ECE606: Solid State Devices
Lecture 21
MOS Electrostatics

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MOS - Electrostatics - Outline

1. **Background**
2. Band diagram in equilibrium and with bias
3. Qualitative Q-V characteristics of MOS capacitor
4. Intermediate Summary
5. Induced charges in depletion and inversion
6. Exact solution of electros
7. Conclusion

MOS ; metal oxide semiconductor

Can be capacitors or transistor

>90% of the markets

REF: Chapters 15-18 from SDF

Differences between MOS FET and Bipolar:
MOS FET have insulator

Outline of the Course

Device-specific system design ← Application specific device operation ← Physical Principle of device Operation ← Foundation in Physics

- TFT for Displays
- CMOS-based Circuits for mP
- LASERS for Disk Drives
- MEMS for Read heads

EE606

- Resistors (5 wk)
- Diodes (3 wk)
- Bipolar (3 wk)
- **MOSFETs (3 wks)**

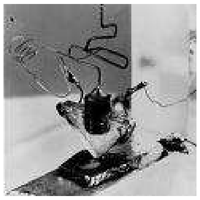
Quantum Mechanics
+
Statistical Mechanics
↓
Transport Equations

Scaling of MOSFETs

Vacuum Tubes

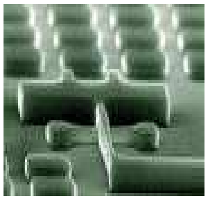
1906-1950s

Bipolar



1947-1980s

MOSFET



1960-until now

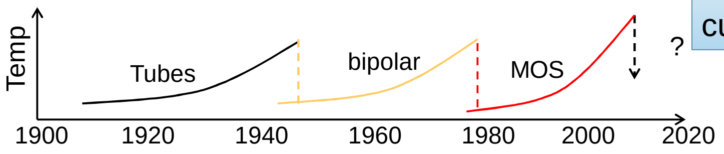
Future??

Spintronics

Bio Sensors

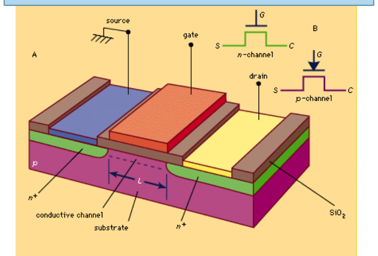
Displays ….

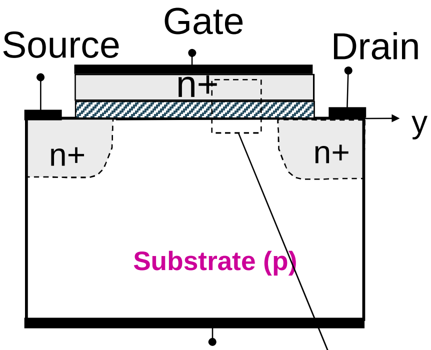
Bipolar has larger leakage current

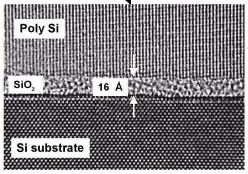


Basic Configuration of a MOSFET

S and D are far away→ could not work like a BJT



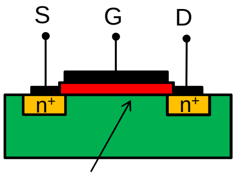




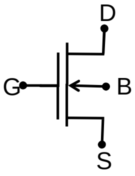
Almost like a lateral bipolar transistor!

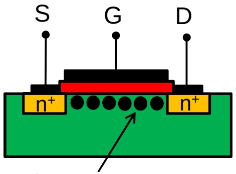
Insulators: Amorphous material →

Symbols

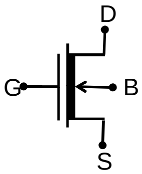


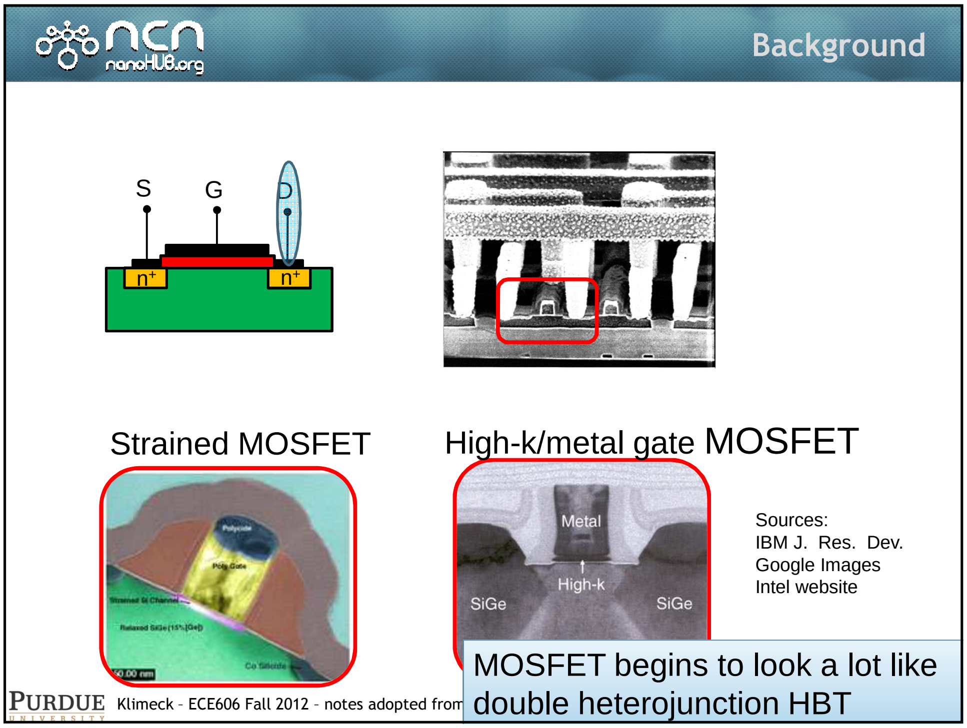
No channel when V_G= 0





Channel when V_G= 0





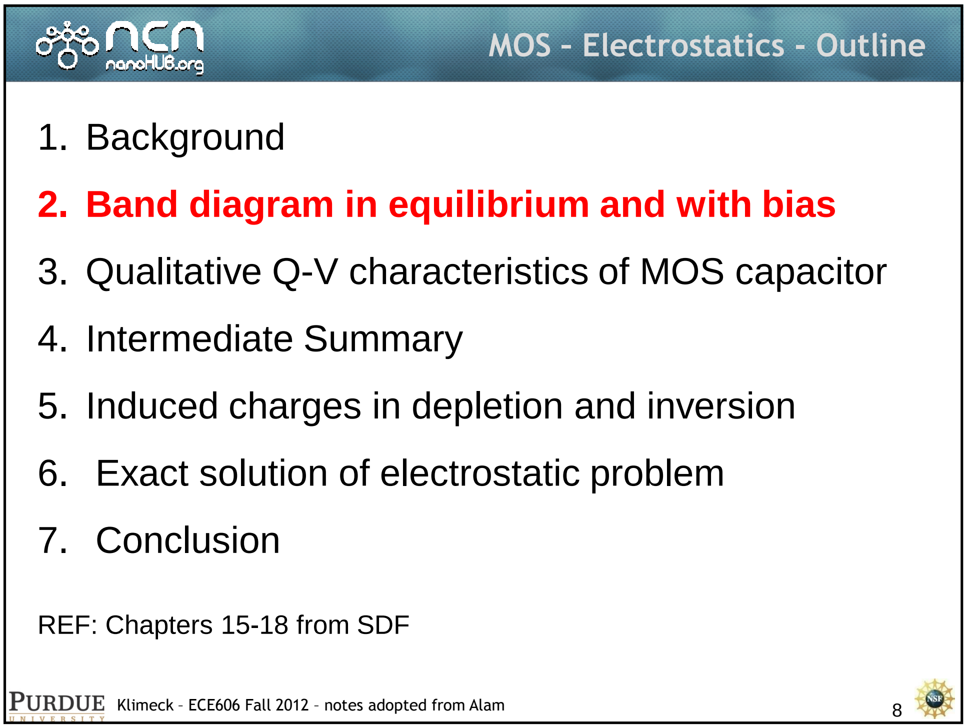
Background

Strained MOSFET

High-k/metal gate MOSFET

Sources:
IBM J. Res. Dev.
Google Images
Intel website

MOSFET begins to look a lot like double heterojunction HBT



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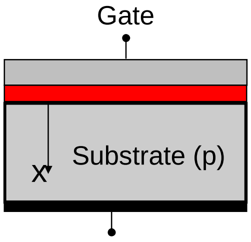
Topic Map

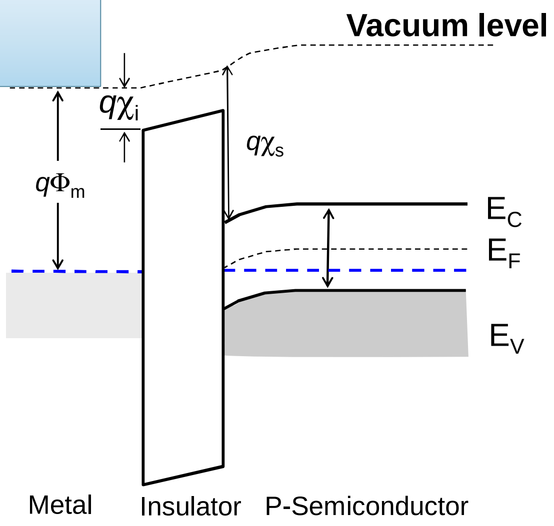
	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOS	◆				

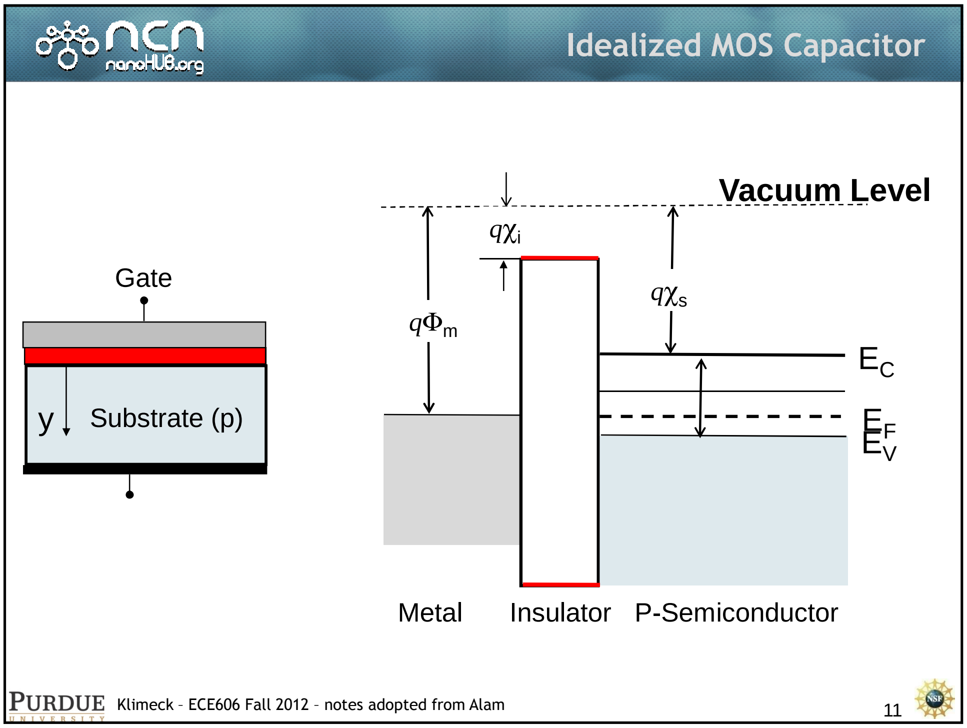
Electrostatics of MOS Capacitor in Equilibrium

No charge in insulator
→ No band bending;
SiO2 band gap: 9eV

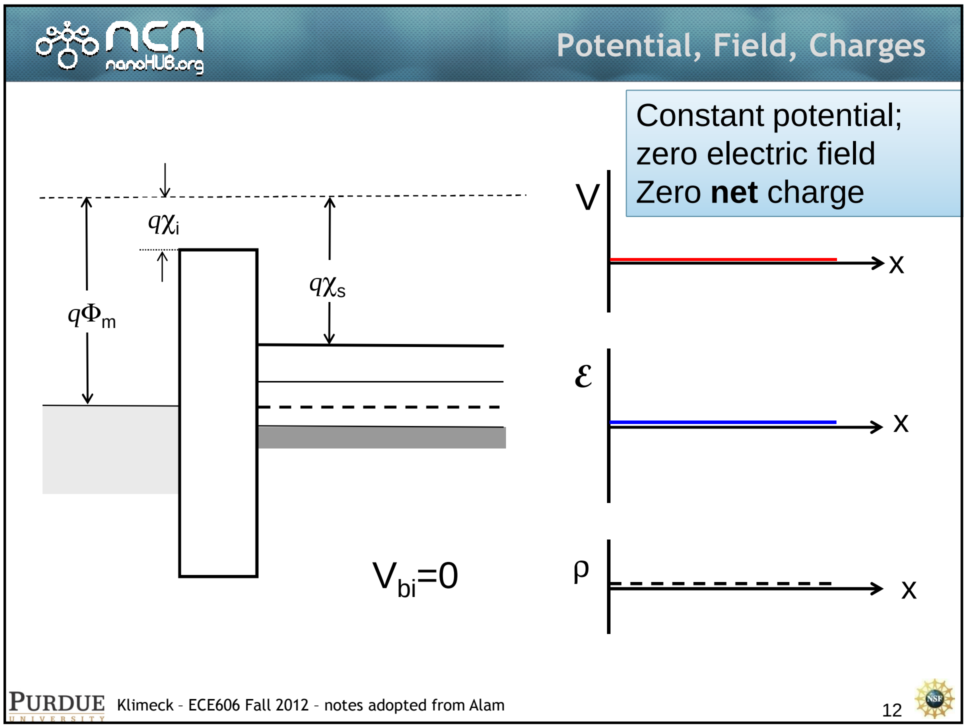


Schottky barrier with an interposed dielectric





Idealized MOS Capacitor



Potential, Field, Charges

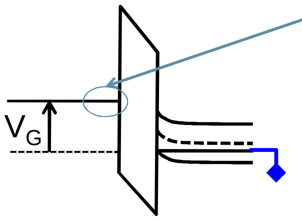
Constant potential;
zero electric field
Zero **net** charge

Topic Map

	Equilibrium	**DC**	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSCAP		◆			

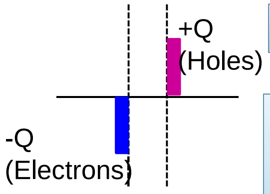
Electrostatics under Bias

Invisible band bending at metal side because of large DOS



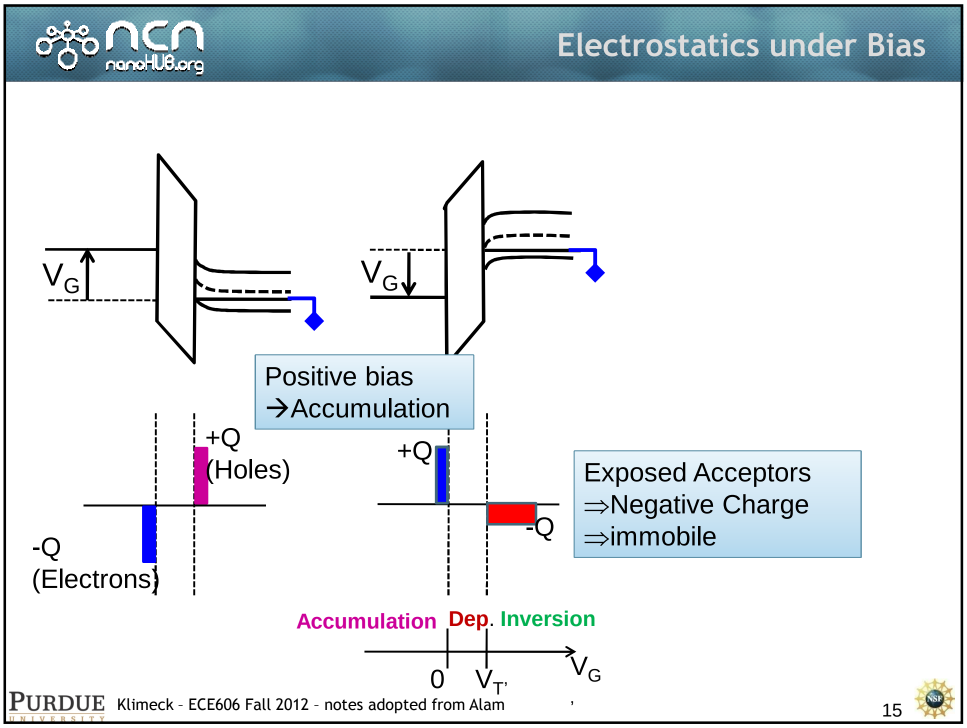
Negative bias
Band bending
Flat Ef → no current

j= n μ dE_f/dx



Accumulation

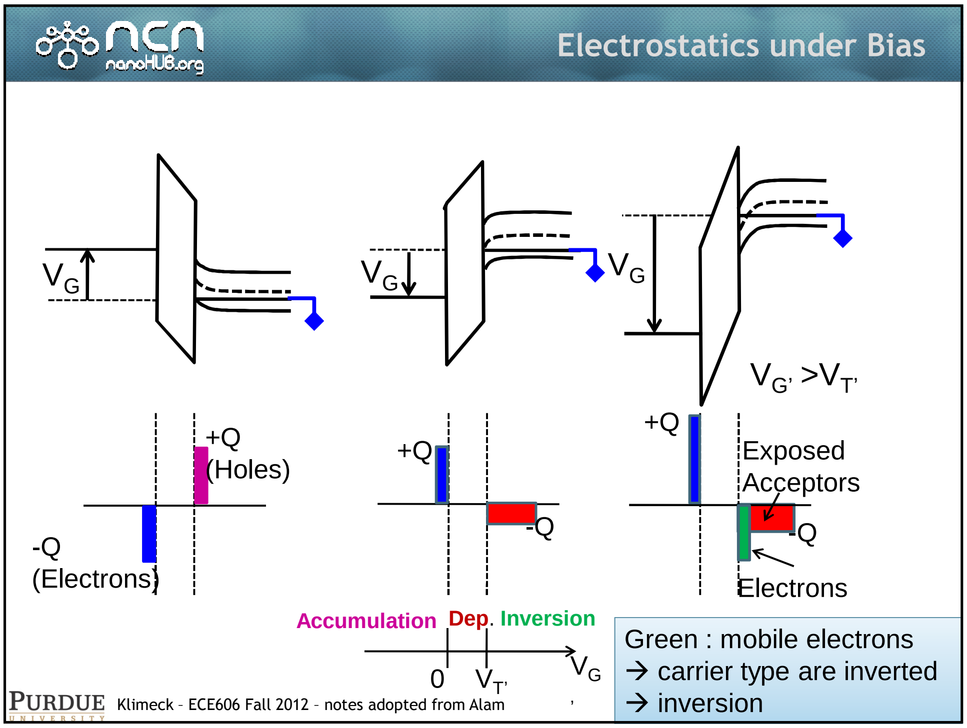
Holes are generated due to band bending/ Valence band come closer to Ef



Electrostatics under Bias

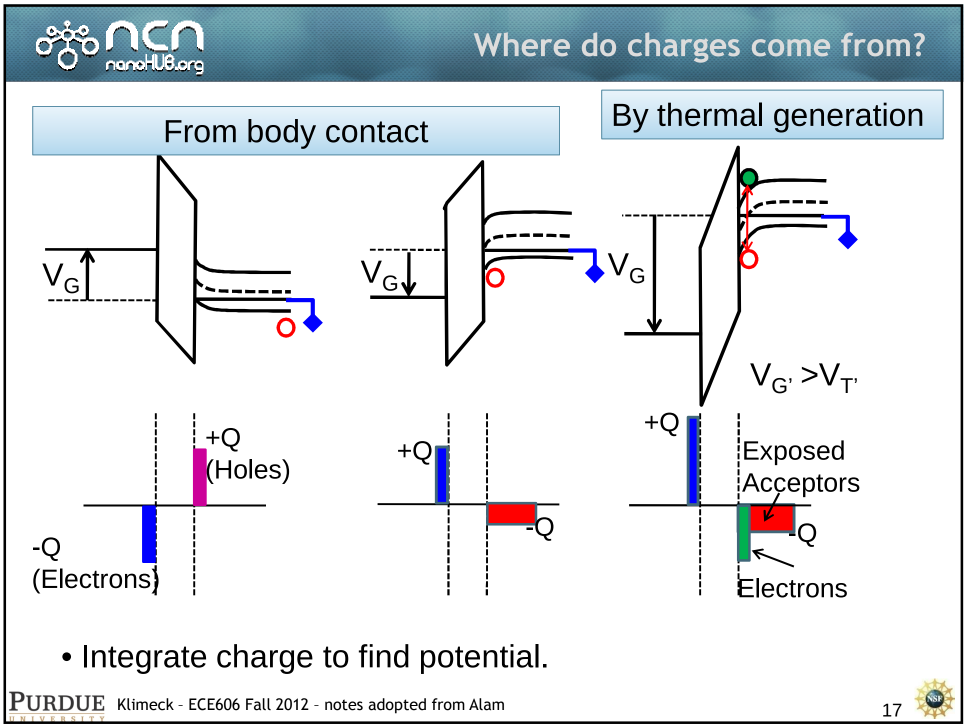
Positive bias
→Accumulation

Exposed Acceptors
⇒Negative Charge
⇒immobile



Electrostatics under Bias

Green : mobile electrons
→ carrier type are inverted
→ inversion

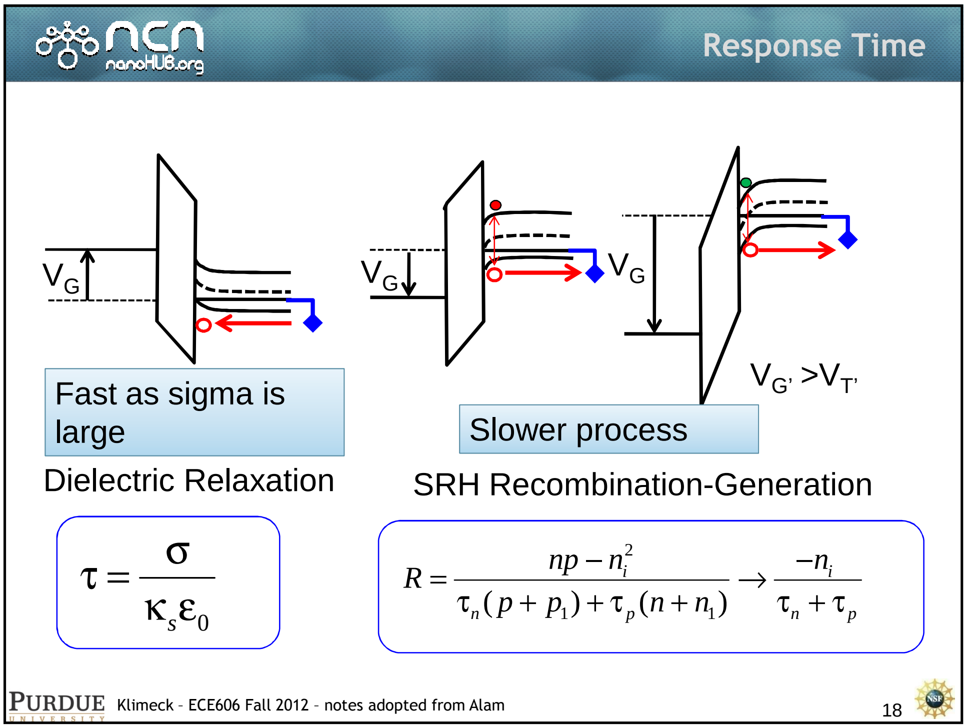


Where do charges come from?

From body contact

By thermal generation

- Integrate charge to find potential.



Response Time

Fast as sigma is large

Dielectric Relaxation

$$\tau = \frac{\sigma}{\kappa_s \varepsilon_0}$$

Slower process

SRH Recombination-Generation

$$R = \frac{np - n_i^2}{\tau_n(p + p_1) + \tau_p(n + n_1)} \rightarrow \frac{-n_i}{\tau_n + \tau_p}$$

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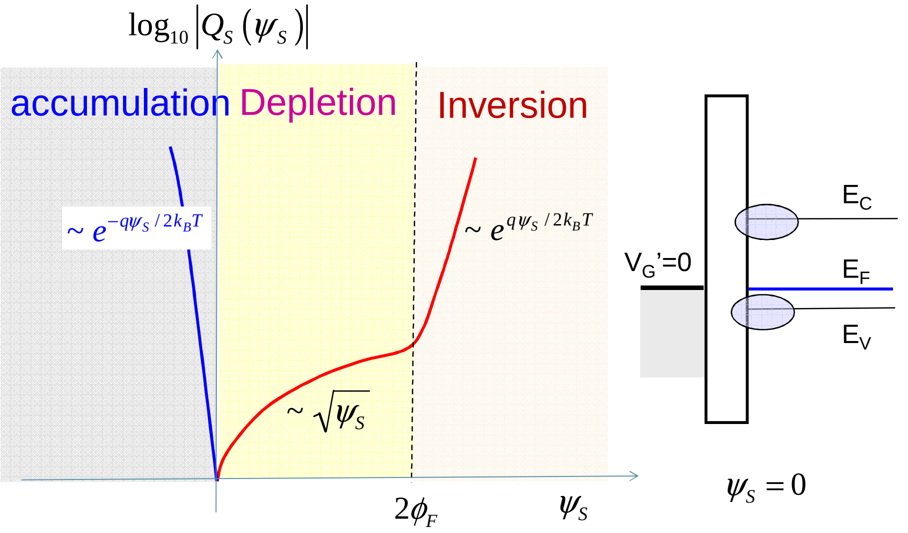
Charges and Surface Potential

Charge in the semiconductor



Solution of $Q_S(\psi_S)$

No current, so only have to solve poisson

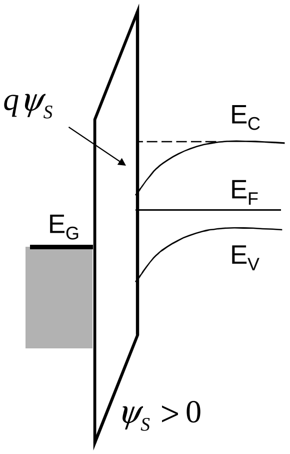
$$\nabla \bullet \vec{D} = \rho$$

$$\cancel{\nabla \bullet \left(\vec{J}_n / -q\right) = (G-R)}$$

$$\cancel{\nabla \bullet \left(\vec{J}_p / q\right) = (G-R)}$$

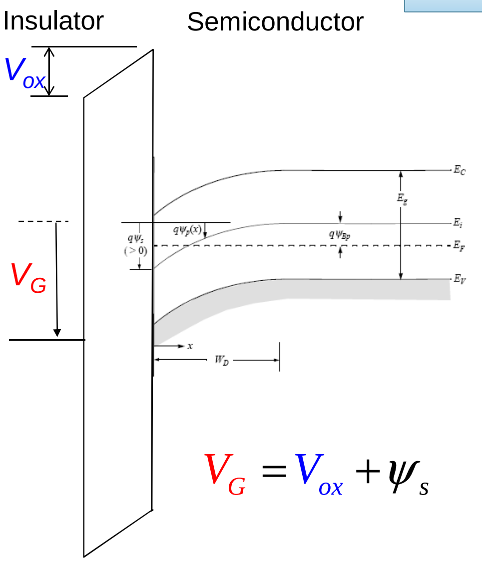
Poisson equation

$$\boxed{\frac{d^2\psi}{dx^2} = \frac{-q}{\kappa_{Si}\varepsilon_0}\left[p_0(x) - n_0(x) + N_D^+ - N_A^-\right]}$$

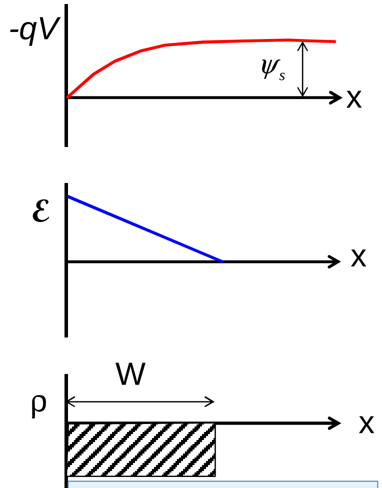


(Depletion) Potential, Field, Charges

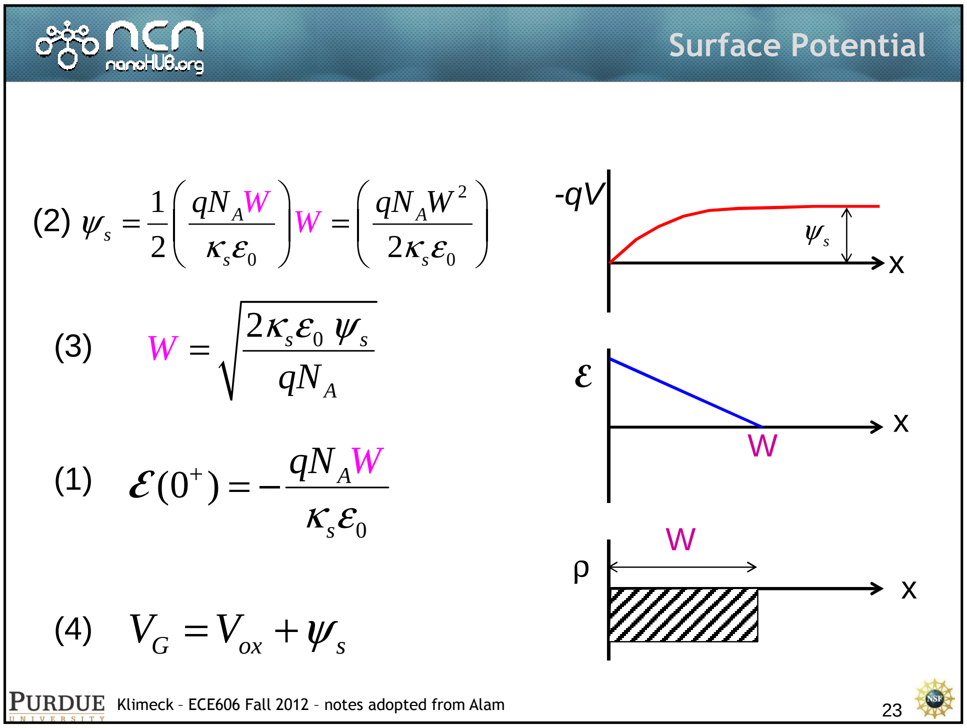
Surface potential



$$V_G = V_{ox} + \psi_s$$



Depleted charge



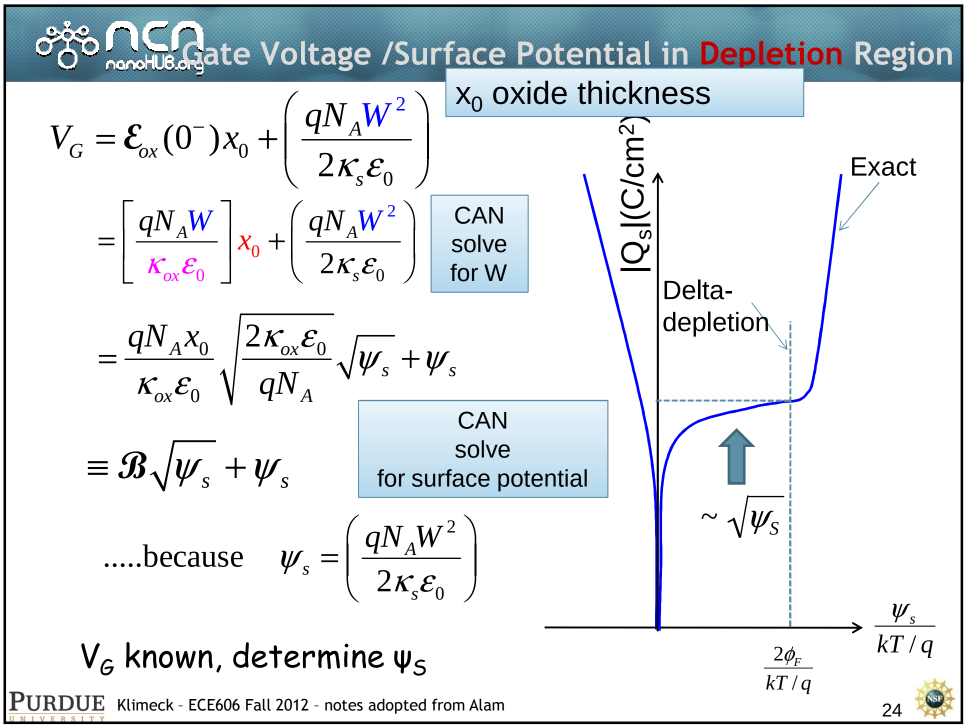
Surface Potential

(2) $\psi_s = \frac{1}{2}\left(\frac{qN_A W}{\kappa_s \varepsilon_0}\right) W = \left(\frac{qN_A W^2}{2\kappa_s \varepsilon_0}\right)$

(3) $W = \sqrt{\frac{2\kappa_s \varepsilon_0 \psi_s}{qN_A}}$

(1) $\mathcal{E}(0^+) = -\frac{qN_A W}{\kappa_s \varepsilon_0}$

(4) $V_G = V_{ox} + \psi_s$



Gate Voltage /Surface Potential in Depletion Region

x_0 oxide thickness

$$V_G = \mathcal{E}_{ox}(0^-)x_0 + \left(\frac{qN_A W^2}{2\kappa_s \varepsilon_0}\right)$$

$$= \left[\frac{qN_A W}{\kappa_{ox}\varepsilon_0}\right] x_0 + \left(\frac{qN_A W^2}{2\kappa_s \varepsilon_0}\right)$$

CAN solve for W

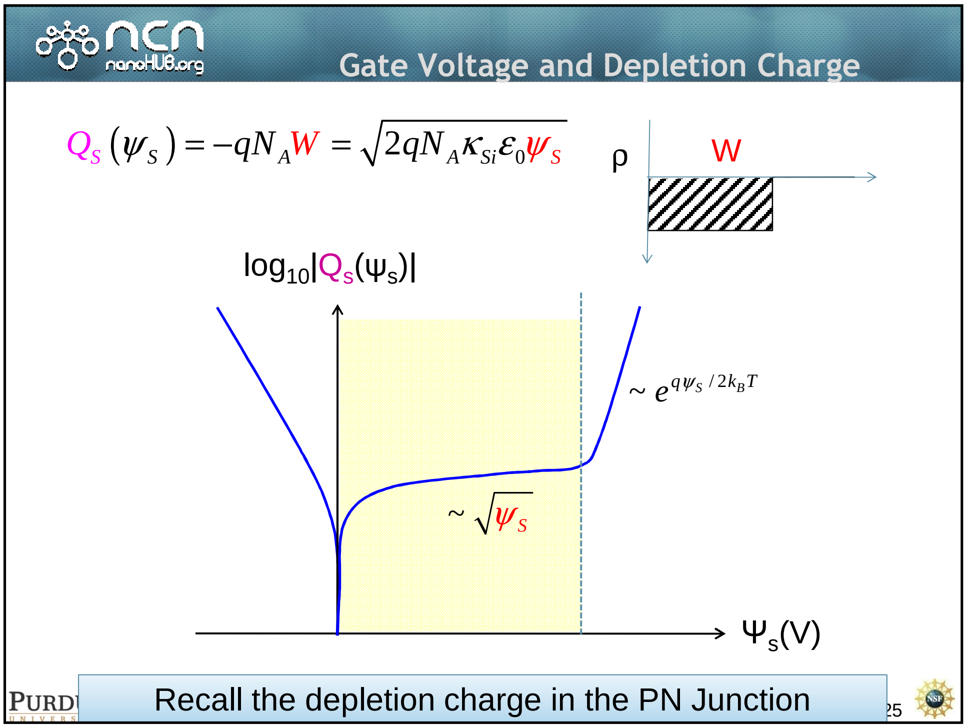
$$= \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

$$\equiv \mathcal{B}\sqrt{\psi_s} + \psi_s$$

CAN solve for surface potential

.....because $\psi_s = \left(\frac{qN_A W^2}{2\kappa_s \varepsilon_0}\right)$

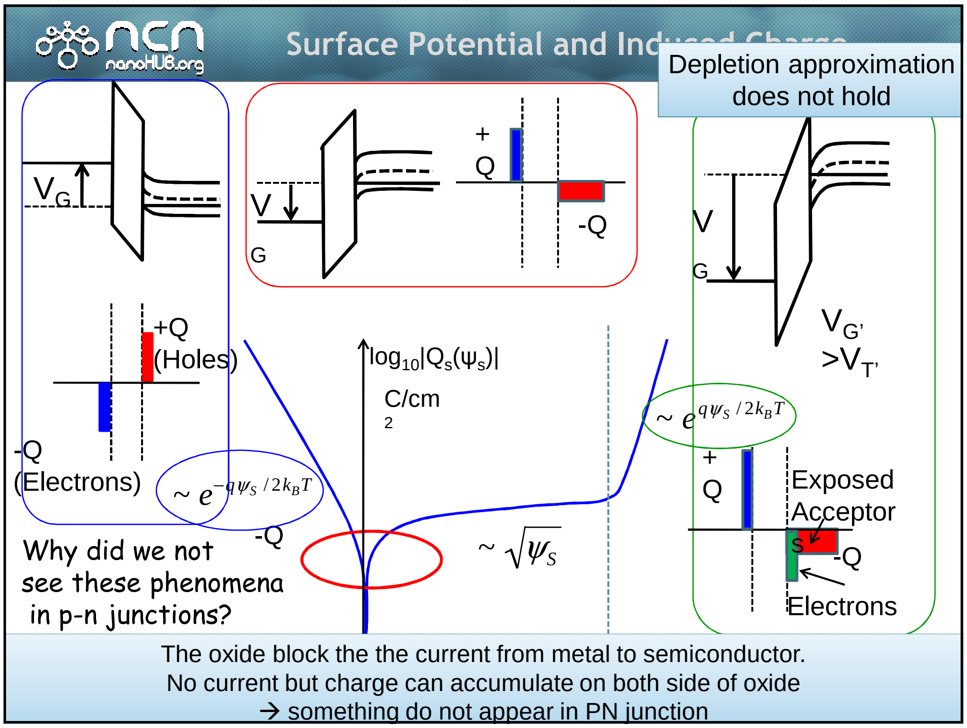
V_G known, determine ψ_S



Gate Voltage and Depletion Charge

$$Q_S(\psi_S) = -qN_A W = \sqrt{2qN_A\kappa_{Si}\varepsilon_0\psi_S}$$

Recall the depletion charge in the PN Junction



Surface Potential and Induced Charge

Depletion approximation does not hold

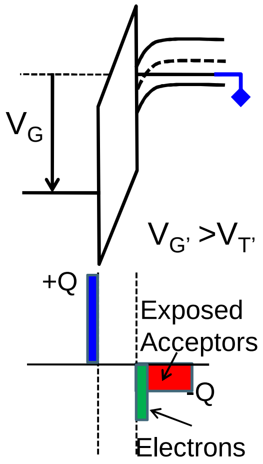
Why did we not see these phenomena in p-n junctions?

The oxide block the the current from metal to semiconductor.
No current but charge can accumulate on both side of oxide
→ something do not appear in PN junction

Intermediate Summary

MOSFET is the dominant electronic device now, not because it is superior to BJTs in terms of performance, but because it consumes far less power and allow denser integration.

MOSFET is an inherently 2D device. We separate out the vertical and horizontal components to qualitatively explore the mechanics of its operation.



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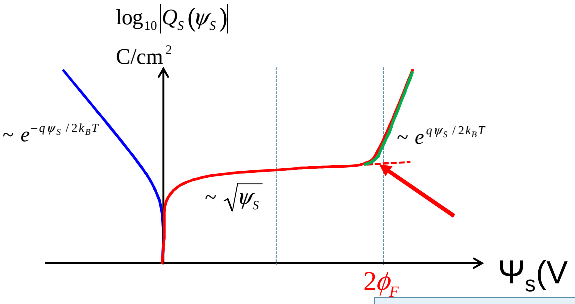
Threshold for Inversion

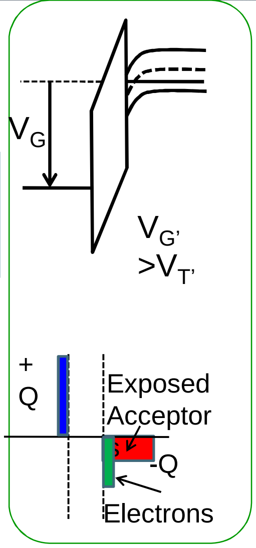
Ψ_F : distance between Ef and intrinsic Ef

$$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$

It very difficult to change Ψs beyond $2\Psi_F$





Threshold voltage

What happens when surface potential is $2\phi_F$?

$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$

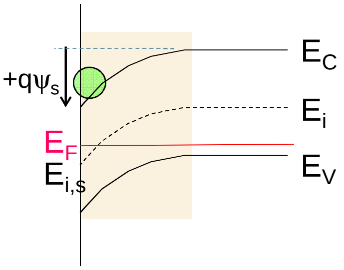
$$n_{1s} = n_i e^{(E_F - E_{is})\beta}$$

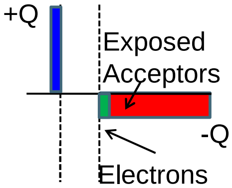
$$= n_i e^{(E_F - E_{i(bulk)})\beta} \times e^{(E_{i(bulk)} - E_{is})\beta}$$

$$= n_i e^{-\phi_F \beta} e^{(E_{i(bulk)} - E_{is})\beta}$$

$$n_{1s} = n_i e^{-\phi_F \beta} e^{2\phi_F \beta}$$

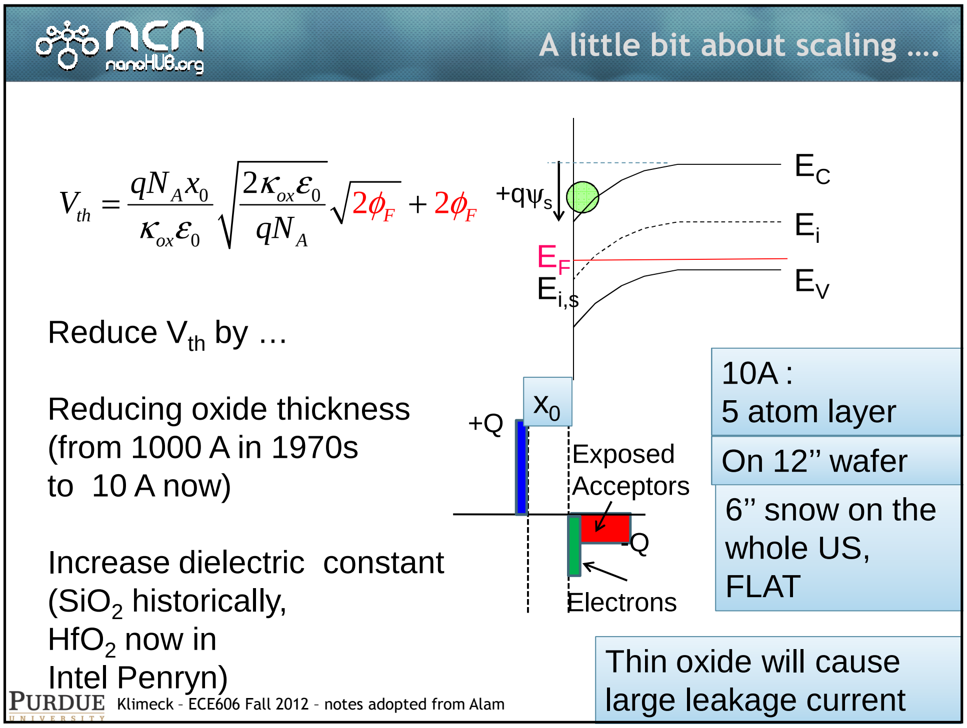
$$= n_i e^{\phi_F \beta} = N_A$$





Total amount of electrons in green are much less than red charge

Electron concentration equals background acceptor concentration



A little bit about scaling ….

$$V_{th} = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{2\phi_F} + 2\phi_F$$

Reduce V_{th} by …

Reducing oxide thickness
(from 1000 A in 1970s
to 10 A now)

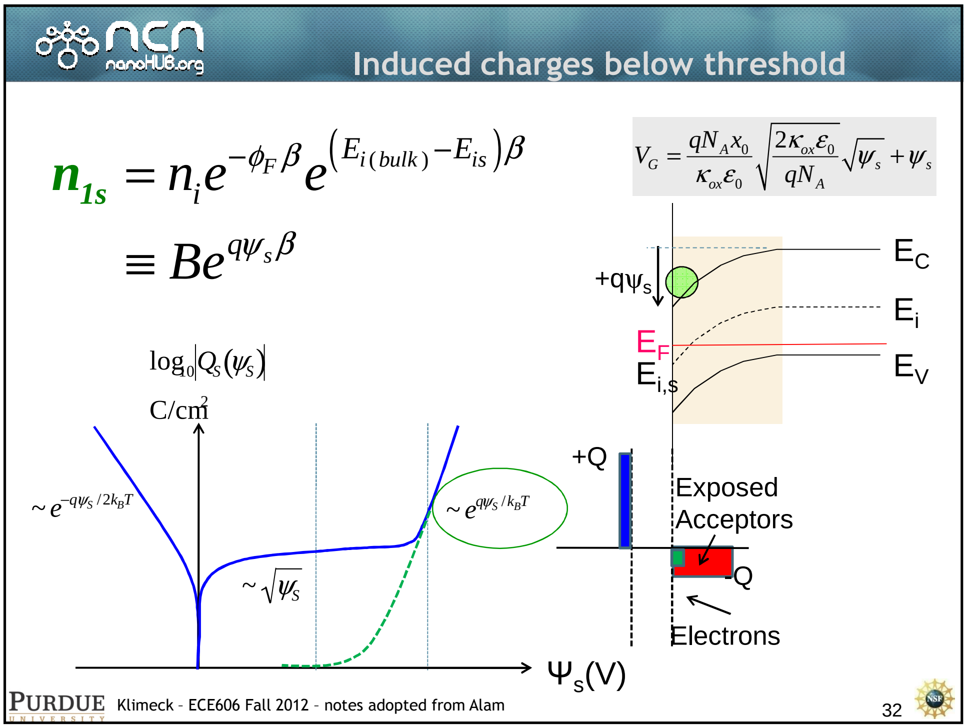
Increase dielectric constant
(SiO_2 historically,
HfO_2 now in
Intel Penryn)

10A :
5 atom layer

On 12" wafer

6" snow on the
whole US,
FLAT

Thin oxide will cause
large leakage current



Induced charges below threshold

$$n_{1s} = n_i e^{-\phi_F\beta} e^{\left(E_{i(bulk)} - E_{is}\right)\beta}$$

$$\equiv B e^{q\psi_s\beta}$$

$$V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

Integrated charges above Threshold

Like a triangle region→ constant E field

$$\frac{Q_i}{q} = \int_0^\infty n(x)\,dx = \int_0^\infty \frac{n_i^2}{N_B} e^{q\psi(x)\beta}\,dx$$

$$= \frac{n_i^2}{N_B}\int_0^\infty e^{q\psi(x)\beta}\frac{1}{\frac{d\psi}{dx}}\,d\psi$$

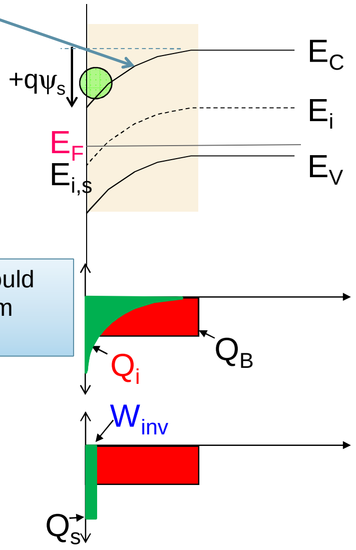
$$= \frac{n_i^2}{N_B}\int_0^\infty e^{q\psi(x)\beta}\frac{1}{\mathcal{E}(x)}\,d\psi$$

Now we should integrate from ψ(0) to ψ(∞)

$$\approx \frac{1}{\langle\mathcal{E}(x)\rangle}\frac{n_i^2}{N_B}\int_0^\infty e^{q\psi(x)\beta}\,d\psi$$

voltage

$$= \frac{\left(\frac{k_BT}{q}\right)}{\langle\mathcal{E}(x)\rangle}\times\frac{n_i^2}{N_B}e^{q\psi_s\beta} \equiv W_{inv}\times n_s$$



Charges above Threshold

Assume there is no charge in the oxide

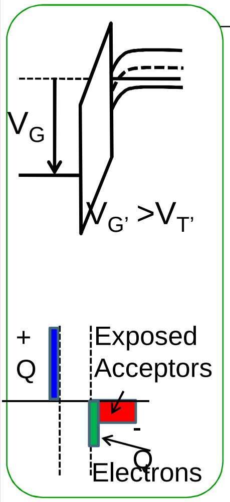
$$V_G = \psi_s + \mathcal{E}_{ox}x_o = \psi_s - \left[\frac{Q_i(\psi_s)+Q_F}{\kappa_{ox}\varepsilon_0}\right]x_o$$

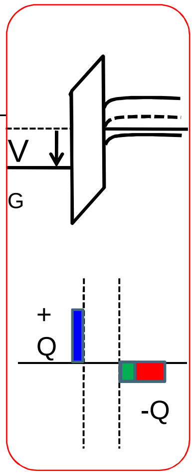
$$V_{th} = 2\phi_F + \mathcal{E}_{ox}x_o = 2\phi_F - \left(\frac{Q_i(2\phi_F)+Q_F}{\kappa_{ox}\varepsilon_0}\right)x_o$$

$$V_G - V_{th} = (\psi_s - 2\phi_F) + \frac{Q_i(\psi_s)-Q_i(2\phi_F)}{\kappa_{ox}\varepsilon_0}x_o$$

This term is small, ψ_s is pinched

The blue region is thin, But the height increase exponentially

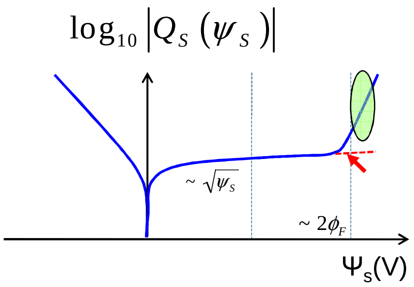


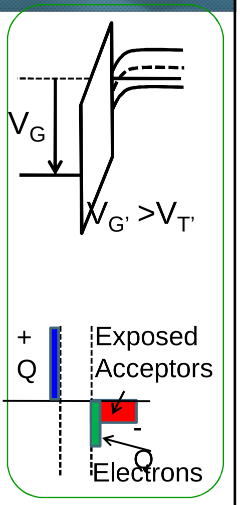


$$\boxed{Q_i = C_{ox}(V_G - V_{th})}$$

Charge has exponential behavior on Psi$_S$
But LINEARLY with gate voltage!

Linear Charge Build-up Above Threshold?





- Small changes ψ_s in changes Q_i a lot ..
- Change in Q_i changes E_{ox}, because $E_{ox}=Q_i/\kappa_0 e_0$
- V_{ox} is large because $V_{ox}=E_{ox}\,x_{0,}$ i.e. most of the drop above $2\psi_F$ goes to V_{ox}.
- Acts like a parallel plate capacitor, hence the inversion equation.

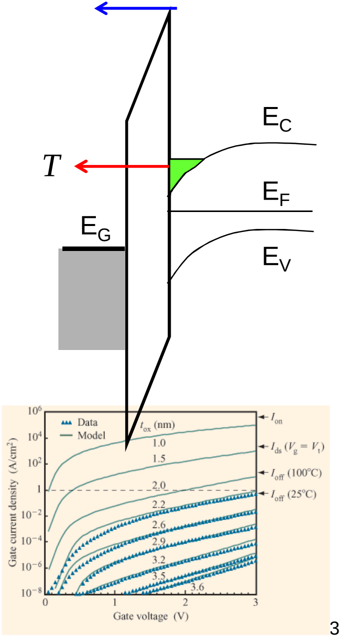
Tunneling Current

Thermionic current

$$J_T = J_{s\to g} - J_{g\to s}$$
$$= \left[Q_i(V_G)e^{-\Delta E_C\beta} - qn_m e^{-\Delta E_C\beta}e^{-qV_{ox}\beta}\right]\upsilon_{th}$$
$$= \left[Q_i(V_G) - qn_m e^{-qV_{ox}\beta}\right]\upsilon_{th}T \qquad T \equiv e^{-\Delta E_C\beta}$$

Negligible!!!

$$J_T = \left[Q_i(V_G) - qn_m e^{-qV_G\beta}\right]\upsilon_{th}\langle T(E)\rangle$$



Oxide can not be too thin because of the tunneling current

Starts to look like base current in a BJT

MOS - Electrostatics - Outline

1. Background
2. Band diagram in equilibrium and with bias
3. Qualitative Q-V characteristics of MOS capacitor
4. Intermediate Summary
5. Induced charges in depletion and inversion
6. **Exact solution of electrostatic problem**
7. Conclusion

REF: Chapters 15-18 from SDF

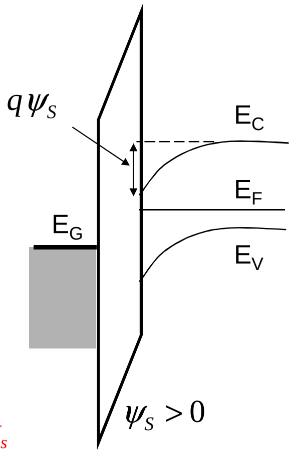
A step back: 'Exact' Solution of $Q_S(\psi_S)$

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \left(\vec{J}_n / -q\right) = (G - R)$$

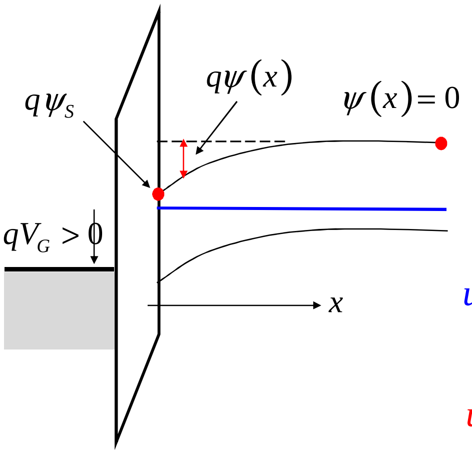
$$\nabla \bullet \left(\vec{J}_p / \; q\right) = (G - R)$$

$$\boxed{\frac{d^2\psi}{dx^2} = \frac{-q}{\kappa_{si}\varepsilon_0}\left[p_0(x) - n_0(x) + N_D^+ - N_A^-\right]}$$



Approximate … $V_G = \frac{qN_A x_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$

Normalized Variable (to save some writing)…



$$E_C(x) = \text{constant} - q\psi(x)$$

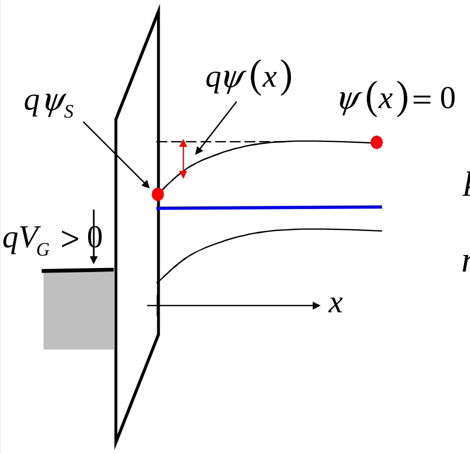
$$\psi(x) = \frac{E_{C,bulk} - E_C(x)}{q}$$

$$u = \frac{\psi(x)}{k_BT/q} = \frac{E_{i(bulk)} - E_{i(x)}}{k_BT}$$

$$u_S = \frac{\psi_S}{k_BT/q} = \frac{E_{i(bulk)} - E_{i(surface)}}{k_BT}$$

$$u_F = \frac{\phi_F}{k_BT/q} = \frac{E_{i(bulk)} - E_F}{k_BT}$$

Normalized Variable (to save some writing!)



$$p(x) = n_i e^{[E_i(x)-E_F]\beta} = n_i e^{+(U_F - U)}$$

$$n(x) = n_i e^{-[E_i(x)-E_F]\beta} = n_i e^{-(U_F - U)}$$

$$N_D^+ = n_i e^{[E_F - E_{i,bulk}]\beta} = n_i e^{-(U_F)}$$

$$N_A^- = n_i e^{-[E_F - E_{i,bulk}]\beta} = n_i e^{(U_F)}$$

Assuming either donor or acceptor is present

Poisson-Boltzmann Equation

$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\kappa_s\varepsilon_0}\left[p(x) - n(x) + N_D^+ - N_A^-\right]$$

$$\frac{q}{k_BT}\frac{d^2U}{dx^2} = \frac{-qn_i}{\kappa_s\varepsilon_0}\left[e^{+(U_F-U)} - e^{-(U_F-U)} + n_ie^{-U_F} - n_ie^{U_F}\right] \equiv g(U,U_F)$$

Can be evaluated at any U

$$\left(2\frac{dU}{dx}\right)\times\frac{d^2U}{dx^2} = -\left(\frac{n_ik_BT}{\kappa_s\varepsilon_0}\right)g(U,U_F)\times\left(2\frac{dU}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{dU}{dx}\right)^2 dx = -\frac{1}{2L_D^2}g(U,U_F)\left(2\frac{dU}{dx}\right)dx$$

Debye Length

$$\int_0^{-q\mathcal{E}(x)/kT} d\left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)dU$$

Exact Solution (continued)

$$\int_0^{-q\mathcal{E}(x)/kT} d\left(\frac{dU}{dx}\right)^2 = -\frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)dU$$

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

At the surface

$$\mathcal{E}_s = \frac{k_BT}{qL_D}F(U_S,U_F)$$

Compare …

$$V_G = \frac{qN_Ax_0}{\kappa_{ox}\varepsilon_0}\sqrt{\frac{2\kappa_{ox}\varepsilon_0}{qN_A}}\sqrt{\psi_s} + \psi_s$$

V_{ox}

$$V_G = \psi_s + \left[\frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s\right]x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_BT}{qL_D}F(U_s,U_F)x_0$$

How does the calculation go …

$$\left[\frac{q\mathcal{E}(x)}{kT}\right]^2 = \frac{1}{L_D^2}\int_0^{U(x)} g(U,U_F)\,dU \equiv \frac{F^2(U,U_F)}{L_D^2}$$

$$V_G = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\mathcal{E}_s x_0 = \psi_s + \frac{\kappa_s}{\kappa_{ox}}\frac{k_B T}{qL_D}F(U_s,U_F)x_0$$

Begin with a surface potential

Calculate U_s and then divide U_s by N points.

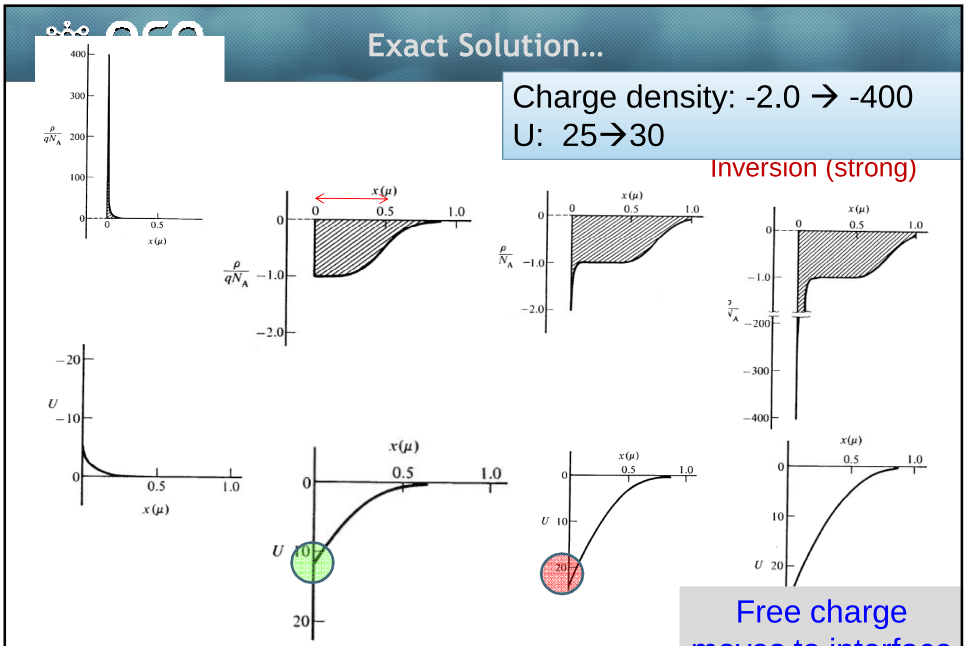
Calculate $g(U, U_F)$ at those points
and integrate to find $F(U_s,U_F)$

Find V_G.

Exact Solution…

Charge density: -2.0 → -400
U: 25→30

Inversion (strong)



Accumulation

Depletion

Inversion (we

Free charge moves to interface

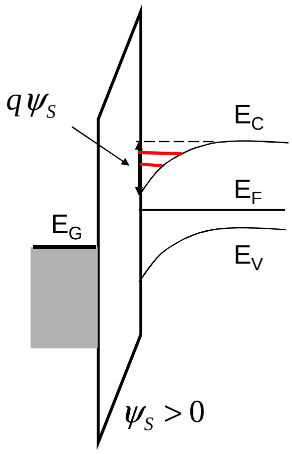
Huge spike! Hard to resolve numerically! Realistic?

“Exact” solution is not really exact …

$$\left.\frac{d^2\psi}{dx^2}\right| = \frac{-q}{\varepsilon}\left[p(x) - n(x)\left|\psi(x)\right|^2 + N_D^+ - N_A^-\right]$$

$\downarrow$

wavefunction, not potential !



Wave function should be accounted for

Bandgap widening near the interface must also should be accounted for.

Assumption of nondegeneracy may not always be valid

Conclusion

Our discussion today was focused on calculating the induced charge in the depletion and inversion region as a function of gate bias.

We found that we could calculate the tunneling current from the inversion changes by using the thermionic emission theory.

We also discussed the “exact” solution of the MOS-capacitor electrostatics. The “exact” solution is mathematically exact, but not necessarily physically exact solution of the electrostatic problem.