ECE606: Solid State Devices
Lecture 19
Bipolar Transistors
Design

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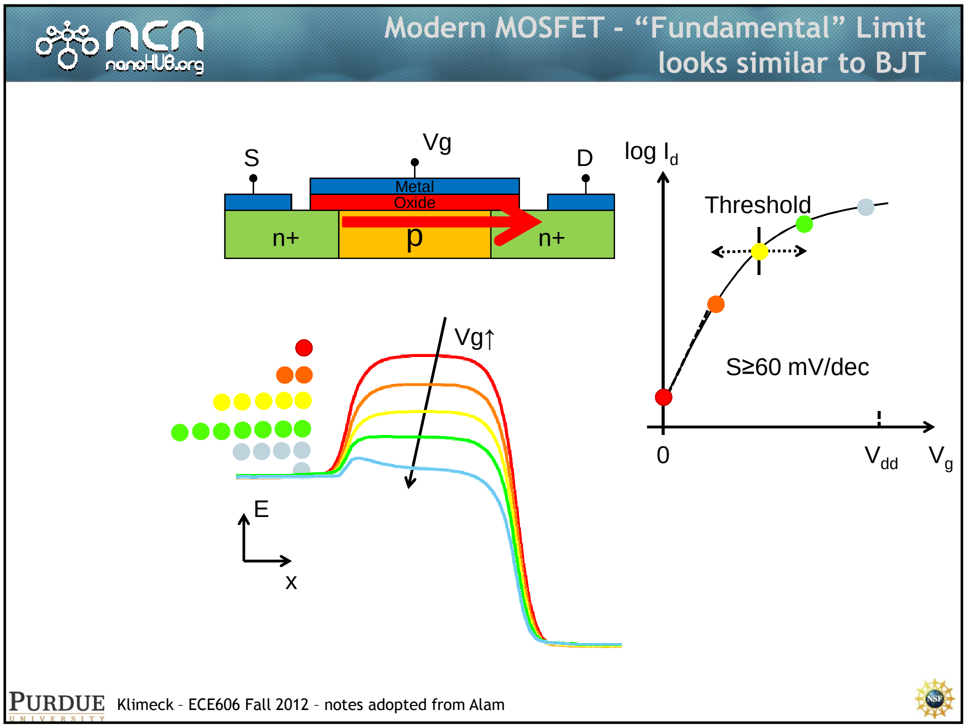
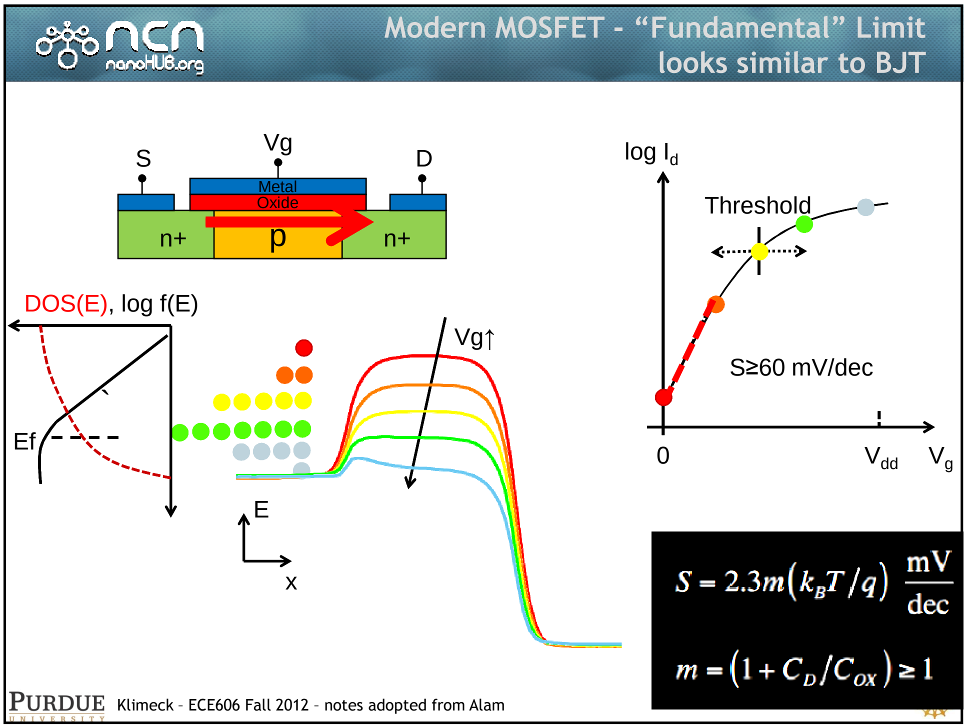
Outline

1) Current gain in BJTs
2) Considerations for base doping
3) Considerations for collector doping
4) Intermediate Summary
5) Problems of classical transistor
6) Poly-Si emitter
7) Short base transport
8) High frequency response
9) Conclusions

REF: SDF, Chapter 10



Modern MOSFET - “Fundamental” Limit looks similar to BJT



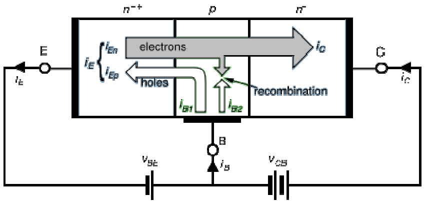
Modern MOSFET - “Fundamental” Limit looks similar to BJT

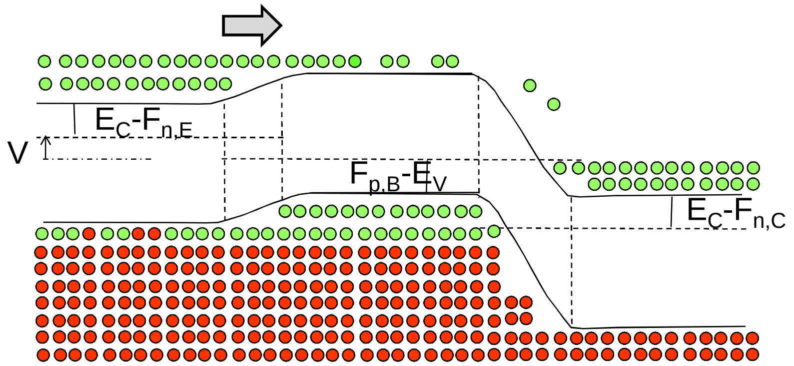
$$S = 2.3m\left(k_B T/q\right)\ \frac{\text{mV}}{\text{dec}}$$

$$m = \left(1 + C_D/C_{OX}\right) \geq 1$$

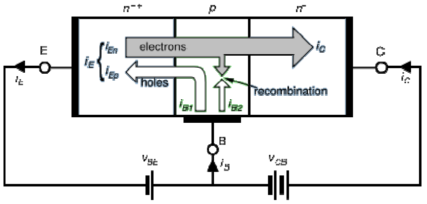
Current flow with Bias

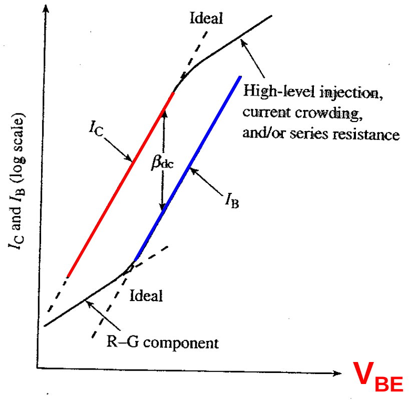
Input small amount of holes results in large amount of electron output





Gummel Plot and Output Characteristics





$$\beta_{DC} = \frac{I_C}{I_B}$$

$\beta_{DC} \longrightarrow$ Common emitter Current Gain

$$\beta_{DC} \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}$$

How to make a Good Silicon Transistor

For a given Emitter length

~1, same material primarily determined by bandgap

$$\beta_{DC} \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}$$

Make-Base short …
(few mm in 1950s, 200 A now)
Want high gradient of carrier density

Emitter doping higher than Base doping

Base doping hard to control
Emitter doping easier

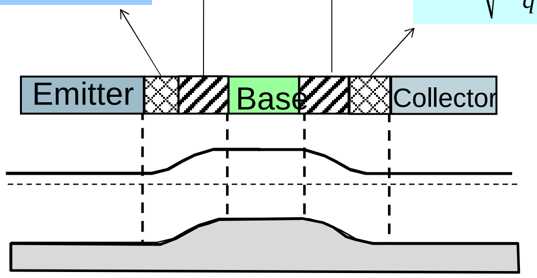
Electrostatics in Equilibrium

$$x_{p,BE} = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_E}{N_B\left(N_E+N_B\right)}V_{bi}}$$

$$x_{p,BC} = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_C}{N_B\left(N_C+N_B\right)}V_{bi}}$$

$$x_{n,E} = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_B}{N_E\left(N_B+N_E\right)}V_{bi}}$$

$$x_{n,C} = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_B}{N_C\left(N_C+N_B\right)}V_{bi}}$$



Two back to back p-n junction

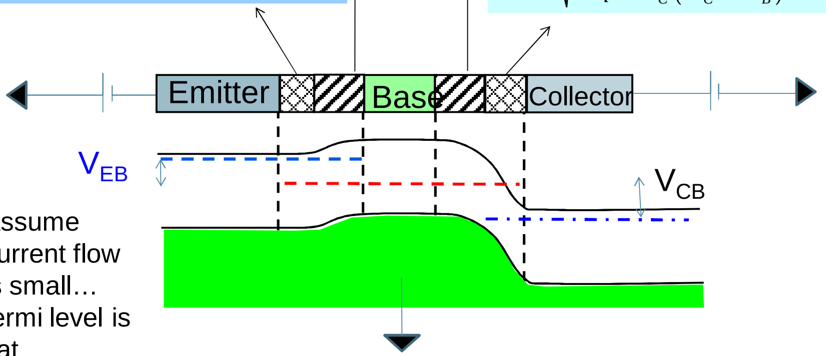
Electrostatics in Equilibrium

$$x_{p,BE}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_E}{N_B\left(N_E+N_B\right)}\left(V_{bi}-V_{EB}\right)}$$

$$x_{p,BC}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_C}{N_B\left(N_C+N_B\right)}\left(V_{bi}-V_{CB}\right)}$$

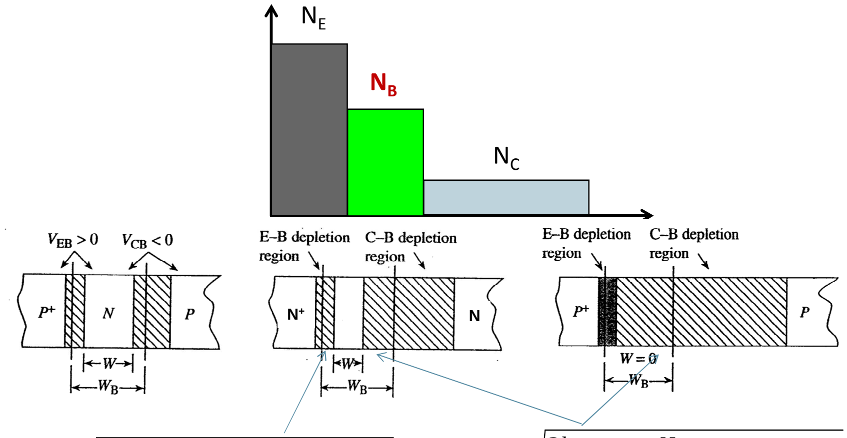
$$x_{n,E}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_B}{N_E\left(N_B+N_E\right)}\left(V_{bi}-V_{EB}\right)}$$

$$x_{n,C}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_B}{N_C\left(N_C+N_B\right)}\left(V_{bi}-V_{CB}\right)}$$



Assume current flow is small… fermi level is flat

Problem of Low Base Doping: Punch-through



$$x_{p,BE}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_E}{N_B\left(N_E+N_B\right)}\left(V_{bi}-V_{BE}\right)}$$

$$x_{p,BC}=\sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_C}{N_B\left(N_C+N_B\right)}\left(V_{bi}-V_{BC}\right)}$$

V_{BE} is positive (forward bias)

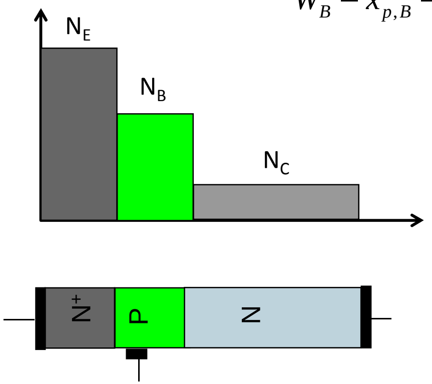
V_{BC} is negative (reverse bias) => $x_{p,BC}$ grows

Low base doping is not a good idea!

Problem of low Base-doping: Base Width Modulation

Electrical base region is smaller than the metallurgical region!

$$\beta_{DC} \approx \frac{D_n}{W_B - x_{p,B} - x_{p,c}} \frac{W_E}{D_p} \cancel{\frac{n^2_{i,B}}{n^2_{i,E}}} \frac{N_E}{N_B}$$

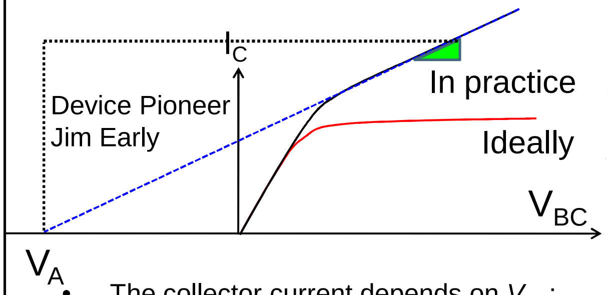


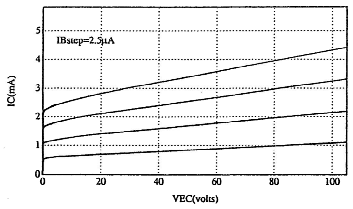
$$x_{p,BE} = \sqrt{\frac{2k_s\varepsilon_0}{q} \frac{N_E}{N_B(N_E + N_B)} (V_{bi} - V_{BE})}$$

$$x_{p,BC} = \sqrt{\frac{2k_s\varepsilon_0}{q} \frac{N_C}{N_B(N_C + N_B)} (V_{bi} - V_{BC})}$$

Gain depends on collector voltage (bad) …
Depletion region width modulation

The Early Voltage





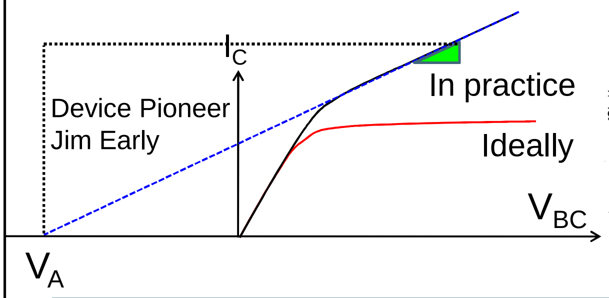
V_{BC} about 1V
V_A ideally infinity

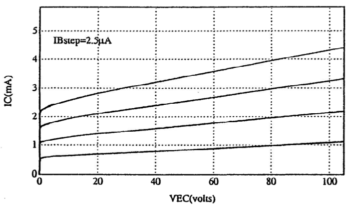
- The collector current depends on V_{CE}:
- For a fixed value of V_{BE}, as V_{CE} increases, the reverse bias on the collector-base junction increases, hence the width of the depletion region increases.
  - The quasi-neutral base width decreases → collector current increases.

collector current increases with increasing V_{CE}, for a fixed value of V_{BE}.

Gain depends on collector voltage (bad) …
Depletion region width modulation

The Early Voltage





V_{BC} about 1V
V_A ideally infinity

- The Early voltage is obtained by drawing a line tangential to the transistor *I-V* characteristic at the point of interest.
- The Early voltage equals the horizontal distance between the point chosen on the *I-V* characteristics and the intersection between the tangential line and the horizontal axis.
- Early voltage is indicated on the figure by the horizontal dotted line

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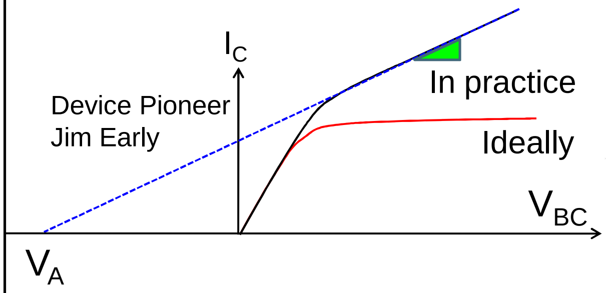
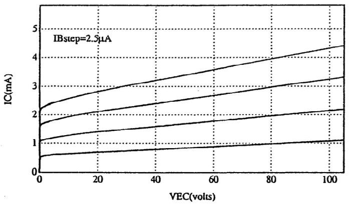
Problem of Low Base-doping: Early Voltage





V_{BC} about 1V
V_A ideally infinity

$$\frac{dI_C}{dV_{BC}} = \frac{I_C}{V_{BC} + V_A} \approx \frac{I_C}{V_A}$$

$$\beta_{DC} \approx \frac{D_n}{W_B - x_{p.B} - x_{p.C}} \frac{W_E}{D_p} \cancel{\frac{n^2_{i,B}}{n^2_{i,E}}} \frac{N_E}{N_B}$$

$$I_{n,C} = -\frac{qD_n}{W_B'} \frac{n^2_{i,B}}{N_B} \left(e^{(qV_{BE}/kT)} - 1\right) + \frac{qD_n}{W_B'} \frac{n^2_{i,B}}{N_B} \left(e^{(qV_{BC}/kT)} - 1\right)$$

Punch-through and Early Voltage

$$\boxed{\frac{dI_C}{dV_{BC}} = \frac{I_C}{V_{BC}+V_A} \approx \frac{I_C}{V_A}}$$

$$\begin{aligned}\frac{dI_C}{dV_{BC}} &= \frac{dI_C}{d\left(qN_BW_B\right)}\frac{d\left(qN_BW_B\right)}{dV_{BC}}\\ &= \frac{1}{qN_B}\left(\frac{dI_C}{dW_B}\right)\left[\frac{dQ_B}{dV_{BC}}\right]\\ &= -\frac{1}{qN_B}\left(\frac{I_C}{W_B}\right)C_{CB}\end{aligned}$$

$$-\frac{C_{CB}}{qN_B}\frac{I_C}{W_B} \approx \frac{I_C}{V_A}$$

$$\Rightarrow V_A = -\frac{qN_BW_B}{C_{CB}} \quad \rightarrow \infty$$

Need higher N_B and W_B or …

$$I_C = \frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B}\left(e^{qV_{BE}\beta}-1\right)+\frac{qD_n}{W_B}\frac{n_{i,B}^2}{N_B}\left(e^{qV_{BC}\beta}-1\right)$$

$$= \frac{\xi}{W_B}$$

$$\frac{dI_C}{dW_B} = \frac{d}{dW_B}\left(\frac{\xi}{W_B}\right) = -\frac{\zeta}{W_B{}^2}$$

$$= -\frac{I_C}{W_B}$$

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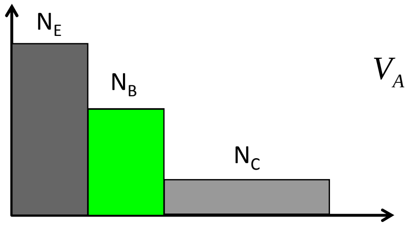
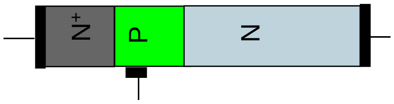
Collector Doping

$$\beta \approx \frac{D_n}{W_B - x_{p,B} - x_{p,C}} \frac{W_E}{D_p} \cancel{\frac{n_{i,B}^2}{n_{i,E}^2}} \frac{N_E}{N_B}$$





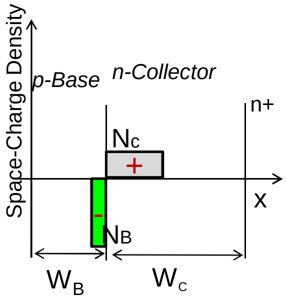
$$V_A = -\frac{qN_B W_B}{C_{CB}} \qquad C_{CB} = \frac{\kappa_s \varepsilon_0}{x_{n,C} + x_{p,B}}$$

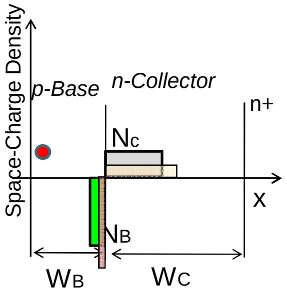
Base-Collector in reverse bias
⇒Majority carriers only
⇒No diffusion capacitance

⇒Reduce capacitance
⇒Increase x_{nC}

If you want low base doping then reduce collector doping even more to increase Collector depletion…..

… but (!) Kirk Effect and Base Pushout





$$J_C = q\upsilon_{sat} n$$

Additional charge! Can be large compared to low doping

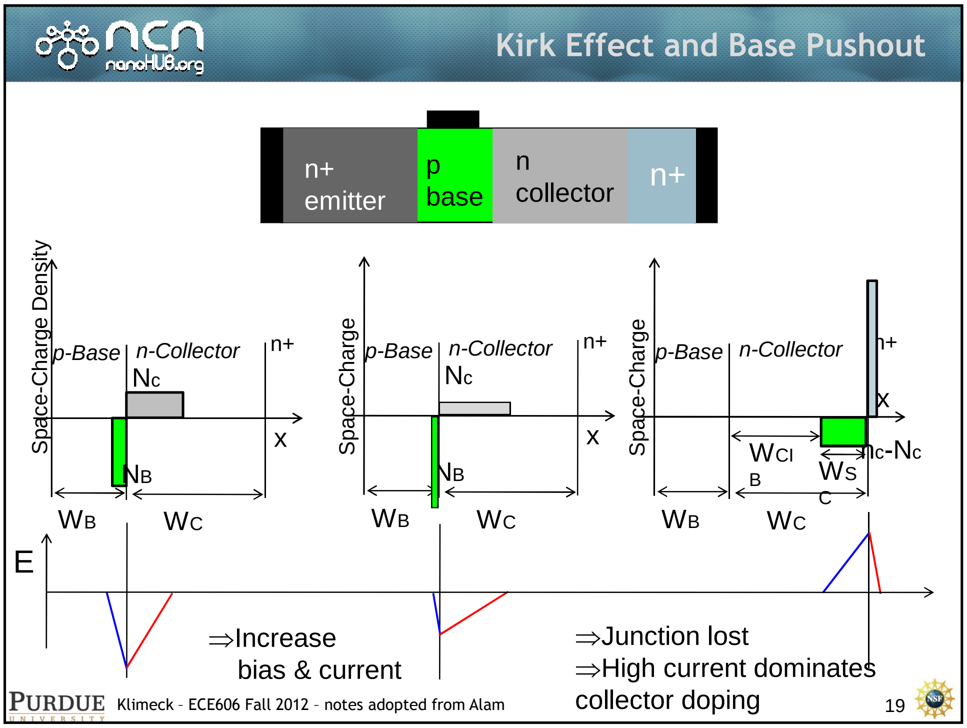
$$N_B x_B = N_C x_C$$

$$V_{bi} - V_{BC} = \frac{q}{2\kappa_s \varepsilon_0}\left[N_B x_B^2 + N_C x_C^2\right]$$

$$(N_B + n)x_B' = (N_C - n)x_C'$$

$$V_{bi} - V_{BC} = \frac{q}{2\kappa_s \varepsilon_0}\left[(N_B + n)x_B^2{}' + (N_C - n)x_C^2{}'\right]$$

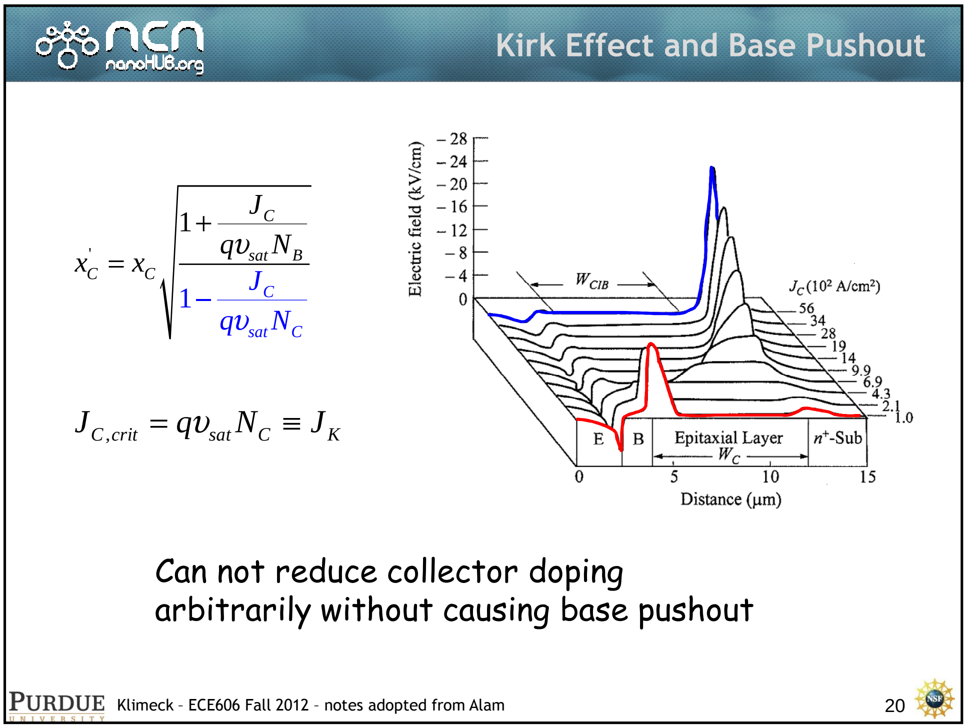
$$x_C' = x_C \sqrt{\frac{1 + \frac{n}{N_B}}{1 - \frac{n}{N_C}}} = x_C \sqrt{\frac{1 + \frac{J_C}{q\upsilon_{sat} N_B}}{1 - \frac{J_C}{q\upsilon_{sat} N_C}}}$$



Kirk Effect and Base Pushout

⇒Increase
bias & current

⇒Junction lost
⇒High current dominates collector doping



Kirk Effect and Base Pushout

$$x_C' = x_C\sqrt{\frac{1+\frac{J_C}{qv_{sat}N_B}}{1-\frac{J_C}{qv_{sat}N_C}}}$$

$$J_{C,crit} = qv_{sat}N_C \equiv J_K$$

Can not reduce collector doping arbitrarily without causing base pushout

Kirk Effect

The Kirk effect occurs at high current densities in a bipolar transistor. The effect is due to the charge density associated with the current passing through the base-collector region. As this charge density exceeds the charge density in the depletion region the depletion region ceases to exist. Instead, there will be a build-up of majority carriers from the base in the base-collector depletion region. The dipole formed by the positively and negatively charged ionized donors and acceptors is pushed into the collector and replaced by positively charged ionized donors and a negatively charged electron accumulation layer, which is referred to as base push out. This effect occurs if the charge density associated with the current is larger than the ionized impurity density in the base-collector depletion region. Assuming full ionization, this translates into the following condition on the collector current density.

Key point : Under high current and low collector doping the depletion approximation is invalid in the C-B junction!

Perhaps High Doping in Emitter?

$$\beta \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B} = \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{N_C N_V e^{-E_{g,B}/kT}}{N_C N_V e^{-E_{g,E}/kT}}\frac{N_E}{N_B} \approx e^{-\Delta E_g/kT}\frac{N_E}{N_B}$$

Very high doping can narrow the bandgap of a semiconductor!

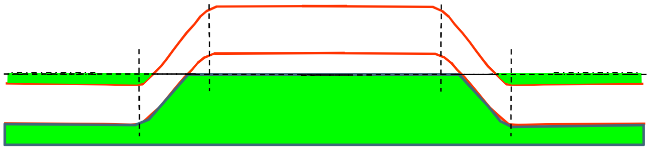
If the emitter is extremely highly doped,
then the bandgap in the emitter may be smaller
than the base

=> Reduction in gain

Band-gap narrowing reduces gain significantly …

Perhaps High Doping in Emitter?

(Easki-like) Tunneling cause loss of base control …



Intermediate Summary

While basic transistor operation is simple, its optimum design is not.

In general, good transistor gain requires that the emitter doping be larger than base doping, which in turn should be larger than collector doping.

If the base doping is too low, however, the transistor suffers from current crowding, Early effects. If the collector doping is too low, then we have Kirk effect (base push out) with reduced high-frequency operation and if the emitter doping is too high then the gain is reduced.

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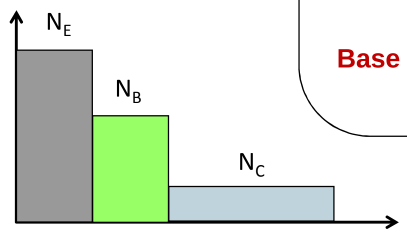
Doping for Gain

$$\beta_{dc} \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}$$



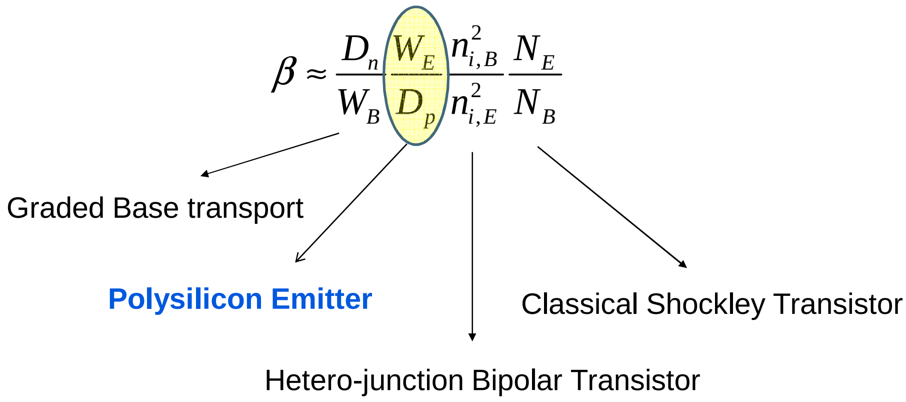
Emitter doping: As high as possible without *band gap narrowing*

Base doping: As low as possible, without *current crowding, Early effect*

Collector doping: Lower than base doping *without Kirk Effect*

Base Width: As thin as possible without *punch through*

How to make better Transistor



$$\beta \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}$$

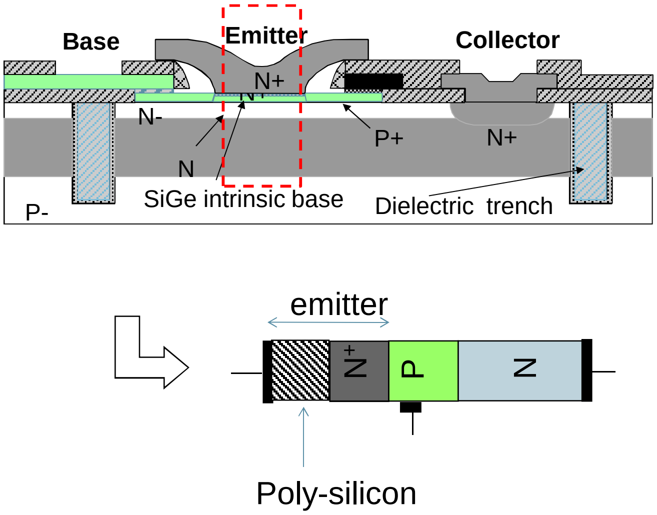
Graded Base transport

Polysilicon Emitter

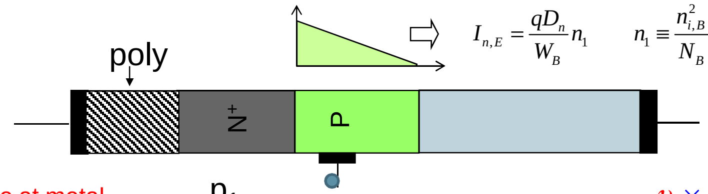
Classical Shockley Transistor

Hetero-junction Bipolar Transistor

Poly-silicon Emitter



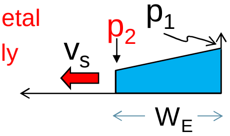
Poly-silicon Emitter



$$I_{n,E} = \frac{qD_n}{W_B} n_1 \qquad n_1 \equiv \frac{n_{i,B}^2}{N_B}\left(e^{qV_{BE}\beta} - 1\right)$$

V_S Infinite at metal

Finite at poly



$$I_{p,E,poly} = -qD_p \frac{p_1 - p_2}{W_E} = -q\upsilon_s p_2$$

$$\frac{p_2}{p_1} = \frac{D_p/W_E}{D_p/W_E + \upsilon_s}$$

$$I_{p,E,poly} = -q\upsilon_s p_2 = -qp_1 \frac{\upsilon_s \times D_p/W_E}{D_p/W_E + \upsilon_s}$$

$$I_{p,E,si} = -q\left(D_p/W_E\right) p_1$$

$$\frac{I_{p,E,poly}}{I_{p,E,si}} = \frac{\upsilon_s}{D_p/W_E + \upsilon_s}$$

Question: Why does poly only suppress the hole current, not electron current?

Ans. Polysilicon is not a ohmic contact and acts as rectifying contact. It blocks the easy passage of holes but lets electrons pass through

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Gain in Poly-silicon Transistor

$$I_{p,E,poly} = -qp_1 \frac{\upsilon_s \times D_p/W_E}{D_p/W_E + \upsilon_s} = I_{p,B,poly}$$

$$I_{p,E,si} = -q\left(D_p/W_E\right) p_1$$

$$\frac{I_{p,B,poly}}{I_{p,B,si}} = \frac{\upsilon_s}{D_p/W_E + \upsilon_s} \simeq \frac{I_{B,poly}}{I_{B,si}}$$

$$\beta_{poly} = \frac{I_C}{I_{B,poly}} = \left(\frac{I_C}{I_{B,si}}\right) \times \left[\frac{I_{B,si}}{I_{B,poly}}\right] \approx \left(\frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}\right) \times \left[\frac{D_p/W_E + \upsilon_s}{\upsilon_s}\right]$$

$$\rightarrow \frac{D_n}{W_B}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B} \times \frac{1}{\upsilon_s} \qquad (\because \upsilon_s << D_p/W_E)$$

Poly suppresses base current, increases gain …

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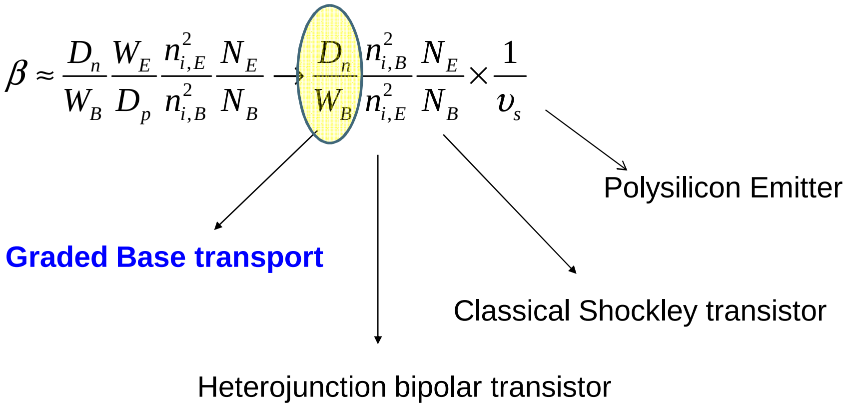
How to make better Transistor



$$\beta \approx \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,E}^2}{n_{i,B}^2}\frac{N_E}{N_B} \rightarrow \frac{D_n}{W_B}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}\times\frac{1}{\upsilon_s}$$

Polysilicon Emitter

Graded Base transport

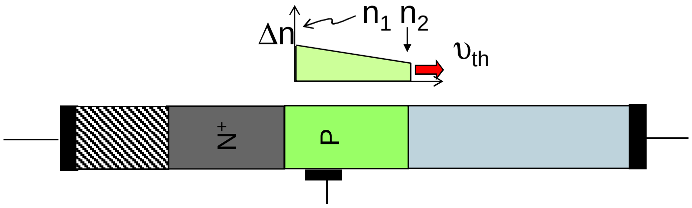
Classical Shockley transistor

Heterojunction bipolar transistor

Short-base Quasi-ballistic Transistor

$$I_{n,E} = -qD_n \frac{n_1 - n_2}{W_B} = -q\upsilon_{th} n_2$$

$$\frac{n_2}{n_1} = \frac{D_n / W_B}{D_n / W_B + \upsilon_{th}} \qquad \frac{I_{n,E,ballistic}}{I_{n,E,si}} = \frac{\upsilon_{th}}{D_n / W_B + \upsilon_{th}}$$



Gain in short-base Poly-silicon Transistor

$$\frac{I_{p,B,poly}}{I_{p,B,si}} = \frac{\upsilon_s}{D_p / W_E + \upsilon_s} \simeq \frac{I_{B,poly}}{I_{B,si}} \qquad \frac{I_{n,E,ballistic}}{I_{n,E,si}} = \frac{\upsilon_{th}}{D_n / W_B + \upsilon_{th}}$$

$$\beta_{poly,ballistic} = \frac{I_{C,ballistic}}{I_{B,poly}} = \left[\frac{I_{C,ballistic}}{I_{C,si}}\right] \times \left[\frac{I_{C,si}}{I_{B,si}}\right] \times \left[\frac{I_{B,si}}{I_{B,poly}}\right]$$

$$\approx \left[\frac{\upsilon_{th}}{D_n / W_B + \upsilon_{th}}\right] \times \left[\frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B}\right] \times \left[\frac{D_p / W_E + \upsilon_s}{\upsilon_s}\right]$$

V_S Assume small Compared to diffusion velocity

$$\rightarrow \frac{n_{i,B}^2}{n_{i,E}^2} \times \frac{N_E}{N_B} \times \frac{\upsilon_{th}}{\upsilon_s}$$

V_S Assume small

Large devices, dinite diffusion length => small diffusion velocity => thermal velocity is large => neglect diffusion velocity
Quasi-Ballistic transport in very short base limits the gain …

Outline

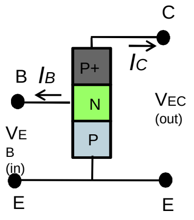
1) Current gain in BJTs
2) Considerations for base doping
3) Considerations for collector doping
4) Intermediate Summary
5) Problems of classical transistor
6) Poly-Si emitter
7) Short base transport
8) **High frequency response**
9) Conclusions

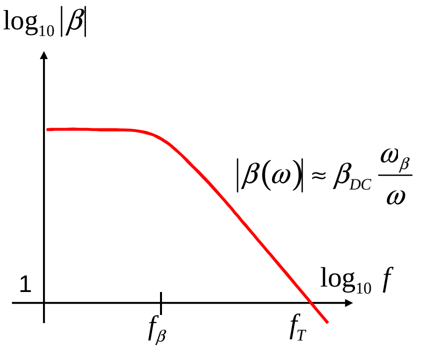
REF: SDF, Chapter 10

Topic Map

	Equilibrium	DC	**Small signal**	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT			◆		
MOS					

Small Signal Response

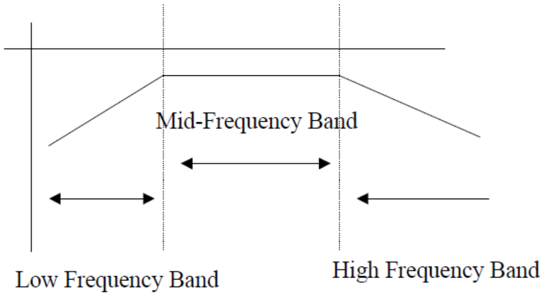




$$\frac{1}{2\pi f_T} = \boxed{\left[\frac{W_B^2}{2D_n} + \frac{W_{BC}}{2\upsilon_{sat}}\right]} + \boxed{\frac{k_B T}{qI_C}\left[C_{j,BC} + C_{j,BE}\right]}$$

Desire high f_T
⇒High IC
⇒Low capacitances
⇒Low widths

Frequency Response

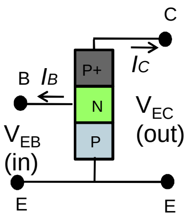


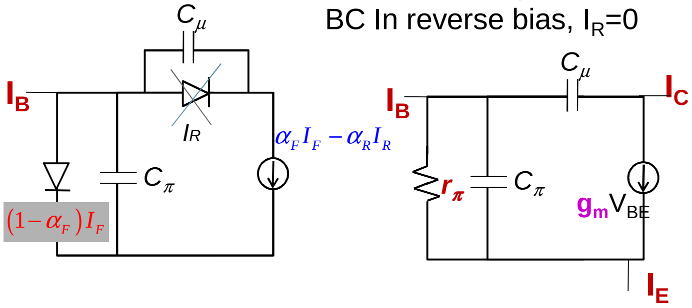
- The gain of an amplifier is affected by the capacitance associated with its circuit.
- This capacitance reduces the gain in both the low and high frequency ranges of operation.
- The reduction of gain in the low frequency band is due to the coupling and bypass capacitors selected. They are essentially short circuits in the mid and high bands.
- The reduction of gain in the high frequency band is due to the internal capacitance of the amplifying device, e.g., BJT, FET, etc.
- This capacitance is represented by capacitors in the small signal equivalent circuit for these devices. They are essentially open circuits in the low and mid bands.

Small Signal Response (Common Emitter) From Ebers Moll Model

DC Circuit

=> AC small signal Circuit
BC In reverse bias, I_R=0





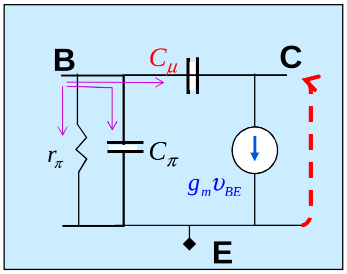
$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{d\left[(1-\alpha_F) I_F\right]}{dV_{BE}} = \frac{qI_B}{k_B T} = \frac{1}{\beta_{DC}} \frac{qI_C}{k_B T}$$

$$I_F = I_{F0}\left(e^{qV_{BE}/kT} - 1\right)$$

$$g_m = \frac{d(\alpha_F I_F)}{dV_{BE}} = \frac{qI_C}{k_B T}$$

$$\delta(\alpha_F I_F) = g_m \delta V_{BE} = g_m \upsilon_{BE}$$

Short Circuit Current Gain



$$\beta(f) = \frac{i_C}{i_B} = \frac{g_m \upsilon_{BE} + j\omega C_\mu \upsilon_{CB}}{\left(\frac{1}{r_\pi}\upsilon_{BE} + j\omega C_\pi \upsilon_{BE}\right) + j\omega C_\mu \upsilon_{BC}}$$

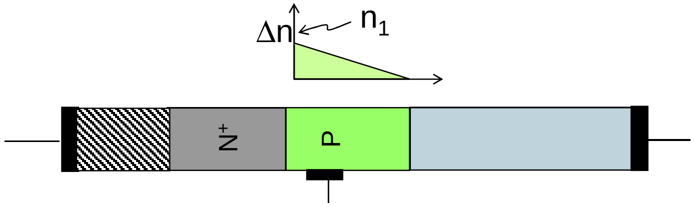
$$\beta(f_T) \equiv 1 = \left| \frac{g_m - j\cancel{\omega_T C_\mu}}{\left(\cancel{\frac{1}{r_\pi}} + j\omega_T C_\pi\right) + j\omega C_\mu} \right| \approx \left| \frac{g_m}{j\omega_T\left(C_\pi + C_\mu\right)} \right|$$

$$\frac{1}{\omega_T} \equiv \frac{1}{2\pi f_T} = \frac{C_\pi + C_\mu}{g_m} :$$

$$\frac{k_B T}{qI_C} C_{d,BC} \overset{\downarrow}{=} \frac{C_{d,BC}}{dI_C / dV_{BE}} = \frac{dQ_B}{dI_C}$$

Base Transit Time

Ref. Charge control model



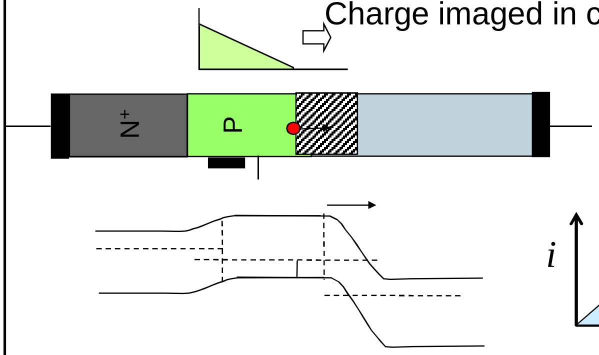
$$\frac{dQ_B}{dI_C}=\frac{Q_B}{I_C}=\frac{q\frac{1}{2}n_1W_B}{q\frac{n_1}{W_B}}=\frac{W_B^2}{2D_n}$$

Base transit time

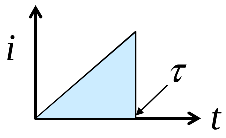
Collector Transit Time

Electrons injected into collector depletion region – very high fields more than diffusion => drift => acceleration of carriers

Charge imaged in collector



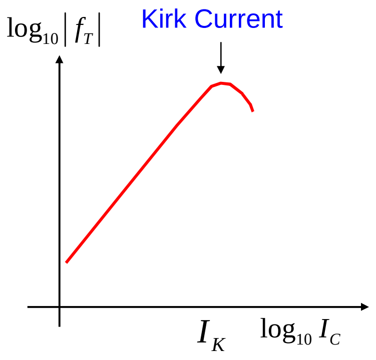
$$\tau=\frac{W_{BC}}{\upsilon_{sat}}?$$



$$\tau_{eff,BC}=\frac{q}{i}=\frac{\tau}{2}=\frac{W_{BC}}{2\upsilon_{sat}}$$

$$\frac{1}{2}\times i\times\tau=q$$

Putting the Terms Together



Collector transit time

Base transit time

$$\frac{1}{2\pi f_T} = \left[\boxed{\frac{W_B^2}{2D_n} + \frac{W_{BC}}{2\upsilon_{sat}}}\right] + \boxed{\frac{k_B T}{q I_C}\left[C_{j,BC} + C_{j,BE}\right]}$$

Junction charging time

Do you see the motivation to reduce WB and W_{BC} as much as possible? What problem would you face if you push this too far ?

Increasing I_C too high reduces W_{BC} and increases the overall capacitance => frequency rolls off….

High Frequency Metrics

(current-gain cutoff frequency, f_T)

$$\tau = \frac{1}{2\pi f_T} = \frac{W_B^2}{2D_n} + \frac{W_{BC}}{2\upsilon_{sat}} + \frac{k_B T/q}{I_C}\left(C_{j,BE} + C_{j,BC}\right) + \left(R_{ex} + R_c\right)C_{cb}$$

(power-gain cutoff frequency, f_{max})

$$f_{\max} = \sqrt{\frac{f_T}{8\pi R_{bb} C_{cbi}}}$$

Summary

We have discussed various modifications of the classical BJTs and explained why improvement of performance has become so difficult in recent years.

The small signal analysis illustrates the importance of reduced junction capacitance, resistances, and transit times.

Classical **homojunctions** BJTs can only go so far, further improvement is possible with **heterojunction** bipolar transistors.