ECE-606 Homework No. 4 Assigned: ~~Feb. 2~~ Sep 11 Due: ~~Feb. 9~~ Sep 18

1a) $$E = \frac{\hbar^2}{2m_x}k_x^2 + \frac{\hbar^2}{2m_y}k_y^2 + \frac{\hbar^2}{2m_z}k_z^2$$

Can be rewritten into:

$$1 = \frac{k_x^2}{a^2} + \frac{k_y^2}{b^2} + \frac{k_z^2}{c^2}$$

Where $a^2 = \frac{2m_x E}{\hbar^2}$ and so on

Volume of ellipsoid is then $\frac{8\pi\sqrt{2}}{3\hbar^3}\sqrt{m_x m_y m_z}E^{3/2}$

$$DOS = \frac{\partial}{\partial E}\left(\frac{V}{8\pi^3}\right) = \frac{1}{\sqrt{2}\pi^2\hbar^3}\sqrt{m_x m_y m_z}E^{1/2}$$

Note spin not included.

Density of state mass $(m_x m_y m_z)^{1/3}$

1b) First, we write $v_j = \frac{1}{\hbar}\frac{\partial E}{\partial k_j} = \frac{\hbar}{m_j}k_j$ then substitute into $\frac{d\vec{k}}{dt} = -\frac{e}{\hbar}\vec{v} \times \vec{B}$

We get the following set of equations of interest,

$$\frac{dk_x}{dt} = -\frac{e}{m_y}k_y B \quad \text{------(1)}$$

$$\frac{dk_y}{dt} = \frac{e}{m_x}k_x B \quad \text{-------(2)}$$

Substitute (2) into (1)

$$\frac{d^2k_x}{dt^2} = -\frac{eB}{m_y}\frac{dk_y}{dt} = -\frac{e^2B^2}{m_y m_x}k_x$$

The solution for this differential equation is simply,

$k_x = \exp(i\omega_c t)$ where $\omega_c = \frac{eB}{\sqrt{m_x m_y}}$

Hence cyclotron mass is $m_c = \sqrt{m_x m_y}$

1c) since we know the solution has the form $\propto \exp\left(-i\omega_c t\right)$, we write:

$$i\omega_c k_1 - \frac{eB_3}{m_2}k_2 + \frac{eB_2}{m_3}k_3 = 0$$

$$i\omega_c k_2 - \frac{eB_1}{m_3}k_3 + \frac{eB_3}{m_1}k_1 = 0$$

$$i\omega_c k_3 - \frac{eB_2}{m_1}k_1 + \frac{eB_1}{m_2}k_2 = 0$$

The condition for nontrivial solutions is

$$\begin{vmatrix} i\omega_c & -\frac{eB_3}{m_2} & +\frac{eB_2}{m_3} \\ +\frac{eB_3}{m_1} & i\omega_c & -\frac{eB_1}{m_3} \\ -\frac{eB_2}{m_1} & +\frac{eB_1}{m_2} & i\omega_c \end{vmatrix} = 0$$

This yields,

$$i\omega_c\left[-\omega_c^2 + \frac{(eB_1)^2}{m_2 m_3}\right] + \frac{eB_3}{m_2}\left[i\omega_c\frac{eB_3}{m_1} - \frac{e^2 B_1 B_2}{m_1 m_3}\right] + \frac{eB_2}{m_3}\left[i\omega_c\frac{eB_2}{m_1} - \frac{e^2 B_1 B_3}{m_1 m_2}\right] = 0$$

The first solution is zero, the other solutions are given by

$$\omega_c^2 = e^2\left[\frac{B_1^2}{m_2 m_3} + \frac{B_2^2}{m_1 m_3} + \frac{B_3^2}{m_2 m_1}\right]$$

This gives us the final answer,

$$\frac{1}{m_c} = \sqrt{\frac{m_x\alpha^2 + m_y\beta^2 + m_z\gamma^2}{m_x m_y m_z}}$$

1d) Let z be the axis along the longitudinal direction. From

$$\frac{1}{m_c} = \sqrt{\frac{m_x\alpha^2 + m_y\beta^2 + m_z\gamma^2}{m_x m_y m_z}}$$, then set $\alpha = \sin\theta\cos\phi,\ \beta = \sin\theta\sin\phi,\ \alpha = \cos\theta$

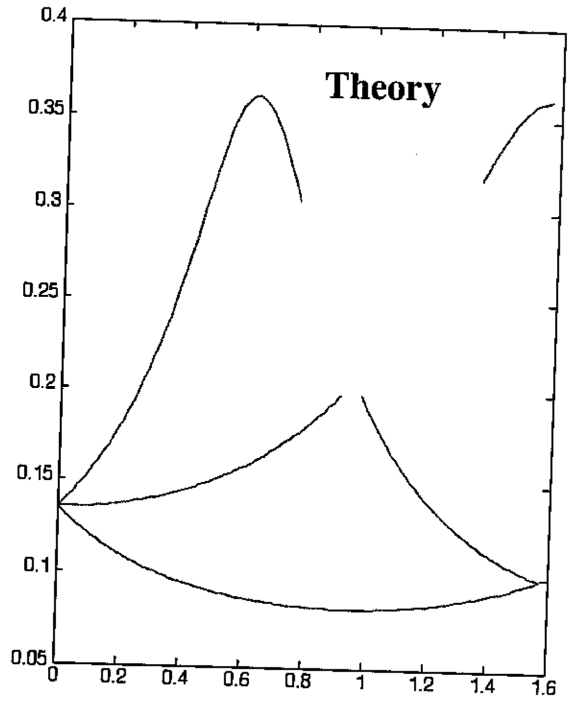
Also, make the replacement $\left(m_x, m_y, m_z\right) = \left(m_t, m_t, m_l\right)$

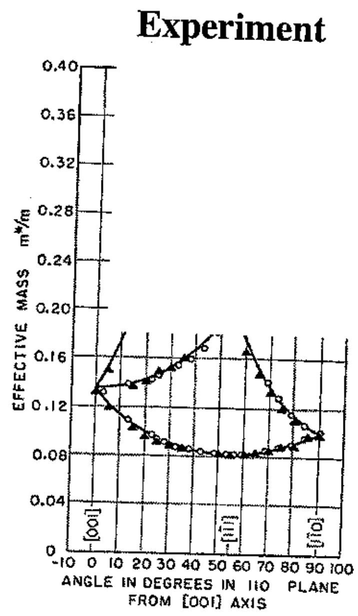
$$\frac{1}{m_c} = \sqrt{\frac{m_t\sin^2\theta\cos^2\phi + m_t\sin^2\theta\sin^2\phi + m_l\cos^2\theta}{m_t^2 m_l}}$$

$$\frac{1}{m_c} = \sqrt{\frac{\cos^2\theta}{m_t^2} + \frac{\sin^2\theta}{m_t m_l}}$$

MATLAB script: reproduce experiment by Dresselhaus.

```
clear;

ml=1.59;
mt=0.082;
N=100;

detaA=linspace(-pi/2,pi/2,N);

for j=1:N
    deta=detaA(j);
    Bvec=[cos(deta); sin(deta)*cos(pi/4); sin(deta)*sin(pi/4)];
    Bvec=Bvec/norm(Bvec);

    % [111]
    vec=[1; 1; 1];
    vec=vec/norm(vec);
    ang=acos(dot(Bvec,vec));
    dump=(cos(ang)^2/mt^2+sin(ang)^2/ml/mt)^0.5;
    mc1(j)=1/dump;
    % [-111]
    vec=[-1; 1; 1];
    vec=vec/norm(vec);
    ang=acos(dot(Bvec,vec));
    dump=(cos(ang)^2/mt^2+sin(ang)^2/ml/mt)^0.5;
    mc2(j)=1/dump;
    % [1-11]
    vec=[1; -1; 1];
    vec=vec/norm(vec);
    ang=acos(dot(Bvec,vec));
    dump=(cos(ang)^2/mt^2+sin(ang)^2/ml/mt)^0.5;
    mc3(j)=1/dump;
    % [-1-11]
    vec=[-1; -1; 1];
    vec=vec/norm(vec);
    ang=acos(dot(Bvec,vec));
    dump=(cos(ang)^2/mt^2+sin(ang)^2/ml/mt)^0.5;
    mc4(j)=1/dump;
end


figure;
plot(detaA,mc1)
hold on;
plot(detaA,mc2)
plot(detaA,mc3)
plot(detaA,mc4)
```





2) Answer to ASF 4.14

2a) At T=300K in Si, $n_i = 10^{10}/cm^3$. Thus here both $N_D \gg N_A$ and $N_D \gg n_i$. Thus per equation 4.80 a and b in textbook,

$$n = N_D = 10^{14}/cm^3$$
$$p = n_i^2 / N_D = 10^6 / cm^3$$

2b) Here $N_A \gg N_D$ and $N_A \gg n_i$. Thus per Eq. 4.80c and d,

$$p = N_A = 10^{15}/cm^3$$
$$n = n_i^2 / N_A = 10^5 / cm^3$$

2c) Here we must retain both N_A, N_D, but $N_D - N_A \gg n_i$

$$n = N_D - N_A = 10^{15}/cm^3$$
$$p = n_i^2 / n = 10^5 / cm^3$$

2d) We deduce from fig 4.17 that at T=470K, $n_i(Si) = 10^{14}/cm^3$. Clearly n_i is comparable to N_D and we must use Eq. 4.79a to compute n,

$$n = \frac{N_D}{2} + \left[\left(\frac{N_D}{2}\right)^2 + n_i^2\right]^{1/2} = 1.62 \times 10^{14}/cm^3$$

$$p = n_i^2 / n = 6.18\times10^{13} / cm^3$$

2e) We conclude from Fig 4.17 that at T=645K, $n_i(Si) = 10^{16} / cm^3$. Here $n_i \gg N_D$. Thus, $n \approx p \approx n_i = 10^{16} / cm^3$

3) Answer to ASF 4.16

3a) As deduced from fig 4.17, $n_i \approx 10^{15} / cm^3$ when T=545K

$N_D(cm^{-3})$	Eq used	$n(cm^{-3})$	$p(cm^{-3})$
10^{13}	$n_i \gg N_D, n \approx n_i$	10^{15}	10^{15}
10^{14}	$n_i \approx N_D$, Eq 4.79a	1.05×10^{15}	9.51×10^{14}
10^{15}	$n_i \approx N_D$, Eq 4.79a	1.62×10^{15}	6.17×10^{14}
10^{16}	$n_i \approx N_D$, Eq 4.79a	1.01×10^{16}	9.9×10^{13}
10^{17}	$N_D \gg n_i, n \approx N_D$	10^{17}	10^{13}

3b) $$E_i - \left(\frac{E_c + E_v}{2}\right) = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}kT\ln\left(\frac{0.81}{1.182}\right) = -0.0133eV$$

$$E_F - E_i = kT\ln\left(\frac{n}{n_i}\right) = 0, 0.00229eV, 0.0227eV, 0.109eV, 0.216eV$$

5a) The band gap is 1.13 eV

5b) With a 3 sq.nm wire the band gap work out to 1.47 eV, while a 4sq. nm gives a corresponding value of 1.36 eV. Note that the band gap is the difference in energy states between the highest valence band and lowest conduction band The band gap has decreased for a wire with larger cross sectional area

5c) If a simple particle box in a model is assumed, the energy states are inversely proportional to the square of the dimensions of confinement. Consequently, a wire with larger cross section has eigen states close to the bottom of the conduction band in a bulk structure.

5d) Three physical parameters that would change are 1. Band gap 2. Effective mass 3. Density of states. The last two parameters change on account of the difference in curvature of the bands (compared to bulk) produced under confinement. Band gap changes because confinement pushes up (down) the CB(VB).

5e) The changes observed in the eigen spectrum arise because of confinement. As the confinement was altered in the previous cases, the energy bands shifted in accord with the inverse square law that operates for particle in a box.

5f) Nano-structures would begin to resemble bulk when the dimensions are increased several fold.

4a) See page 121 of ASF.

4b)

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = \sqrt{\frac{3.23\times10^{19}\times1.83\times10^{19}}{300^3}} T^{1.5} \exp\left(-\frac{1.12\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times T}\right) = 1\times10^{15}$$

$$T^{1.5} \exp\left(-\frac{6493}{T}\right) = 0.2137$$

Discard

By some iteration, we can find $T \approx 585K$. This is consistent, as we can see that after this temperature, the semiconductor behaves like intrinsic, with $n \approx p$

4c) Yes it is reasonable. The match is surprisingly well.

4d) At low temperature, it starts from near the conduction band edge and as temperature increase, it position near the mid gap, consistent with the picture that the semiconductor turns intrinsic.

4e) As temperature increase, more donors get ionized and eventually become fully ionized at high enough temperature.

Q4 According to classical statistics, a free electron with three translational degrees of freedom must obey the equipartition theorem.

$$E = \frac{3}{2} KT = \frac{3}{2}\left(300 \times 8.61 \times 10^{-5}\right) = 0.039 \text{ eV}$$

From Fermi-Dirac statistics, average energies are of the order of 5.0 eV. So the classical prediction is wrong. The difference is due to the fact that electrons obey the exclusion principle and cannot fit all the low-lying states near KT.

Sub-part 1). Generally, classical statistics are appropriate when the particles are far enough. They can be regarded as distinguishable then. This occurs when the average separation is greater than the de-Broglie wavelength.

Sub-part 2).

$$f_{MB} = \exp\left(-(E-E_f)/kT\right) \quad \text{for } E-E_f > 4kT$$

$$f_{FD} = \frac{1}{1+\exp(E-E_f)/kT}$$

$$\text{error} = \frac{f_{MB} - f_{FD}}{f_{FD}} = \exp\left(-(E-E_f)/kT\right) < 5/100$$

$$\Rightarrow E-E_f > kT \ln\left(\frac{100}{5}\right) = kT \ln 20 = \underline{\underline{2.99\,kT}}$$