e^{qV_{BE}\beta} - 1)$$

$$I_R = I_{R0} (e^{qV_{BC}\beta} - 1)$$

Temperature dependent

$$I_E = -A \left[ \frac{qD_p n_{i,E}^2}{W_E N_E} + \frac{qD_n n_{i,B}^2}{W_B N_B} \right] (e^{qV_{BE}\beta} - 1) + A \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BC}\beta} - 1)$$

$$\equiv I_{F0} (e^{qV_{BE}\beta} - 1) - \alpha_R I_{R0} (e^{qV_{BC}\beta} - 1)$$

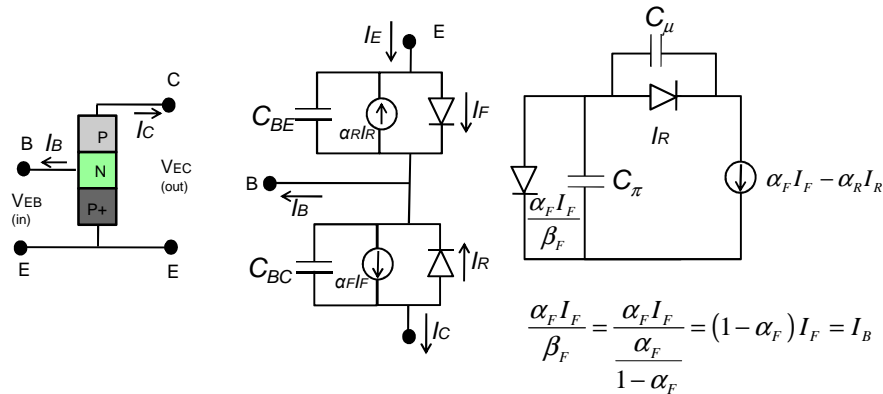


Junction capacitance and diffusion capacitance

How would the model change if this was a Schottky barrier BJT?

The original transistor was a metal/ semicond / metal device  
No minority carriers, no diffusion capacitance but the "rest" about the same.

Common base configuration provides power gain, but no current gain.  
=> Emitter and collector current are identical => no current gain  
=> Collector current  $I_C$  can be driven through large resistor => power gain  
Is there another configuration that can deliver current gain?



This is a practice problem ...



- The physics of BJT is most easily understood with reference to the physics of junction diodes.
- The equations can be encapsulated in simple equivalent circuit appropriate for dc, ac, and large signal applications.
- Design of transistors is far more complicated than this simple model suggests => the next lecture elements
- For a terrific and interesting history of invention of the bipolar transistor, read the book "Crystal Fire".



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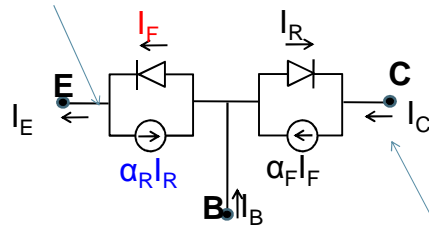
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$$I_B = A \frac{qD_p n_{i,E}^2}{W_E N_E} (e^{qV_{BE}\beta} - 1) + A \frac{qD_p n_{i,C}^2}{W_C N_C} (e^{qV_{BC}\beta} - 1)$$

$$I_E = -A_E \left( \frac{qD_n n_{i,B}^2}{W_B N_B} + \frac{qD_p n_{i,E}^2}{W_E N_E} \right) (e^{qV_{BE}\beta} - 1) + A_E \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BC}\beta} - 1)$$

$$= I_{F0} (e^{qV_{BE}\beta} - 1) - \alpha_R I_{R0} (e^{qV_{BC}\beta} - 1)$$



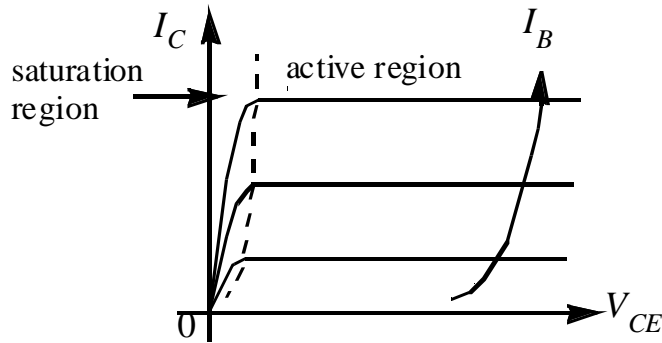
$$I_F = I_{F0} (e^{qV_{BE}\beta} - 1)$$

$$I_R = I_{R0} (e^{qV_{BC}\beta} - 1)$$

$$I_C = -A_C \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BE}\beta} - 1) + A_C \left[ \frac{qD_n n_{i,B}^2}{W_B N_B} + \frac{qD_n n_{i,C}^2}{W_C N_C} \right] (e^{qV_{BC}\beta} - 1)$$

$$= \alpha_F I_{F0} (e^{qV_{BE}\beta} - 1) - I_{R0} (e^{qV_{BC}\beta} - 1)$$

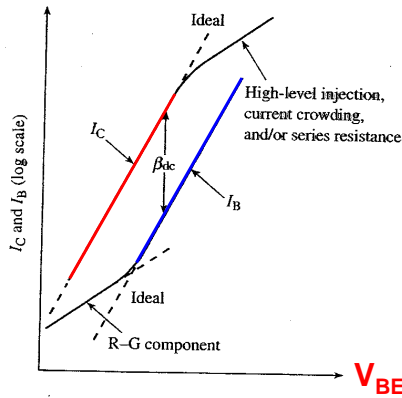




The Ebers-Moll model describes both the active and the saturation regions of BJT operation.

- The simultaneous plot of collector and base current vs. the base-emitter voltage on a semi-logarithmic scale is known as a Gummel Plot.
- This plot is extremely useful in device characterization because it reflects on the quality of the emitter-base junction while the base-collector bias is kept at a constant.
- A number of other device parameters can be ascertained either quantitatively or qualitatively directly from the Gummel plot because of its semi-logarithmic nature
  - For example the d.c gain  $\beta$ , base and collector ideality factors, series resistances and leakage currents.

$$\frac{I_C}{A} \approx -\frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BE}/kT} - 1) + \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BC}/kT} - 1)$$



$$\frac{I_B}{A} = \frac{qD_p n_{i,E}^2}{W_E N_E} (e^{qV_{BE}/kT} - 1)$$

$$\beta_{DC} = \frac{I_C}{I_B}$$

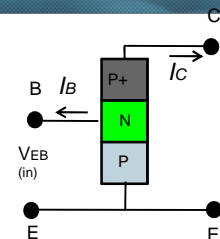
$\beta_{DC} \rightarrow$  Common emitter Current Gain

Common Emitter current gain ..

$$\beta_{DC} = \frac{I_C}{I_B}$$

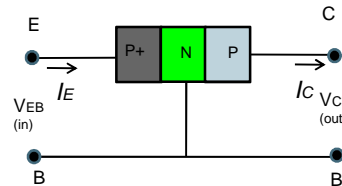
$$\frac{\frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BE}/kT} - 1) + \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV_{BC}/kT} - 1)}{\frac{qD_n n_{i,E}^2}{W_E N_E} (e^{qV_{BE}/kT} - 1)} \approx \frac{D_n W_E n_{i,B}^2 N_E}{W_B D_p n_{i,E}^2 N_B}$$

Will examine



Common Base current gain ..

$$\alpha_{DC} = \frac{I_C}{I_E} \Rightarrow \beta_{DC} = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{\alpha_{DC}}{1 - \alpha_{DC}}$$

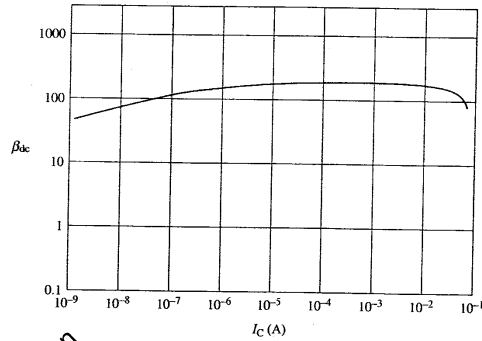


DC transfer gain

Properties are related – (transistor did not change ☺)



$$\beta_{DC} \approx \frac{D_n W_E n_{i,B}^2 N_E}{W_B D_p n_{i,E}^2 N_B}$$



Base recombination  
=> roll-off

High injection  
collector current  
=> roll-off  
Base current  
does not roll off

For a given Emitter length

$$\beta_{DC} \approx \frac{D_n W_E n_{i,B}^2 N_E}{W_B D_p n_{i,E}^2 N_B}$$

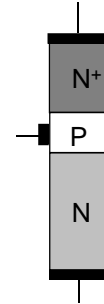
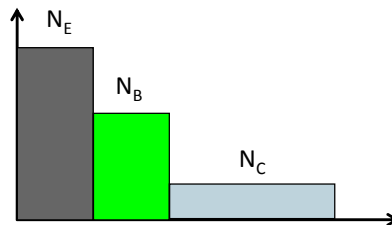
~1, same material  
primarily determined  
by bandgap

Make-Base short ...  
(few mm in 1950s, 200 A now)  
Want high gradient of carrier density

Emitter doping higher  
than Base doping

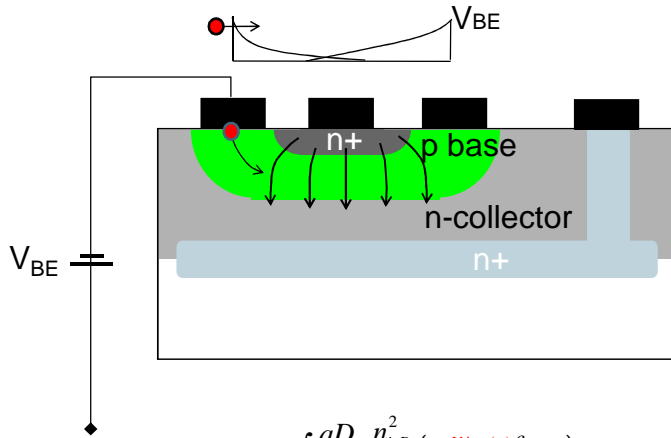
Base doping hard to control  
Emitter doping easier

$$\beta_{DC} \approx \frac{D_n}{D_p} \frac{W_E}{W_B} \frac{n_{i,B}^2}{n_{i,E}^2} \frac{N_E}{N_B}$$



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**what's wrong with the previous recipe?**
- 7) Considerations for collector doping
- 8) Conclusions

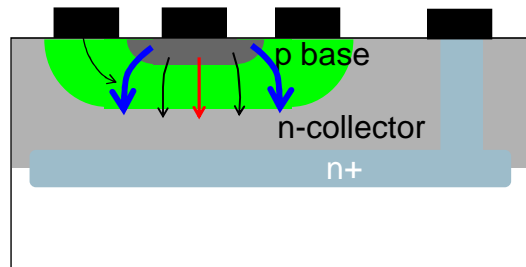
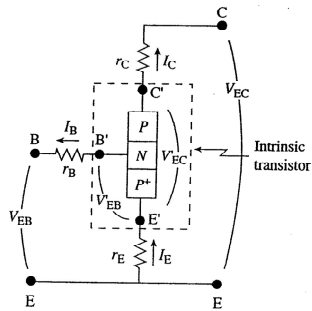
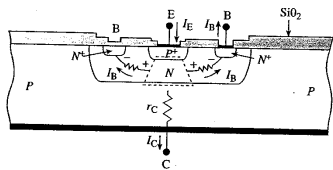




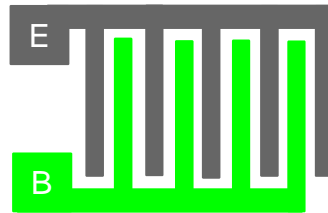
Double diffused junction configuration: Emitter doping must compensate / overcome the base doping  
 Low doping in base => resistance along the current path  
 => potential drop  
 => Determines the injection  
 => Spatially dependent  
 => More current in the corners

$$\beta = \frac{I_C}{I_B} = \frac{\int J_C(x) dx}{\int J_B(x) dx} = \frac{\int \frac{qD_n n_{i,B}^2}{W_B N_B} (e^{qV'_{BE}(x)\beta} - 1) dx}{\int \frac{qD_p n_{i,E}^2}{W_E N_E} (e^{qV'_{BE}(x)\beta} - 1) dx}$$

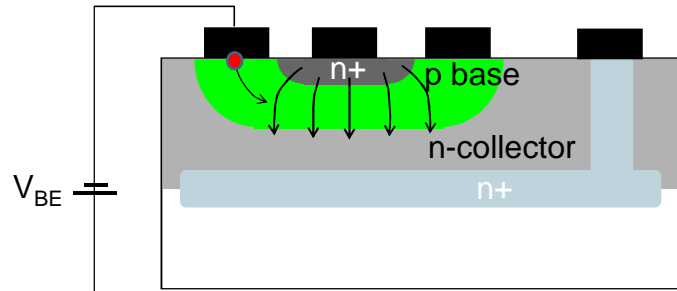
Sketches from text book



Non-uniform current inefficient  
 High current at the edge can cause burn-out

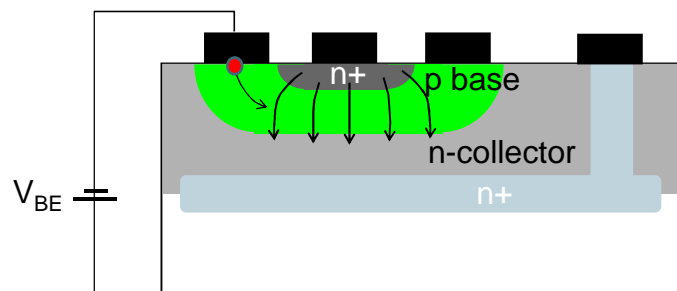


Interdigitated designs for almost all high power transistors (E-B distance minimized)



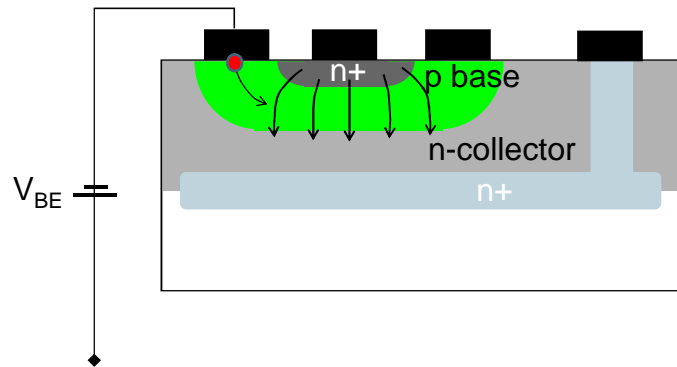
We talked about how low doping for the base enhances the current gain.  
But there is a potential downside to this approach

If the base doping is kept to small values, it will have a high resistance: Lesser ability to conduct means higher resistance



Non-zero base resistance results in a lateral potential difference under the emitter region

For an n-p-n transistor as shown, the potential decreases from edge of the emitter towards the centre (the emitter is highly doped and can be considered an equipotential region)



The number of electrons injected from emitter to base is exponentially dependent on base-emitter voltage

With the lateral drop in the voltage in the base between the edge and centre of emitter, more carriers will be injected at the edge than the emitter centre.



Key facts:

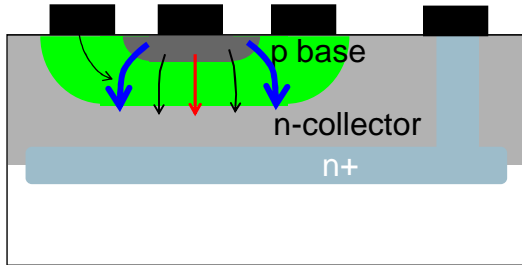
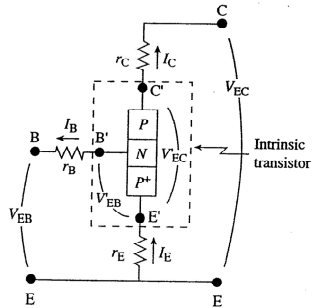
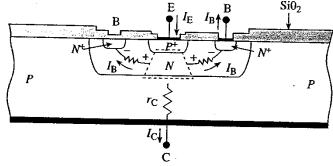
1. Current crowding is due to 2D nature of BJTs
2. It is a function of the doping concentration
3. As doping concentration increases, resistivity decreases
  - Consequence: Current gain goes smaller → Emitter current injection efficiency decreases

The larger current density near the emitter may cause localized heating and high injection effects

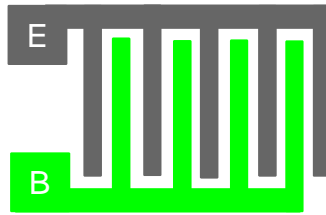
Possible Solution: Emitter widths are fabricated with an inter-digitated design → Many narrow emitters connected in parallel to achieve the required emitter area



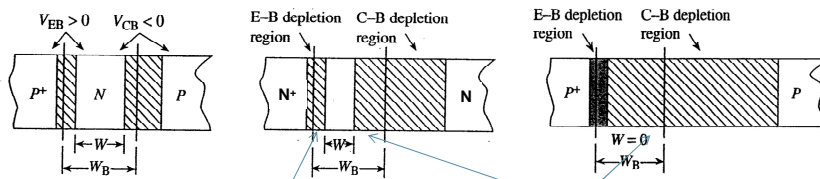
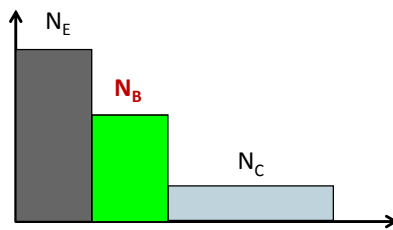
Sketches from text book



Non-uniform current inefficient  
High current at the edge can cause burn-out



Interdigitated designs for almost all high power transistors (E-B distance minimized)



$$x_{p, BE} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_E}{N_B (N_E + N_B)}} (V_{bi} - V_{BE})$$

$$x_{p, BC} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_C}{N_B (N_C + N_B)}} (V_{bi} - V_{BC})$$

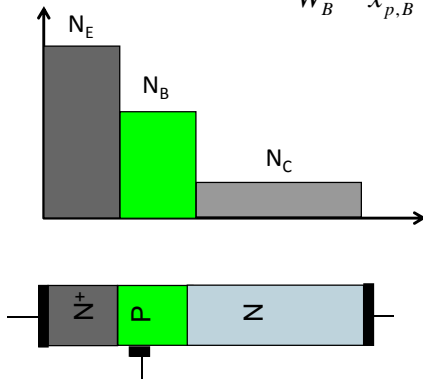
Low base doping is not a good idea!



## Problem of low Base-doping: Base Width Modulation

Electrical base region is smaller than the metallurgical region!

$$\beta_{DC} \approx \frac{D_n}{W_B - x_{p,B} - x_{p,c}} \frac{W_E n_{i,B}^2 N_E}{D_p n_{i,E}^2 N_B}$$



$$x_{p,BE} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_E}{N_B(N_E + N_B)} (V_{bi} - V_{BE})}$$

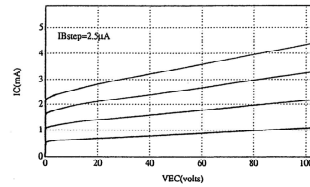
$$x_{p,BC} = \sqrt{\frac{2k_s \epsilon_0}{q} \frac{N_C}{N_B(N_C + N_B)} (V_{bi} - V_{BC})}$$

Gain depends on collector voltage (bad) ...  
Depletion region width modulation



## Problem of Low Base-doping: Early Voltage

$$\beta_{DC} \approx \frac{D_n}{W_B - x_{p,B} - x_{p,c}} \frac{W_E n_{i,B}^2 N_E}{D_p n_{i,E}^2 N_B}$$

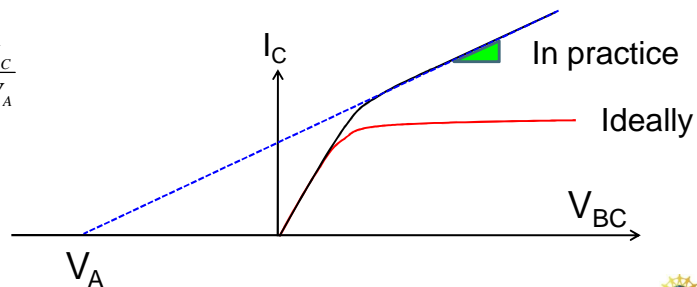


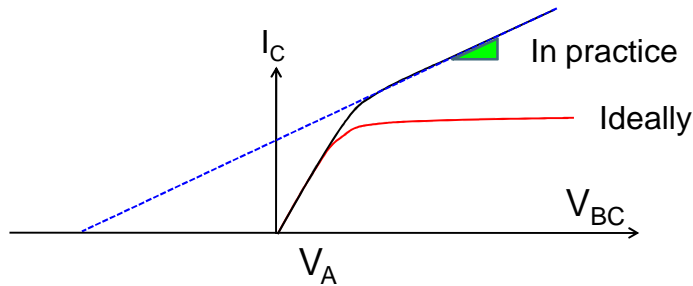
$$I_{n,C} = -\frac{qD_n n_{i,B}^2}{W_B' N_B} (e^{(qV_{BE}/kT)} - 1) + \frac{qD_n n_{i,B}^2}{W_B' N_B} (e^{(qV_{BC}/kT)} - 1)$$

$$\frac{dI_C}{dV_{BC}} = \frac{I_C}{V_{BC} + V_A} \approx \frac{I_C}{V_A}$$

$V_{BC}$  about 1V  
 $V_A$  ideally infinity

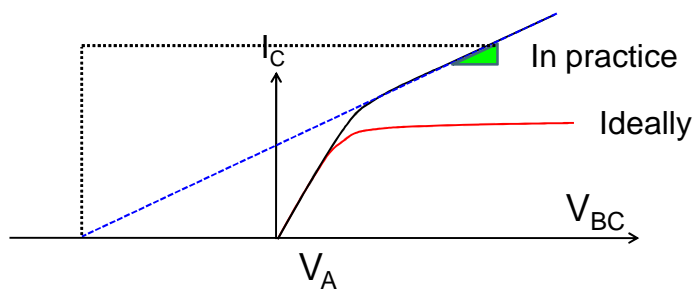
Jim Early  
device pioneer





- The collector current depends on  $V_{CE}$ :
- For a fixed value of  $V_{BE}$ , as  $V_{CE}$  increases, the reverse bias on the collector-base junction increases, hence the width of the depletion region increases.
  - The quasi-neutral base width decreases  $\rightarrow$  collector current increases.

Due to the Early effect, collector current increases with increasing  $V_{CE}$ , for a fixed value of  $V_{BE}$ .



- The Early voltage is obtained by drawing a line tangential to the transistor  $I$ - $V$  characteristic at the point of interest.
- The Early voltage equals the horizontal distance between the point chosen on the  $I$ - $V$  characteristics and the intersection between the tangential line and the horizontal axis.
- Early voltage is indicated on the figure by the horizontal dotted line





$$\frac{dI_C}{dV_{BC}} = \frac{I_C}{V_{BC} + V_A} \approx \frac{I_C}{V_A}$$

$$\begin{aligned} \frac{dI_C}{dV_{BC}} &= \frac{dI_C}{d(qN_B W_B)} \frac{d(qN_B W_B)}{dV_{BC}} \\ &= \frac{1}{qN_B} \left( \frac{dI_C}{dW_B} \right) \left[ \frac{dQ_B}{dV_{BC}} \right] \\ &= -\frac{1}{qN_B} \left( \frac{I_C}{W_B} \right) C_{CB} \end{aligned}$$

$$\begin{aligned} -\frac{C_{CB}}{qN_B} \frac{I_C}{W_B} &\approx \frac{I_C}{V_A} \\ \Rightarrow V_A &= -\frac{qN_B W_B}{C_{CB}} \rightarrow \infty \end{aligned}$$

Need higher  $N_B$  and  $W_B$  or ...

$$\begin{aligned} I_C &= \frac{qD_n}{W_B} \frac{n_{i,B}^2}{N_B} (e^{qV_{BE}\beta} - 1) + \frac{qD_n}{W_B} \frac{n_{i,B}^2}{N_B} (e^{qV_{BC}\beta} - 1) \\ &= \frac{\zeta}{W_B} \end{aligned}$$

$$\begin{aligned} \frac{dI_C}{dW_B} &= \frac{d}{dW_B} \left( \frac{\zeta}{W_B} \right) = -\frac{\zeta}{W_B^2} \\ &= -\frac{I_C}{W_B} \end{aligned}$$

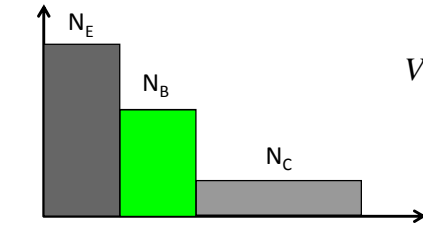


- 1) Equilibrium and forward band-diagram
- 2) Currents in bipolar junction transistors
- 3) Eber's Moll model
- 4) Intermediate Summary
- 5) Current gain in BJTs
- 6) Considerations for base doping
- 7) Considerations for collector doping**
- 8) Conclusions

**REF:** SDF, Chapter 10

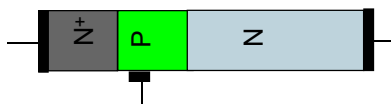


$$\beta \approx \frac{D_n}{W_B - x_{p,B} - x_{p,C}} \frac{W_E \cancel{N_B^2} N_E}{D_p \cancel{n_{i,E}^2} N_B}$$

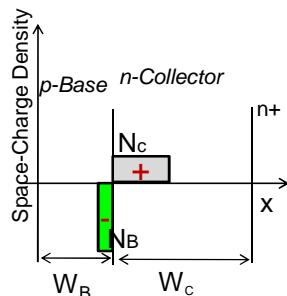


$$V_A = -\frac{qN_B W_B}{C_{CB}} \quad C_{CB} = \frac{\kappa_s \epsilon_0}{x_{n,C} + x_{p,B}}$$

- Base-Collector in reverse bias
- ⇒ Majority carriers only
- ⇒ No diffusion capacitance
- ⇒ Reduce capacitance
- ⇒ Increase  $x_{nC}$

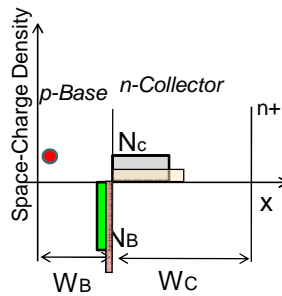


If you want low base doping then reduce collector doping even more to increase Collector depletion.....



$$N_B x_B = N_C x_C$$

$$V_{bi} - V_{BC} = \frac{q}{2\kappa_s \epsilon_0} [N_B x_B^2 + N_C x_C^2]$$



$$J_C = qv_{sat} n$$

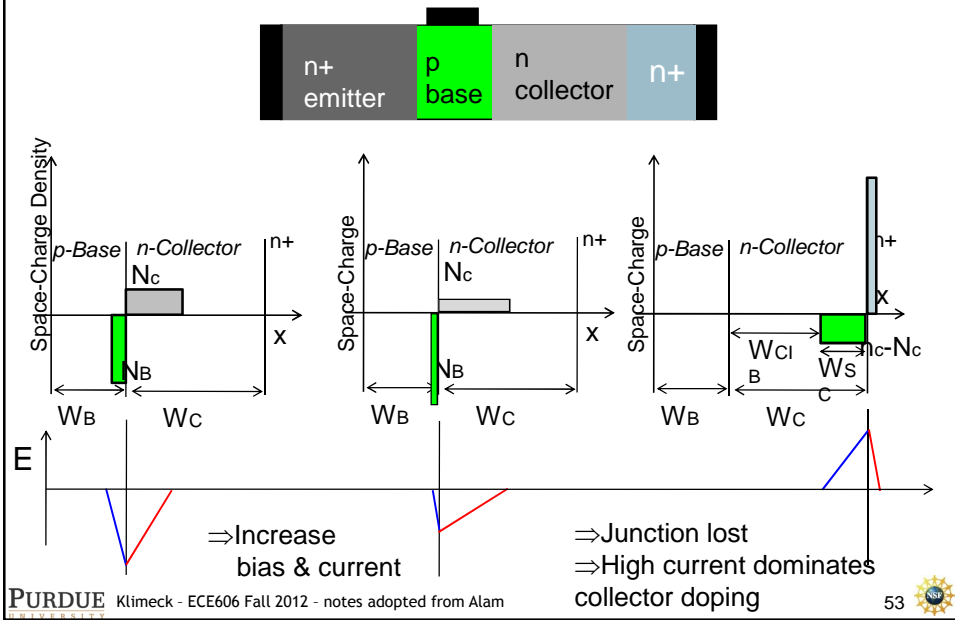
Additional charge!  
Can be large compared to low doping

$$(N_B + n) x'_B = (N_C - n) x'_C$$

$$V_{bi} - V_{BC} = \frac{q}{2\kappa_s \epsilon_0} [(N_B + n) x_B'^2 + (N_C - n) x_C'^2]$$

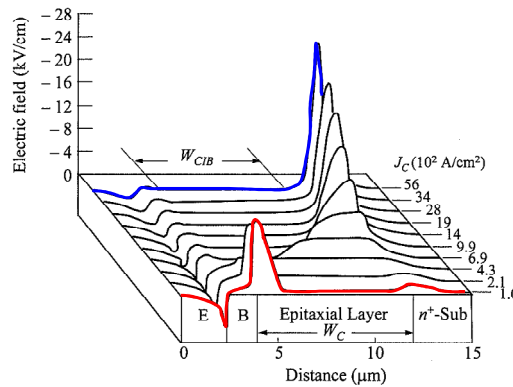
$$x'_C = x_C \sqrt{\frac{1 + \frac{n}{N_B}}{1 - \frac{n}{N_C}}} = x_C \sqrt{\frac{1 + \frac{J_C}{qv_{sat} N_B}}{1 - \frac{J_C}{qv_{sat} N_C}}}$$





$$x'_C = x_C \sqrt{\frac{1 + \frac{J_C}{qv_{sat} N_B}}{1 - \frac{J_C}{qv_{sat} N_C}}}$$

$$J_{C,crit} = qv_{sat} N_C \equiv J_K$$



Can not reduce collector doping arbitrarily without causing base pushout

The Kirk effect occurs at high current densities in a bipolar transistor. The effect is due to the charge density associated with the current passing through the base-collector depletion region. As this charge density exceeds the charge density in the depletion region the depletion region ceases to exist. Instead, there will be a build-up of majority carriers from the base in the base-collector depletion region. The dipole formed by the positively and negatively charged ionized donors and acceptors is pushed into the collector and replaced by positively charged ionized donors and a negatively charged electron accumulation layer, which is referred to as base push out. This effect occurs if the charge density associated with the current is larger than the ionized impurity density in the base-collector depletion region. Assuming full ionization, this translates into the following condition on the collector current density.

**Key point : Under high current and low collector doping the depletion approximation is invalid in the C-B junction!**



Band-gap narrowing reduces gain significantly ...

$$\beta \approx \frac{D_n}{W_B} \frac{W_E}{D_p} \frac{n_{i,B}^2}{n_{i,E}^2} \frac{N_E}{N_B} = \frac{D_n}{W_B} \frac{W_E}{D_p} \frac{N_C N_V e^{-E_{g,B}/kT}}{N_C N_V e^{-E_{g,E}/kT}} \frac{N_E}{N_B} \approx e^{-\Delta E_g/kT} \frac{N_E}{N_B}$$

(Easki-like) Tunneling cause loss of base control ...



While basic transistor operation is simple, its optimum design is not.

In general, good transistor gain requires that the emitter doping be larger than base doping, which in turn should be larger than collector doping.

If the base doping is too low, however, the transistor suffers from current crowding, Early effects. If the collector doping is too low, then we have Kirk effect (base push out) with reduced high-frequency operation and if the emitter doping is too high then the gain is reduced.

