ECE606: Solid State Devices
Lecture 7

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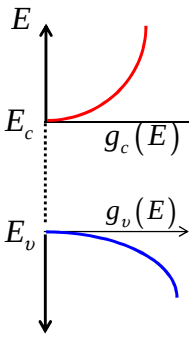
Presentation Outline

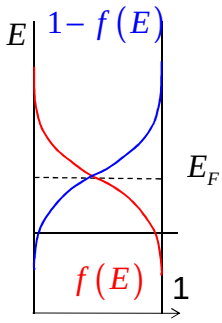
- Intrinsic carrier concentration
- Potential, field, and charge
- E-k diagram vs. band-diagram
- Basic concepts of donors and acceptors
- Law of mass-action & intrinsic concentration
- Statistics of donors and acceptor levels
- Intrinsic carrier concentration
- Temperature dependence of carrier concentration
- Multiple doping, co-doping, and heavy-doping
- Conclusions

Reference: Vol. 6, Ch. 3 & 4

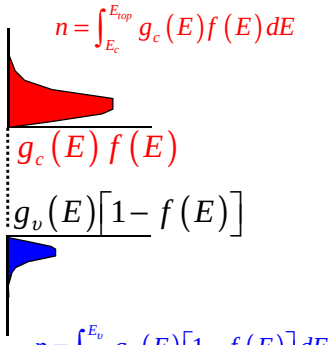
DOS



F-D



concentration



$$n=\int_{E_c}^{E_{top}} g_c(E)f(E)dE$$

$$=\int_{E_c}^{E_{top}} 2\times\frac{m_n^*\sqrt{2m_n^*(E-E_C)}}{2\pi^2\hbar^3}\frac{1}{1+e^{\beta(E-E_F)}}dE$$

Include spin factor of 2

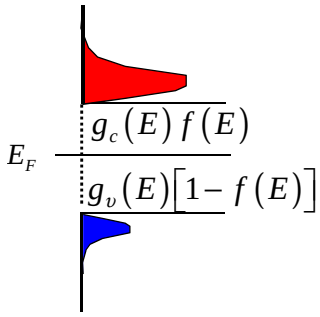
$$\simeq\int_{E_c}^{\infty}\frac{m_n^*\sqrt{2m_n^*(E-E_C)}}{\pi^2\hbar^3}\frac{1}{1+e^{\beta(E-E_c)}e^{\beta(E_c-E_F)}}dE$$

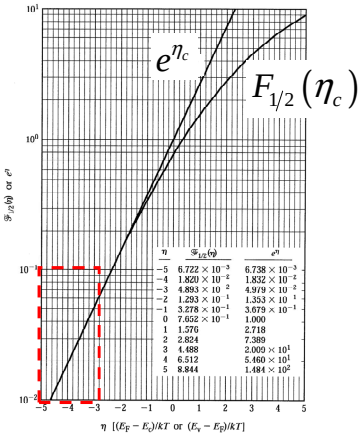
Assume wide bands

$$=N_C\frac{2}{\sqrt{\pi}}F_{1/2}(\eta_c)\qquad \eta_c\equiv\beta(E_F-E_C)$$

$$N_C\equiv 2\left(\frac{2\pi m_n^*\beta}{h^2}\right)^{3/2}\qquad F_{1/2}(\eta)=\int_0^{\infty}\frac{\sqrt{\xi}d\xi}{1+e^{\xi-\eta}}$$

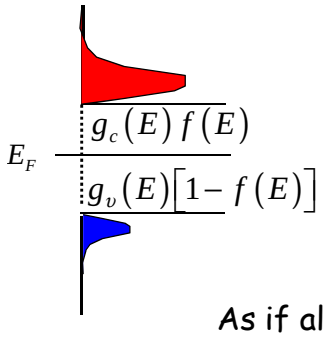
$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{\eta_c} \quad if \quad -\eta_c \equiv \beta(E_C - E_F) > 3$$

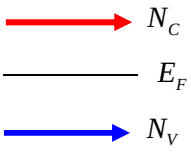




η	$\mathcal{F}_{1/2}(\eta)$	e^{η}
−5	6.722×10^{-3}	6.738×10^{-3}
−4	1.820×10^{-2}	1.832×10^{-2}
−3	4.893×10^{-2}	4.979×10^{-2}
−2	1.293×10^{-1}	1.353×10^{-1}
−1	3.278×10^{-1}	3.679×10^{-1}
0	7.652×10^{-1}	1.000
1	1.576	2.718
2	2.824	7.389
3	4.488	2.009×10^{1}
4	6.512	5.460×10^{1}
5	8.844	1.484×10^{2}

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{-\beta(E_c - E_F)} \quad if \quad E_c - E_F > 3\beta$$





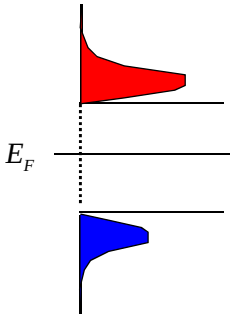
As if all states are at a single level E_C

$$n = N_C e^{-\beta(E_c - E_F)}$$

$$p = N_V e^{+\beta(E_v - E_F)}$$

$$n \times p = N_C N_V e^{-\beta(E_c - E_v)}$$

$$= N_C N_V e^{-\beta E_g}$$



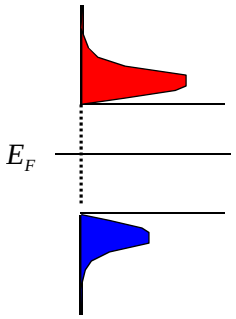
Product is independent of the Fermi level!
Very useful balance equation! Will use it often

$$n = p = n_i$$

$$n_i^2 = N_C N_V e^{-\beta E_g}$$

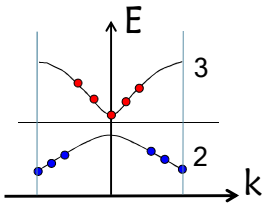
$$n_i = \sqrt{N_C N_V}\, e^{-\beta E_g/2}$$

$$E_F \equiv E_i$$



$$n = p \Rightarrow N_C e^{-\beta(E_c - E_i)} = N_V e^{+\beta(E_v - E_i)}$$

$$E_i = \frac{E_G}{2} + \frac{1}{2\beta} \ln \frac{N_V}{N_C}$$



Summary - DOS and Fermi Functions

- We discussed how electrons are distributed in electronic states defined by the solution of Schrodinger equation.
- Since electrons are distributed according to their energy, irrespective of their momentum states, the previously developed concepts of constant energy surfaces, density of states etc. turn out to be very useful.
  => will not discuss Schroedinger Eq. anymore
  => everything is captured in bandedges and effective masses
- We still do not know where E_F is for general semiconductors … If we did, we could calculate electron concentration.

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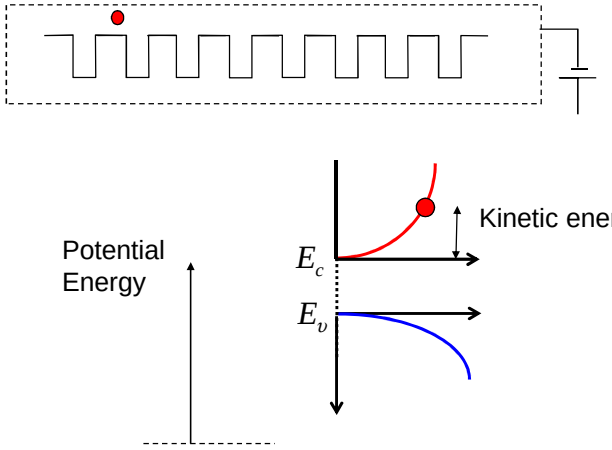
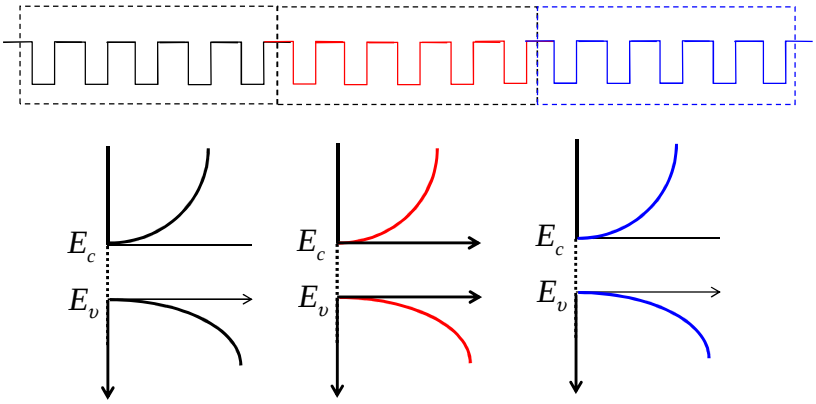
E-k diagram vs. band-diagram

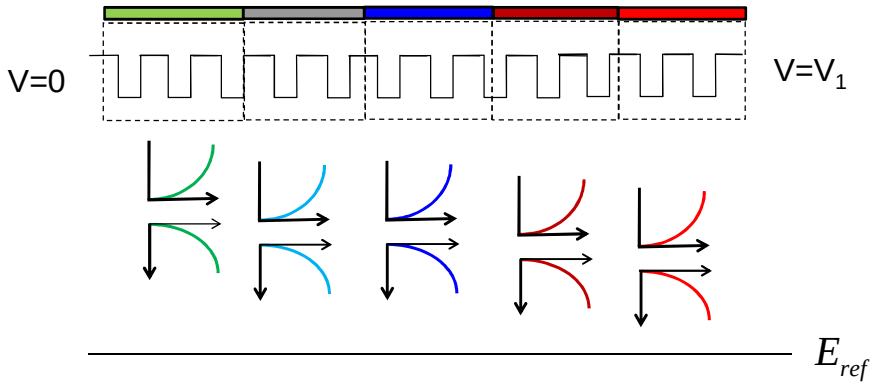


$$P.E. = E_c - E_{ref} = -qV$$

Position Resolved E-k Diagram

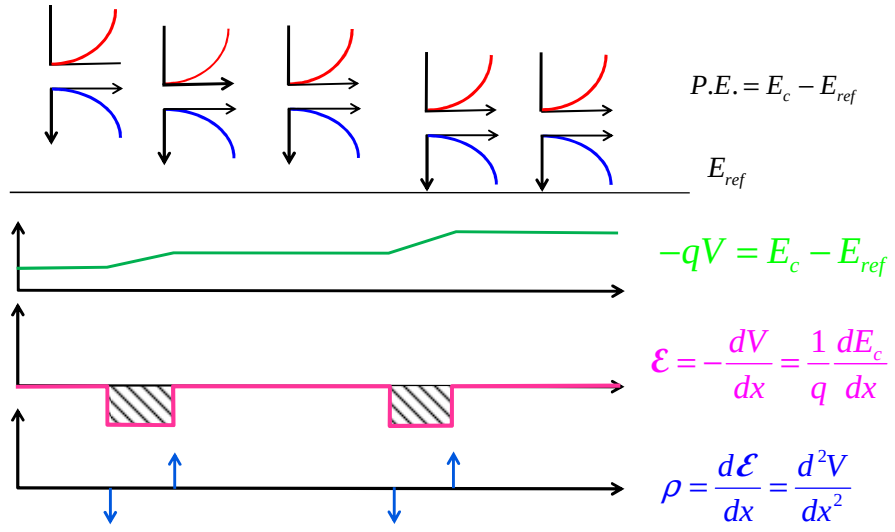


E-k Diagram vs. Band-diagram



$$P.E. = E_c - E_{ref} = -qV(x)$$

Potential, Field and Charge



In most practical cases start from charge and derive potentials!
=> Useful to learn "graphical" integration

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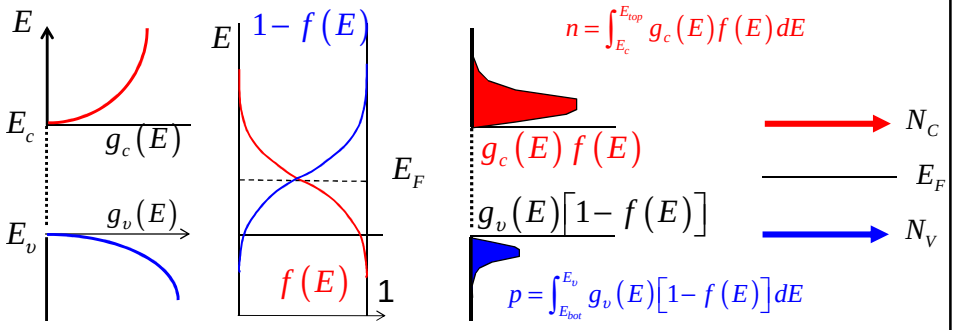
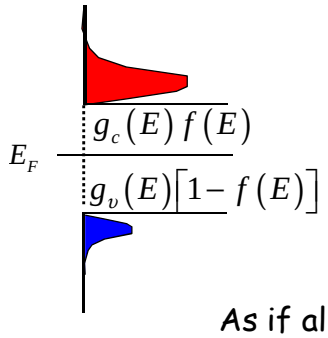
Carrier Distribution

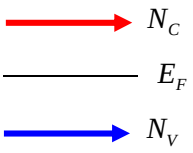
DOS F-D concentration



Critical items here;
Intrinsic semiconductor has VERY few active electrons
n_i is of the order of $10^{10}/cm^3$. In $10^{22}/cm^3$ atoms!
Do not include the coulomb interactions of individual free electrons
Often good enough to forget about the
Distribution of carriers in energy
=> Replace by delta functions at band edge

Effective Density of States





As if all states are at a single level E_C

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_c) \rightarrow N_C e^{-\beta(E_c - E_F)} \quad if \quad \beta(E_c - E_F) > 3$$

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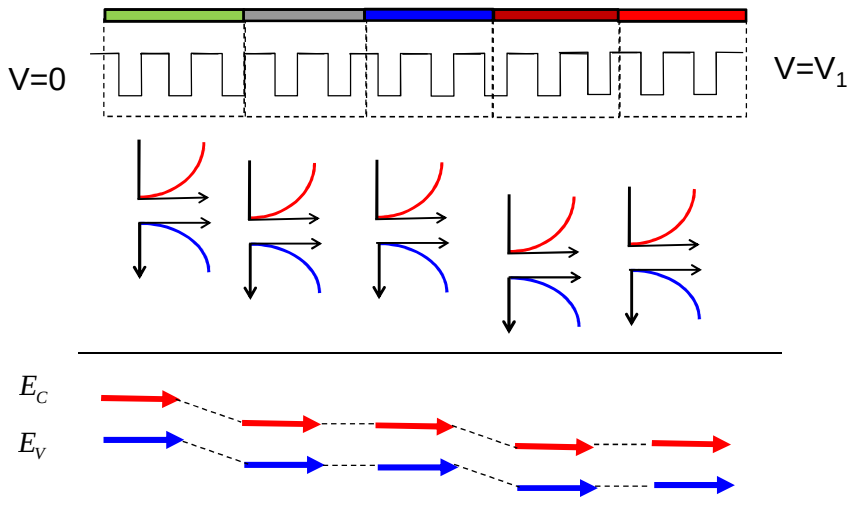
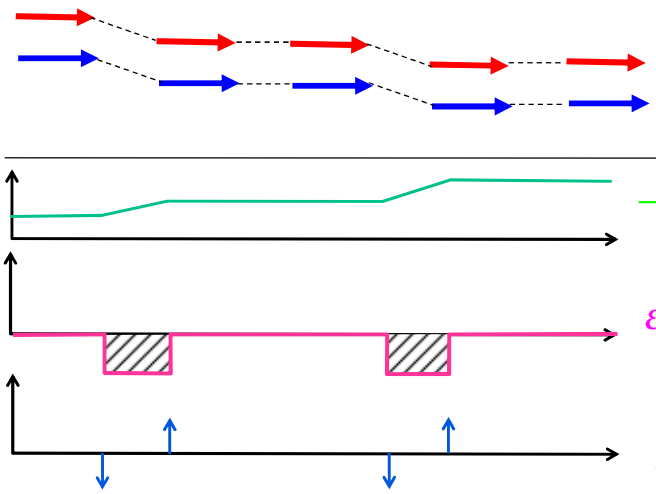
E-k Diagram vs. Band-diagram



All quantum mechanics is now hidden in a single point per band!



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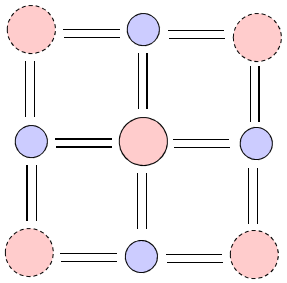
Insulator, Semiconductor, Metal

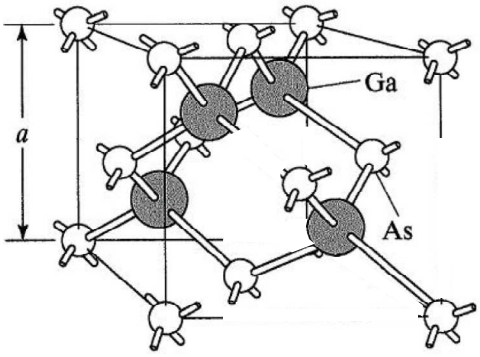
- Metal:
  - »Conducts electrons even at very low temperatures
  - »Fermi Level crosses multiple bands even at very low temperaturs
- Semiconductor
  - »Very weakly conducting
  - »Si: Eg=1.1eV ni~10^{10}/cm^3 in 10^{23}/cm^3 0.1 in a trillion
  - »GaAs: Eg=1.42eV ni~10^{6}/cm^3 in 10^{23}/cm^3
  - »Ge: Eg=0.8eV ni~10^{13}/cm^3 in 10^{23}/cm^3 0.1 in a billion
- Insulator
  - »"Not" conducting
  - »SiO2, Eg= 9eV, ni~10^{-68}/cm3
    - ✓ The whole earch has about 10^{50} atoms! If you made the whole worl out of glass there would be not one electon conductive at room temperature!

$$n_i = \sqrt{N_C N_V}\, e^{-\beta E_g/2}$$

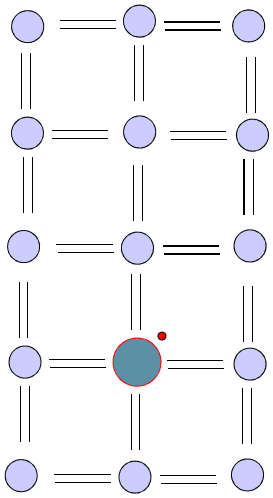
$$E_F \equiv E_i$$

Simplified Planar View of Atoms





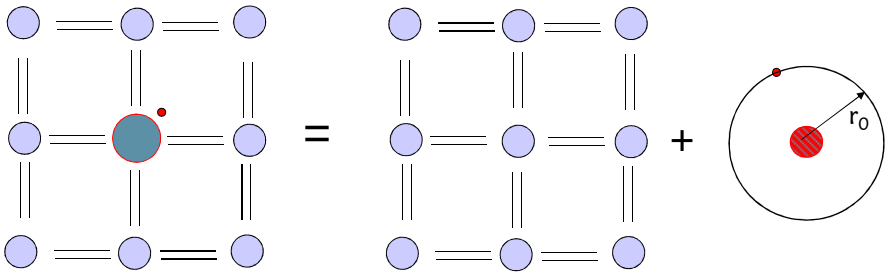
Donor Atoms

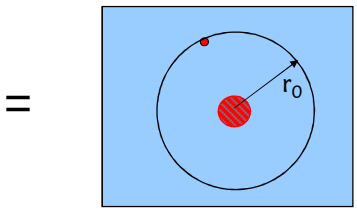


II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 **Al**	14 **Si**	15 **P**	16 S
30 **Zn**	31 **Ga**	32 Ge	33 **As**	34 **Se**
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po

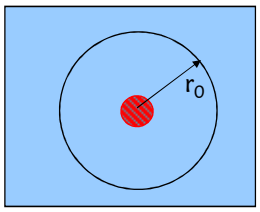
Even with donors, material is charge neutral

Donor Atoms in H_2-analogy





Donor Atoms in Real and Energy Space

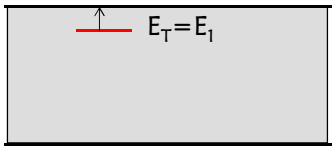


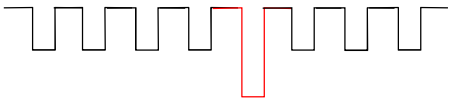
$$E_1 = -\frac{m^*_{host} q^4}{2\left(4\pi\varepsilon_0 K_{s,host}\hbar\right)^2}$$

$$= -\frac{m_0 q^4}{2\left(4\pi\varepsilon_0\hbar\right)^2}\frac{m^*_{host}}{m_0}\frac{1}{K_{s,host}^{\ 2}}$$

$$= -13.6\times\frac{m^*_{host}}{m_0}\frac{1}{K_{s,host}^{\ 2}}$$

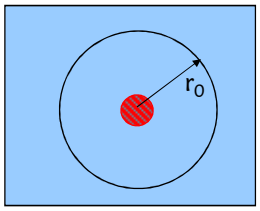
~10s meV





1/β~k_BT~25meV at T=300K

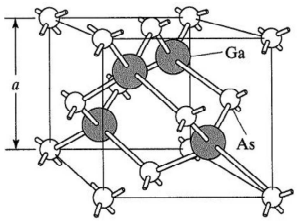
Assumption of Large Radius …



$$r_{1,P} = \frac{4\pi\varepsilon_0 K_{s,host}\hbar^2}{m^*_{host} q^2}$$

$$= \frac{4\pi\varepsilon_0\hbar^2}{m_0 q^2}\frac{m_0 K_{s,host}}{m^*_{host}}$$

$$= r_{1,H}\frac{K_{s,host}}{m^*_{host}/m_0}$$



$$r_{1,P} = 0.53\ A\times\frac{12.9}{0.53} = 12.9\ A$$

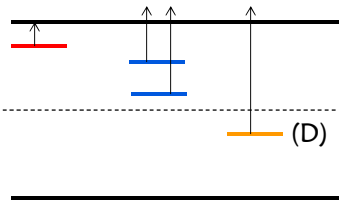
a~0.5nm=5A => hundreds of Si atoms

Characteristics of Donor Atoms

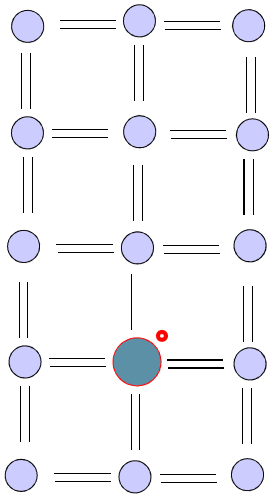
The number of donor atoms is much smaller compared to host atoms. Therefore, the electrons from one donor atom can go to the other donor atoms only via the conduction /valence bands of the host crystal.

Just like a Hydrogen atom, it is possible to have multiple localized level for a given atom (see the blue levels).

Good donors live close to the conduction band, so that they can offer electrons easily. However, if they are below the midgap, the donor levels are marked with (D) to differentiate them from acceptor atoms (which live close to the valence band).



Acceptor Atoms



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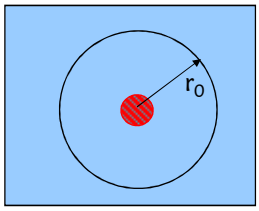

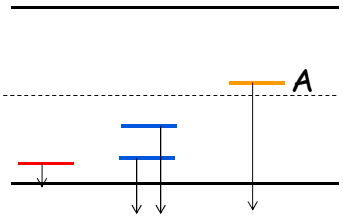

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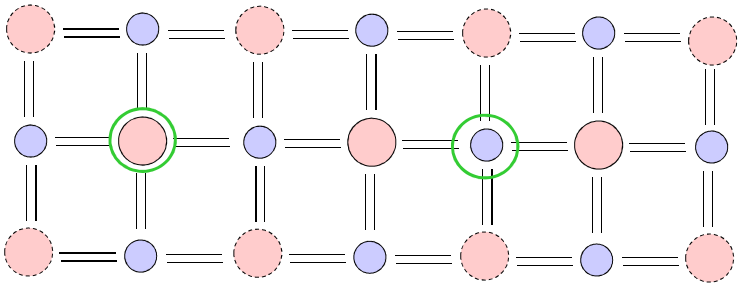
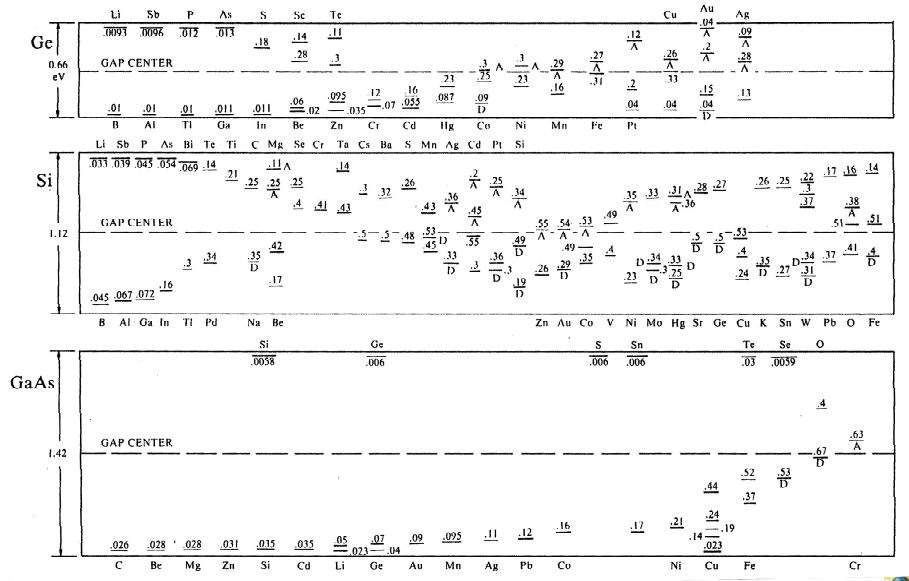
Donor-type

acceptor-type





Intrinsic carrier concentration is so small that semiconductor must be doped to make it useful.

A doping atom behaves like a H-atom, except that the dielectric constant and effective masses are given by by those of the host atom.

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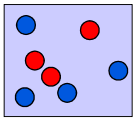
Looking ahead: Carrier-Density w/Doping

A bulk material must be charge neutral over all …

$$\int\left[p-n+N_D^+-N_A^-\right]dV=0$$



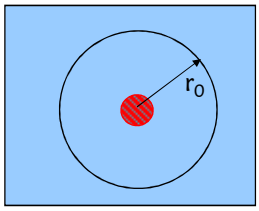
Further if the material is *spatially homogenous*

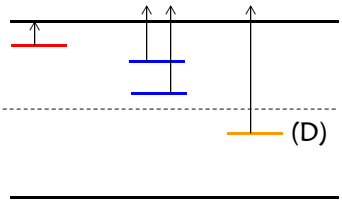
$$p-n+N_D^+-N_A^-=0$$

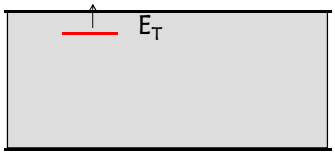
$$N_V e^{-(E_F-E_V)/k_BT}-N_C e^{-(E_C-E_F)/k_BT}+\frac{N_D}{1+2e^{(E_F-E_D)/k_BT}}-\frac{N_A}{1+4e^{(E_A-E_F)/k_BT}}=0$$

Let us see how the formula come about …

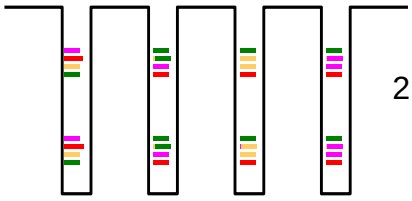
Characteristics of Donor Atoms



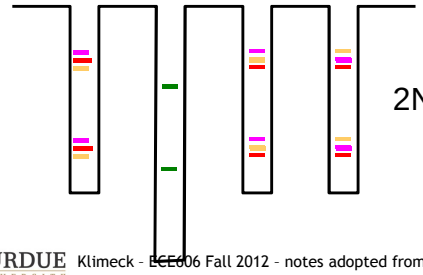




Localized vs. Delocalized States

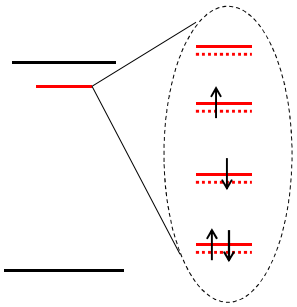


2N states/per-band (with spin)



2N-2 states/per-band (with spin)

$$P_i = \frac{e^{-(E_i - N_i E_F)/k_B T}}{\sum_i e^{-(E_i - N_i E_F)/k_B T}} \equiv \frac{e^{-(E_i - N_i E_F)/k_B T}}{Z}$$



u/d	E_i	N_i	P_i	
$0/0$	0	0	$1/Z$	
1/1	**x**	**x**	x	Coulomb interaction forbids this configuration

u/d	E_i	N_i	P_i
$0/0$	0	0	$1/Z$
$0/1$	1	1	$e^{-\frac{(E_i - E_F)}{k_B T}} \Big/ Z$
$1/0$	1	1	$e^{-\frac{(E_i - E_F)}{k_B T}} \Big/ Z$

Prob. that the donor is empty (charged)

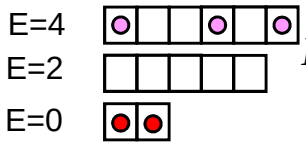
$$f_{00} = \frac{P_{00}}{P_{00} + P_{01} + P_{10}} = \frac{1/Z}{1/Z + 2e^{-(E_i - E_F)/k_B T}/Z} = \frac{1}{1 + 2e^{(E_F - E_i)/k_B T}}$$

Prob. that the donor is filled with at least one electron (neutral)

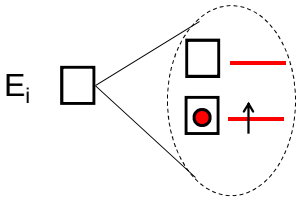
$$1 - f_{00} = 1 - \frac{1}{1 + 2e^{(E_F - E_i)/k_B T}} = \frac{1}{1 + \frac{1}{2} e^{(E_i - E_F)/k_B T}}$$

Note the extra factor ....

Coulomb Exclusion for Band Electrons?

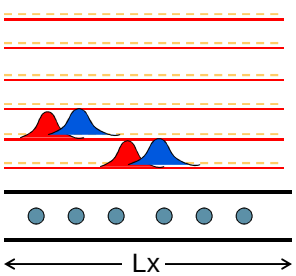


$$P_i = \frac{e^{-(E_i - N_i E_F)/k_B T}}{\sum_i e^{-(E_i - N_i E_F)/k_B T}} \equiv \frac{e^{-(E_i - N_i E_F)/k_B T}}{Z}$$



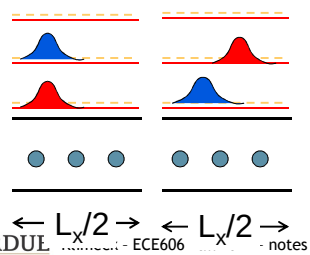
$state$	E_i	N_i	P_i
0	0	0	$1/Z$
1	1	1	$e^{-\frac{(E_i - E_F)}{k_B T}} \Big/ Z$

Localized vs. Band Electrons



E6 ← $12\pi/L_x$
E5 ← $10\pi/L_x$
E4 ← $8\pi/L_x$
E3 ← $6\pi/L_x$
E2 ← $4\pi/L_x$
E1 ← $2\pi/L_x$

Two electrons (even with opposite spin) can not be at the same position and same energy because of electrostatic repulsion

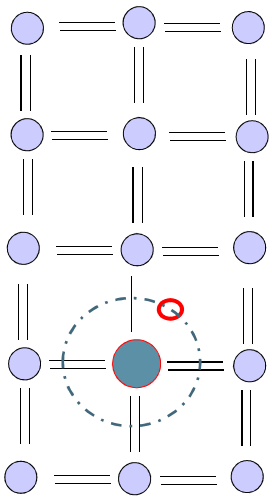


E3' ← $6\pi/(L_x/2)$
E2' ← $4\pi/(L_x/2)$
E1' ← $2p/(L_x/2)$

Band electrons (with opposite spin) need not be at the same position, so they can share occupy same energy level.

When we divide space by a factor of 2, the number of states (e.g. 6 here) does not change.

Acceptor Atoms

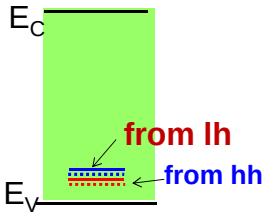


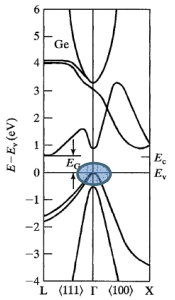
II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 **Al**	14 **Si**	15 **P**	16 S
30 **Zn**	31 **Ga**	32 Ge	33 **As**	34 **Se**
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po

State [1] …. Hole present … N-1 charges

State [0] … Hole filled …. N charges

Statistics of Acceptor Levels in Si and Ge



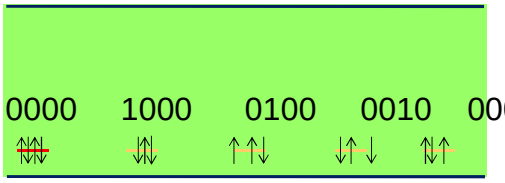


1. Each atom contributes 2 states (up & down spin) to a band, therefore a band has 2N states.

2. Every time a host atom is replaced by a impurity atom, 2 states are disappear per a band and appear as localized states (sort of).

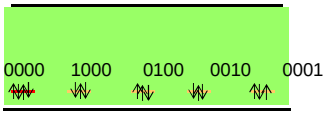
3. Therefore an acceptor atom close to hh and lh bands removes four states from those bands.

4. Because of Coulomb interaction only 1 hole can seat in these 4 states: the states are 0000, 0001, 0010, 0100, 1000.



4N-4 States
In HH/LH bands

1) [0000] is the charged state as it has N electrons, but N-1 protons.
2) Single hole configuration [0001] is uncharged, as we have N-1 electrons, and N-1 protons … same is true for [0010], [0100], [1000] states.
3) Going from [0000] to [0001] states, the number of electrons goes down by 1 (N_i=-1).
4) Going from [0000] to [0001] states energy goes down by $-E_A$, because one electron is no longer occupying the high energy level at E_A.



$$P_{0000} = \frac{e^{-(0-0xE_F)/k_BT}}{\sum_i e^{-(E_i - N_i E_F)/k_BT}} \equiv \frac{1}{Z}$$

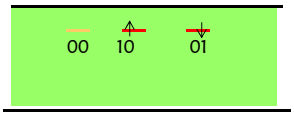
Steps 3 & 4

$$P_{0001} = P_{0010} = P_{0100} = P_{1000} = \frac{e^{-(-E_A-(-1)E_F)/k_BT}}{\sum_i e^{-(E_i - N_i E_F)/k_BT}} \equiv \frac{e^{(E_A - E_F)/k_BT}}{Z}$$

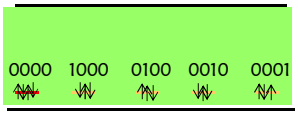
$$f_{0000} = \frac{P_{0000}}{P_{0000} + P_{1000} + P_{0100} + P_{0010} + P_{0001}} = \frac{1}{1 + 4e^{(E_A - E_F)/k_BT}}$$

Filled and empty Donor/Acceptor Levels

2N-2 states



$$N_D^{empty} \equiv N_D^+ = N_D f_{00} = N_D \frac{1}{1+2e^{(E_F - E_i)/k_B T}}$$



$$N_A^{filled} \equiv N_A^- = N_A [f_{0000}] = N_A \frac{1}{1+4e^{(E_A - E_F)/k_B T}}$$

4N-4 States
In HH/LH bands
(Two holes can not seat together)

Distributions are physical ....

$$f_D = \frac{N_D}{1+g_D e^{(E_F - E_D)/k_B T}}$$

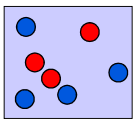
Degeneracy factor …

$$f_D = \frac{N_D}{1+e^{\varepsilon/k_B T} e^{(E_F - E_D)/k_B T}} = \frac{N_D}{1+e^{(E_F - E_D')/k_B T}}$$

Effective donor level

A bulk material must be charge neutral over all …

$$\int \left[p - n + N_D^+ + N_A^- \right] dV = 0$$



Further if the material is *spatially homogenous*

$$p - n + N_D^+ + N_A^- = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_A e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

- Intrinsic carrier concentration
- Potential, field, and charge
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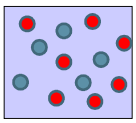
- Multiple doping, co-doping, and heavy-doping
- Conclusions

Reference: Vol. 6, Ch. 3 & 4

A bulk material must be charge neutral over all …

$$\int\left[p-n+N_D^+ + N_A^-\right]dV = 0$$



Further if the doping is spatially homogenous

$$p-n+N_D^+ + N_A^- = 0$$

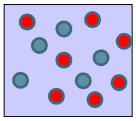
FD integral vs. FD function ?

$$N_V \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_F - E_V\right)\right] - N_A \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_C - E_F\right)\right] + \frac{N_D}{1+2e^{\beta(E_F - E_D)}} - \frac{N_A}{1+4e^{\beta(E_A - E_F)}} = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_A e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1+2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1+4e^{(E_A - E_F)/k_B T}} = 0 \quad \text{(approx.)}$$

Once you know E_F, you can calculate n, p, N_D^+, N_A^-.

$$p-n+N_D^+ + N_A^- = 0$$



$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1+2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1+4e^{(E_A - E_F)/k_B T}} = 0$$

$$n - p = 0 \Rightarrow N_C e^{-\beta(E_c - E_F)} = N_V e^{+\beta(E_v - E_F)}$$

$$E_F \equiv E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

Presentation Outline

- Intrinsic carrier concentration
- Potential, field, and charge
- E-k diagram vs. band-diagram
- Basic concepts of donors and acceptors
- Law of mass-action & intrinsic concentration
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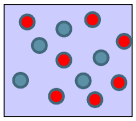
Reference: Vol. 6, Ch. 3 & 4

Carrier Density with Donors

In spatially homogenous field-free region …



$$p - n + N_D^+ + N_A^- = 0$$

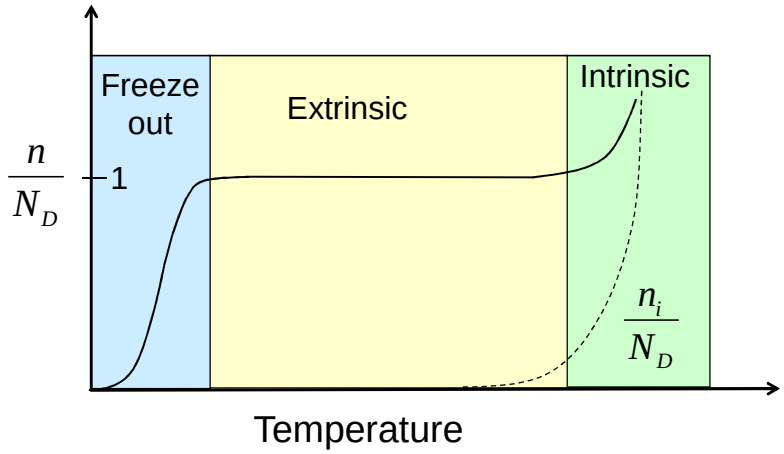
Assume
N-type doping …

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

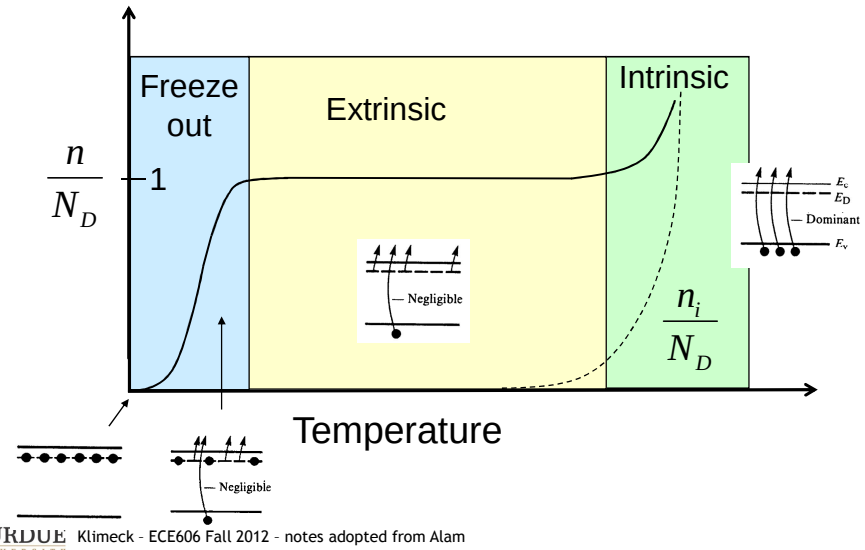
n

(will plot in next slide)

Temperature-dependent Concentration



Physical Interpretation



$$n = N_C e^{-\beta(E_C - E_F)} \Rightarrow \frac{n}{N_C} e^{\beta E_C} = e^{\beta E_F}$$

$$N_D^+ = \frac{N_D}{1 + 2e^{\beta(E_F - E_D)}} = \frac{N_D}{1 + 2\left[\frac{n}{N_C} e^{\beta(E_c - E_D)}\right]} \equiv \frac{N_D}{1 + \frac{n}{N_\xi}}$$

$$p - n + N_D^+ = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} = 0$$

$$p \times n = n_i^2$$

$$\frac{n_i^2}{n} - n + \frac{N_D}{1 + \frac{n}{N_\xi}} = 0$$

No approximation so far ….

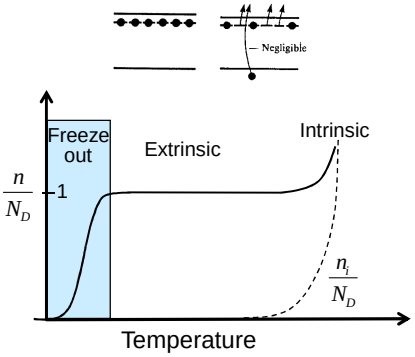
High Donor density/Freeze out T

$$\frac{n_i^2}{n} - n + \frac{N_D}{1+\frac{n}{N_\xi}} = 0$$

$$N_D \gg n_i$$

$$\Rightarrow -n + \frac{N_D}{1+\frac{n}{N_\xi}} \approx 0$$

$$\Rightarrow n^2 + N_\xi n - N_\xi N_D = 0$$



$$N_\xi \equiv \frac{N_C}{2} e^{-\beta(E_C - E_D)}$$

$$n = \frac{N_\xi}{2}\left[\left(1+\frac{4N_D}{N_\xi}\right)^{1/2} - 1\right]$$

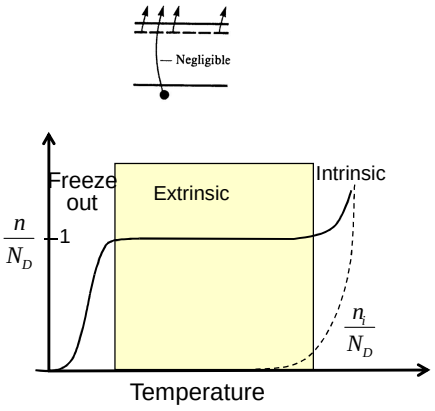
Extrinsic T

$$N_\xi \equiv \frac{N_C}{2} e^{-(E_C - E_D)/kT} \gg N_D$$

$$n = \frac{N_\xi}{2}\left[\left(1+\frac{4N_D}{N_\xi}\right)^{1/2} - 1\right]$$

$$\approx \frac{N_\xi}{2}\left[\left(1+\frac{1}{2}\frac{4N_D}{N_\xi}\right) - 1\right]$$

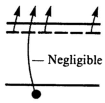
$$\approx N_D$$



Electron concentration equals donor density
hole concentration by nxp=n_i^2

$$N_D^+ = \frac{N_D}{1+2e^{(E_F-E_D)/k_BT}} \approx N_D \quad \text{for } E_F < E_D$$

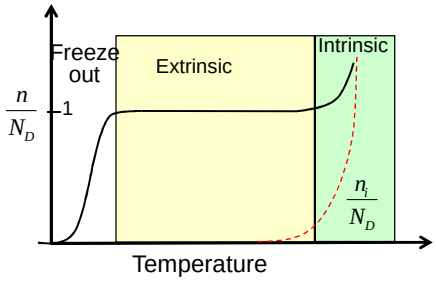
$2e^{(E_F-E_D)/k_BT} \to 0$



$$\frac{n_i^2}{n} - n + \frac{N_D}{1+\frac{n}{N_c}} = 0$$

$$\frac{n_i^2}{n} - n + N_D \approx 0$$

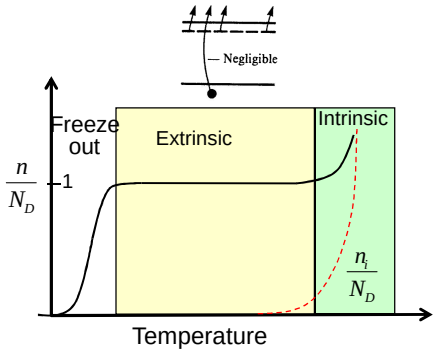
$$\Rightarrow -n_i^2 + n^2 - N_D n = 0$$



$$n = \frac{N_D}{2} + \left[\frac{N_D^{\,2}}{4} + n_i^2\right]^{1/2}$$

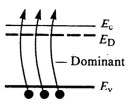
For $N_D \gg n_i$

$$n = \frac{N_D}{2} + \left[\frac{N_D^{\,2}}{4} + n_i^2\right]^{1/2} \approx N_D$$



For $n_i \gg N_D$

$$n = \frac{N_D}{2} + \left[\frac{N_D^{\,2}}{4} + n_i^2\right]^{1/2} \approx n_i$$



What will happen if you use silicon circuits at very high temperatures ?
Bandgap determines the intrinsic carrier density.

$$n = N_C e^{-\beta(E_c - E_F)} \Rightarrow E_F = E_C + \frac{1}{\beta} ln\left(\frac{n}{N_C}\right)$$

$$p - n + N_D^+ = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} = 0$$

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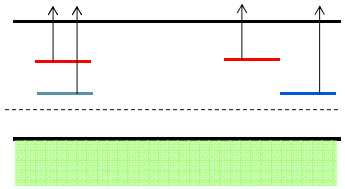
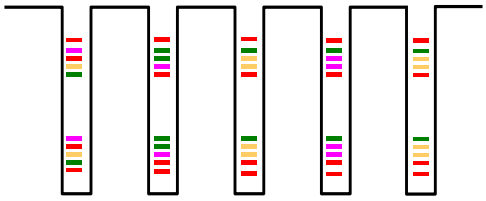


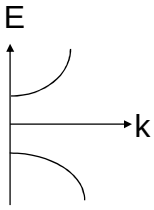
Multiple levels of same donor …

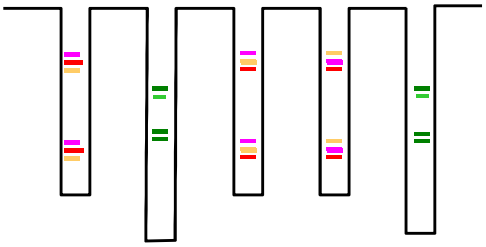
$$p - n + \frac{N_D}{1+2e^{(E_F - E_{D1})/k_BT}} + \frac{N_D}{1+2e^{(E_F - E_{D2})/k_BT}} - \frac{N_A}{1+4e^{(E_A - E_F)/k_BT}} = 0$$

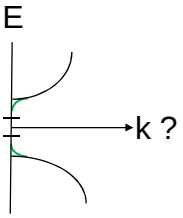
Codoping…

$$p - n + \frac{N_{D1}}{1+2e^{(E_F - E_{D1})/k_BT}} + \frac{N_{D2}}{1+2e^{(E_F - E_{D2})/k_BT}} - \frac{N_A}{1+4e^{(E_A - E_F)/k_BT}} = 0$$

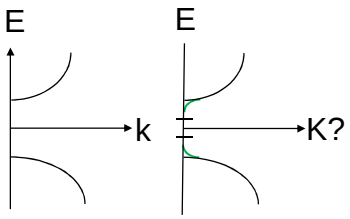








Heavy Doping Effects: Hopping Conduction



Bandgap narrowing

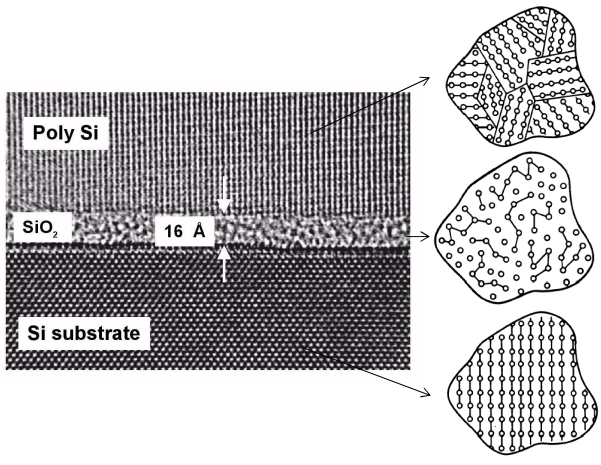
$$p \times n = N_C N_V e^{-\beta E_G^*}$$

e.g. Base of HBTs

Band transport
vs.
hopping-transport

e.g. a-silicon, OLED

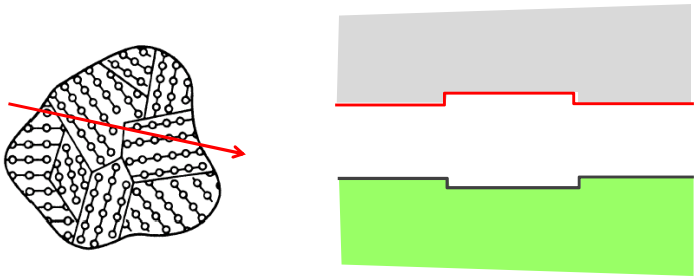
Arrangement of Atoms



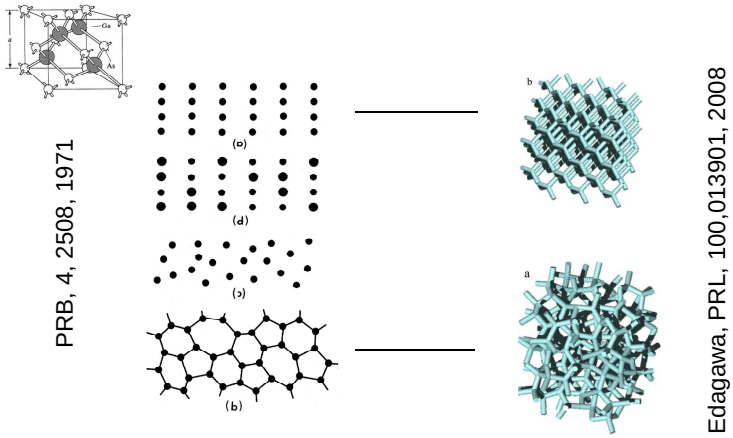
Poly-crystalline
Thin Film
Transistors

Amorphous
Oxides

Crystalline



Isotropic bandgap and increase in scattering



Periodicity is sufficient, but not necessary for bandgap.
Many amorphous material show full isotropic bandgap

1. Charge neutrality condition and law of mass-action allows calculation of Fermi-level and all carrier concentration.

2. For semiconductors with field, charge neutrality will not hold and we will need to use Poisson equation.

3. Heaving doping effects play an important role in carrier transport.