Solutions

HW #5

ECE 606

(2)

**Problem ~~4A~~ a:**

The continuity equation for the excess hole is given by:

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - r_p + g_p$$

For steady state, dark room and low level recombination, we have

$$-\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta p}{\tau_p} = 0$$

The steady state current is given by:

$$J_p = q\mu E \Delta p - qD_p \frac{\partial \Delta p}{\partial x}$$

Substitute back into continuity equation we get,

$$D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau_p} = 0$$

Using the hint, we have the following roots,

$$r = \frac{\mu E \pm \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p}$$

Hence

$$r_1 = \frac{\mu E + \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p} \quad \text{and} \quad r_2 = \frac{\mu E - \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p}$$

$$\Delta p(x) = c_1 \exp(r_1 x) + c_2 \exp(r_2 x)$$

To determine $c_1$ and $c_2$ we make use of boundary condition.

$$\Delta p(0) = c_1 + c_2$$

$$\Delta p(L) = c_1 \exp(r_1 L) + c_2 \exp(r_2 L) = 0$$

$$\Rightarrow c_1 = -c_2 \exp((r_2 - r_1)L)$$

Therefore the final solution is:

$$\Delta p(x) = \frac{\Delta p(0)}{1 - \exp((r_1 - r_2)L)} \exp(r_1 x) + \frac{\Delta p(0)}{1 - \exp((r_2 - r_1)L)} \exp(r_2 x)$$

**Problem 4B:**

$$D_p = \frac{kT}{q}\mu_p = 10 cm^2/s$$

We have also the following constants for

$$\Delta p(x) = c_1 \exp(r_1 x) + c_2 \exp(r_2 x)$$

$r_1 = 1.22\times10^5$

$r_1 = -8.2\times10^4$

$c_1 = -1.5\times10^{20}$

$c_2 = 1.15\times10^{21}$

The drift current at x=0 is given by:

$q\mu E\Delta p = 6.41\times10^3\, A/m^2$

The diffusion current at x=0 is given by;

$-qD_p \frac{\partial \Delta p}{\partial x} = 1.8\times10^4\, A/m^2$

Supposed now we assume E=0,

$r_1 = 1.0\times10^5$

$r_1 = -1.0\times10^5$

$c_1 = -1.57\times10^{20}$

$c_2 = 1.157\times10^{21}$

$-qD_p \frac{\partial \Delta p}{\partial x} = 2.1\times10^4\, A/m^2$

Discard.

The fraction error introduced is approximately 0.167.

**Problem 4C:**

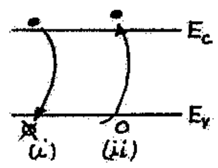
False, True, False, False

②

62

(5.10)

(a)




(b) RECOMBINATION ... We note first of all that $\frac{\partial n}{\partial t}\big|_R = \frac{\partial p}{\partial t}\big|_R$. Also, following a text-like argument, the recombination rate is expected to be proportional to both $n$ and $p$, the electron and hole concentrations. Letting $c_b$ be the proportionality constant, we conclude

$$\frac{\partial n}{\partial t}\Big|_R = \frac{\partial p}{\partial t}\Big|_R = -c_b np \qquad (c_b \geq 0)$$

GENERATION... The generation rate, $\frac{\partial n}{\partial t}\big|_G = \frac{\partial p}{\partial t}\big|_G$, should depend on the number of filled states in the valence band and on the number of empty states in the conduction band. If the semiconductor is taken to be nondegenerate, both of the cited quantities are approximately constant. Thus

$$\frac{\partial n}{\partial t}\Big|_G = \frac{\partial p}{\partial t}\Big|_G = \beta = \text{constant}$$

Combining the above relationships yields

$$r_b = -\left(\frac{\partial n}{\partial t}\Big|_R + \frac{\partial n}{\partial t}\Big|_G\right) = -\left(\frac{\partial p}{\partial t}\Big|_R + \frac{\partial p}{\partial t}\Big|_G\right) = c_b np - \beta$$

Next invoking the equilibrium simplification based on the principle of detailed balance gives

$$r_b\big|_{\text{EQUILIBRIUM}} = 0 = c_{b0} n_0 p_0 - \beta_0$$

or $\quad \beta_0 = c_{b0} n_0 p_0 = c_{b0} n_i^2$

Making the usual assumption that $\beta$ and $c_b$ do not change significantly under non-equilibrium conditions then yields



3

and $\beta \simeq \beta_0 = c_{b0} n_i^2 \simeq c_b n_i^2$

$$\left[ r_b = c_b (np - n_i^2) \right]$$

(c) Letting $n = n_0 + \Delta n$
$p = p_0 + \Delta p = p_0 + \Delta n$
and substituting into the $r_b$ expression gives

$$r_b = c_b(\cancel{n_0 p_0} + n_0 \Delta n + p_0 \Delta n + \underbrace{\Delta n^2}_{\text{neglect for low level injection}} - \cancel{n_i^2})$$
$$= c_b (n_0 + p_0)\Delta n$$

or

$$\left[ \begin{array}{l} r_b = \dfrac{\Delta n}{\tau_b} \\[2ex] \tau_b = \dfrac{1}{c_b(n_0+p_0)} \end{array} \right]$$

(d) A nondegenerate doping in silicon at 300K corresponds to a doping of $N_D$ or $N_A \lesssim 10^{18}/\text{cm}^3$. Thus the typical range of interest for $n_0+p_0$ is $10^{14}/\text{cm}^3 \lesssim n_0+p_0 \lesssim 10^{18}/\text{cm}^3$. Computing $\tau_b$ from the part (c) result with $c_b = 5\times10^{-15}\ \text{cm}^3/\text{sec}$, and comparing against the worst case senario ($\tau_{R\text{-}G\,\text{center}}$ = 1 msec for p-type Si) as deduced from Fig 5.11, we obtain

| $N_D$ or $N_A$ ($\text{cm}^{-3}$) | $\tau_b$ (sec) | $\tau$ (Fig 5.11) (sec) |
|---|---|---|
| $10^{14}$ | 2 | $1\times10^{-3}$ |
| $10^{15}$ | $2\times10^{-1}$ | $1\times10^{-3}$ |
| $10^{16}$ | $2\times10^{-2}$ | $1\times10^{-3}$ |
| $10^{17}$ | $2\times10^{-3}$ | $6\times10^{-4}$ |
| $10^{18}$ | $2\times10^{-4}$ | $1\times10^{-5}$ |

For this worst case senario, $\tau_b$ is $\geq 20\times\tau$ over most of the doping range, only dropping to $\sim 3\times\tau$ near $N_A = 10^{17}/\text{cm}^3$. Clearly, one seldom has to worry about band-to-band recombination in nondegenerately doped Si at room temperature.

**Q3** Intrinsic silicon at 300 K.

$$n_0 = p_0 = n_i = 10^{10}\ \text{cm}^{-3}$$

$$\mu_n = 1360\ \text{cm}^2/\text{V.s}$$

$$\mu_p = 460\ \text{cm}^2/\text{V.s}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)} = \frac{1}{1.6 \times 10^{-19}(1360 + 460) \times 10^{10}} = 3.43 \times 10^5\ \Omega/\text{cm}$$

n- doped silicon $N_D = 10^{16}\ \text{cm}^{-3}$ (300 K)

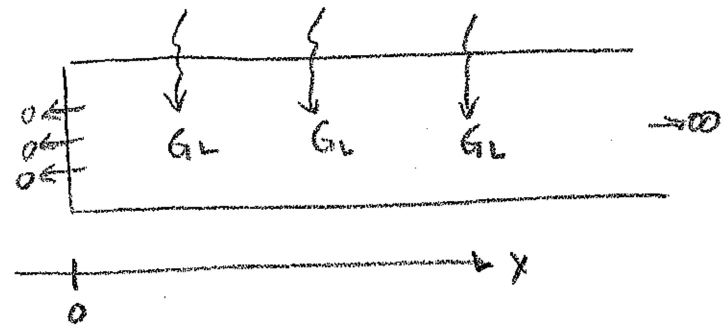
$$\rho = \frac{1}{q\mu_n N_D} = \frac{1}{1.6 \times 10^{-19} \times 1360 \times 10^{10}} = 0.46\ \Omega/\text{cm}$$

Q4



4. A semi-infinite n-type semiconductor bar is subject to uniform penetrating illumination resulting in a generation rate of $G_L$ electron-hole pairs per second per cm$^2$ throughout the bar. $G_L$ is such that sample remains in low-level injection. Minority carriers are extracted at the surface at $x = 0$. Obtain an expression for the maximum hole current that can be drawn from the bar in steady-state.

[20 pts]




MCDE: $\frac{\partial \Delta p_n}{\partial t} = 0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$

GEN'L SOL'N: $\Delta p_n(x) = A e^{-x/L_p} + B e^{x/L_p} + \tau_p G_L$

BC: at $x = \infty$, $\Delta p_n$ must be finite, so set $\boxed{B=0}$.

at $x = 0$, to extract holes at the highest possible rate, set $p_n(0) = 0$. Then

$\Delta p_n(0) = p_n(0) - p_{n0} = -p_{n0}$.

$\therefore \Delta p_n(0) = A e^0 + \tau_p G_L = -p_{n0}$

so $A = -(p_{n0} + \tau_p G_L)$

and $\boxed{\Delta p_n(x) = \tau_p G_L - (p_{n0} + \tau_p G_L) e^{-x/L_p}}$

$J_p(x) = -q D_p \frac{\partial \Delta p_n}{\partial x} = -q \frac{D_p}{L_p}(p_{n0} + \tau_p G_L) e^{-x/L_p}$

and $\boxed{J_p(0) = -q \frac{D_p}{L_p}(p_{n0} + \tau_p G_L)}$

6

16.00

(5)



~~8~~. Consider a region in a semiconductor that is totally depleted of carriers ($n = p = 0$). Obtain an expression for the energy level of the RG centers relative to midgap $\Delta E = (E_T - E_i)$ that results in the highest possible generation rate. Your answer should include the minority carrier lifetimes $\tau_N$ and $\tau_P$.

[20 pts]

$$R = \frac{pn - n_i^2}{\tau_n(p+p_1) + \tau_p(n+n_1)} \qquad \begin{cases} n_1 = n_i e^{(E_T - E_i)/kT} \\ p_1 = n_i e^{(E_i - E_T)/kT} \end{cases}$$

If $n = p = 0$, $\quad -R = G = \dfrac{n_i^2}{\tau_n p_1 + \tau_p n_1}$

$$G = \frac{n_i^2}{\tau_n n_i e^{(E_i - E_T)/kT} + \tau_p n_i e^{(E_T - E_i)/kT}}$$

Let $(E_T - E_i) = \Delta E$, then $G = \dfrac{n_i}{\tau_n e^{-\Delta E/kT} + \tau_p e^{\Delta E/kT}}$

Set $\dfrac{dG}{d\Delta E} = 0$ to find the maximum generation rate.

$$\frac{dG}{d\Delta E} = \frac{(-n_i)}{(\tau_n e^{-\Delta E/kT} + \tau_p e^{\Delta E/kT})^2}\left(-\frac{\tau_n}{kT} e^{-\Delta E/kT} + \frac{\tau_p}{kT} e^{\Delta E/kT}\right) = 0$$

$\therefore \quad \tau_p e^{\Delta E/kT} - \tau_n e^{-\Delta E/kT} = 0$

multiply by $e^{\Delta E/kT}$ gives $\tau_p e^{2\Delta E/kT} = \tau_n$

$$e^{2\Delta E/kT} = \frac{\tau_n}{\tau_p} \longrightarrow \Delta E = E_T - E_i = \boxed{\frac{kT}{2} \ln\left(\frac{\tau_n}{\tau_p}\right)}$$

(7)

9:2n