# Edge Sharpness Perception with Force and Contact Location Information

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**ABSTRACT**

The effect of contact location information on the perception of virtual edges was investigated by comparing human edge sharpness discrimination under the force-alone and force-plus-contact-location conditions. The virtual object consisted of the 2D profile of an edge with two adjoining surfaces. Edge sharpness JNDs for both conditions increased from about 2 to 7 mm as the edge radii increased from 2.5 to 20.0 mm, and no significant difference was found between the two conditions. A follow-up experiment with the contact-location alone condition resulted in higher (worse) edge sharpness discrimination thresholds, especially at higher edge radius values. Our results suggest that contact location cues alone are capable of conveying edge sharpness information, but that force cues dominate edge sharpness perception when both types of cues are available.

**KEYWORDS:** contact location display, edge sharpness perception, curvature discrimination.

**INDEX TERMS:** H.5.1 [INFORMATION INTERFACES AND PRESENTATION]: Multimedia Information System - Artificial augmented, and virtual realities; H.5.2 [INFORMATION INTERFACES AND PRESENTATION]: User Interfaces (D.2.2,H.1.2,I.3.6) - Haptic I/O

## 1 INTRODUCTION AND BACKGROUND

Most haptic systems can be categorized into three groups: vibrotactile displays, force-feedback devices and fingertip displays. Vibrotactile displays use vibrating elements such as resonant-type tactors to stimulate skin surfaces with low-amplitude and high-frequency vibratory signals. Vibrotactile feedback has been widely used for sensory substitution in the past [1] and more recently, in mobile [2-3] and wearable [4-6] applications. Force-feedback devices stimulate the receptors located in muscles, tendons and joints by imparting forces to the user's hand via a manipulandum (e.g., stylus) or thimble interface. Examples of commercially available force-feedback devices include the PHANToM (Sensable Technology, Woburn, MA), the OMEGA series (Force Dimension, Switzerland) and the Maglev (Butterfly Haptics, Pittsburgh, PA). These devices can provide realistic force interactions between an interface tool and objects in a virtual environment. They are used in a wide range of applications including teleoperation [7], medical training [8], and education [9-10]. A major limitation of both vibrotactile displays and force-feedback devices is that they deprive the user of distributed stress and strain information on the fingertips. Numerous studies have demonstrated the importance of cutaneous information on the fingertips for object identification [11] and tactile shape perception [12-13]. Fingertip displays attempt to restore the missing cutaneous information, and the present study belongs to this growing research area.

The term "fingertip haptics" was first coined by Colgate's group at Northwestern University [14]. Fingertip displays include pin-array devices [15], actuated plates that convey surface orientation and curvature [16-17], contact area display [18], slip display [14], contact location display [19-20], skin stretch displays [21-23], thermal displays [24] and variable-friction surfaces modulated mechanically [25] or electrically [26]. The present study uses the contact location display (CLD) that was originally developed at Stanford University [19] to investigate the perception of local features such as corners and small protrusions on object surfaces. The CLD can be mounted on force-feedback devices. As the user moves his/her finger across a surface, an actuated roller moves on the fingertip as, say, an edge of a corner would, thereby augmenting force feedback with tactile contact information. The ability to keep track of the movement of surface features such as edges is expected to facilitate contour following, feature identification and object manipulation.

Initially, a 1 degree-of-freedom (1-DOF) CLD prototype was developed and attached to a PHANToM force-feedback device. It was used in a study to measure human curvature discrimination thresholds using physical and virtual curvature models. The results showed similar levels of discrimination, indicating the usefulness of the CLD in conveying curvature information [19]. In this initial study, the virtual fingertip was modeled as a line segment and the orientation of the real and virtual fingertips remained horizontal due to a hardware limitation. In a subsequent study [20], the virtual fingertip was modeled as a circular arc, and a rotary encoder was added to the CLD for measuring fingertip orientation. The participants were asked to trace a right-angle corner as quickly as possible without breaking contact. The results indicated that the participants were able to perform the contour following task in less time and with fewer failures (loss of contact) with the CLD. More recent studies have shifted the focus from hardware validation to algorithm development for rendering force and contact location information simultaneously with virtual objects. A new shading algorithm for polygonal object models was developed based on re-parameterized Bezier approximations [27]. Participants were asked to discriminate between an ideal smooth cylinder and equivalent polygonal models under various conditions (with and without shading and tactile feedback). The results, reported in terms of the maximum angular changes between adjacent polygons for rendering smooth objects, suggested that the new shading algorithm can significantly reduce a user's sensitivity to discontinuities in polygonal models. Compared to other shading schemes (e.g., [28]), the new shading algorithm can provide both tactile (contact location) and force shading. The thresholds obtained in [27] was incorporated into a

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3D force and contact-location rendering platform and evaluated with a 3D object identification task [29].

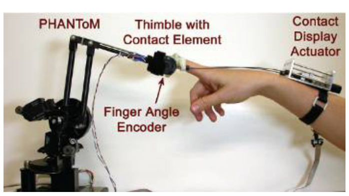
The present study investigates the extent to which the CLD benefits the exploration and perception of shape primitives in touch. As corners of varying degrees of sharpness are representative of most local geometrical surface features, we selected corner sharpness perception as the focus of the present study. Because the current prototype of the CLD is 1-DOF, it is technically more accurate to describe the objects used in the present study as edges adjoining two flat surfaces. Our specific aim is to measure the relative contributions of force and contact location information in sharpness perception of virtual edges. We hypothesized that the availability of contact location information, enabled by the actuated roller on the CLD that simulated the movement of a virtual edge on the fingertip, should enhance a user's ability to discriminate the sharpness of virtual edges. The results of the present study can help us better understand the haptic cues involved in object manipulation and shape perception. They also have important implications for the design of future generations of CLDs.

The remainder of this paper is organized as follows. The next section presents the methods for the edge sharpness discrimination experiment. The results are shown in Sec. 3. In the last section, we discuss the implications of our results and present additional data collected to clarify our findings.

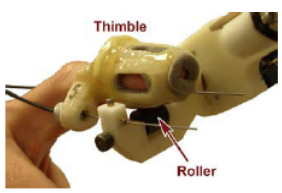
## 2 Methods

### 2.1 The Contact Location Display (CLD) System

The CLD system displays contact location and contact force simultaneously by combining custom-designed CLD hardware with a PHANToM force feedback device. A linear 1-DOF mechanism is attached to a user's fingertip and forearm to provide contact location information as shown in Figure 1(a). A user's fingertip is held by an open-bottom thimble, through which a roller is located as illustrated in Figure 1(b). Readings from the position encoders on the PHANToM and CLD are used to calculate the position of the user's finger. An additional encoder attached to the PHANToM gimbal, shown as the "Finger Angle Encoder" in Figure 1(a), measures the orientation of the user's fingertip.



(a)



(b)

Figure 1. (a) Contact location display system. (b) Side view of the roller in the contact location display. (From Fig.2b in [20], and Fig. 4 in [27], respectively.)

A small cylindrical roller with a radius of 4.8 mm is suspended beneath the user's fingertip. The roller can be moved relative to the fingertip along the distal-proximal direction (i.e., along the length of the fingertip) by two actuated push-pull wires. Contact of the roller with the fingertip occurs passively whenever the user presses the finger down to counterbalance forces exerted by the PHANToM. The roller has a fixed range of motion of 16 mm, and a nominal position resolution of 0.17 µm. No force is applied to the roller when the user's finger is in free space. When a contact with a virtual object is detected, the rendering software calculates both the contact force and location and renders it through the PHANToM and the CLD, respectively. A PID controller moves the roller to the desired location along the length of the user's finger. More details about the hardware and the controller can be found in [19-20].

### 2.2 Haptic Rendering

The haptic rendering software was developed in Visual C++ using the CHAI3D library (www.chai3d.org). For collision detection, the user's fingertip was represented in 2D as a circular arc with a radius of 20 mm (see Figure 2).[1] The position of the virtual fingertip was updated at 1 kHz, based on the user's fingertip position calculated from the PHANToM's position encoders and the gimbal's finger angle encoder. The entire space was divided into three regions similar to Voronoi regions [30]. As can be seen in Figure 2, each region contained one of the three primitives of the virtual object: the edge and the two surfaces. If any part of the virtual finger fell into a region, then the minimum distance between points on the virtual finger and the object surface was calculated. The point on the virtual finger that was closest to the virtual object became a candidate for a possible point of collision. There could be up to three collision-point candidates. The point with the minimum distance to the object surface was selected as the most likely contact point on the fingertip ($x_f$). If this point was on or inside the virtual surface, then a collision was detected, and the virtual finger was replaced by a finger proxy that was constrained to move on the object surface during contact [31]. The collision force was calculated as follows:

$$\mathbf{F} = K\left(\mathbf{x_p} - \mathbf{x_f}\right),$$

where $K$=3 $N/mm$ is the stiffness of the virtual surface; $\mathbf{x_p}$, the proxy position vector, is the most likely point of collision on the object surface; and $\mathbf{x_f}$ is the most likely contact point vector on the fingertip. The target position of the roller was set to $\mathbf{x_f}$. The collision force and the contact location were then displayed through the PHANToM and the CLD devices, respectively.

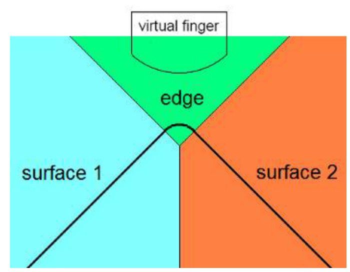



Figure 2. Illustration of collision calculation regions.

When no collision was detected, then no force was exerted through the PHANToM device. The target position of the roller, however, was updated in anticipation of an eventual collision. This way, the roller could be positioned to the point of contact on the user's fingertip without abrupt motion when a collision occurred [20].

[1] It should be noted that the user's fingertip was constrained to move in a 2D plane when exploring a 3D edge with adjoining surfaces. The axis normal to the profile shown in Figure 2 was ignored by the CLD and the contact-location rendering algorithm. Therefore, we do not differentiate between an "edge" and an "arc," or between a "surface" and a "line."

## 2.3 Stimuli

The haptic stimuli consisted of a 2D profile of a radiused edge adjoining two flat surfaces (see Figure 3). The edge was rendered as a smooth circular arc occupying 90°, and the two surfaces as straight lines. The two straight surfaces formed a right angle in all stimuli. The tangent lines at the two ends of the edge had the same slopes as those of the two surfaces, respectively. The radius of the edge (R) varied from 1.0 to 32.0 mm. The virtual surface was rendered such that the top of the edge was at a constant height regardless of its radius. The haptic stimuli were oriented such that the participants explored the different edges by moving their fingers in fore-aft motions between the two surfaces adjoining the edge. The diagram in Figure 3 was shown to the participants during training but not during the main experiment.

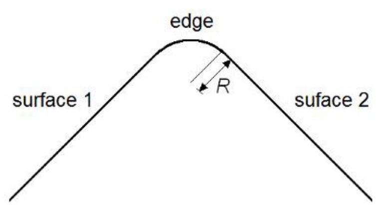



Figure 3. Illustration of the virtual edges used in the experiment.

Two rendering conditions were used in the experiment: force only (F) and force with contact location display (F+CLD). In the F condition, the PHANToM delivered force information calculated from the collision between the virtual finger and the virtual edge and its surrounding surfaces. The roller remained fixed at the center of the user's fingertip, thus information about changes in contact location due to the finger's movement on the virtual edge was unavailable. In the F+CLD condition, the roller moved along the user's fingertip and contacted the fingertip at locations that were consistent with where the virtual edge touched the virtual finger. In the latter case, the user experienced not only the force delivered by the PHANToM, but also the contact location delivered by the CLD.

## 2.4 Participants

Fourteen participants (6 females, 23-45 years old) took part in the main experiment. None of them had any known problems with their sense of touch. All were right handed by self-report.

## 2.5 Procedures

The participants discriminated the sharpness of a pair of virtual edges under two conditions: F and F+CLD as described in Section 2.3. The roller touched the participant's fingertip in both conditions, but did not move in the F condition. The method of constant stimuli was used to estimate the discrimination thresholds for edge sharpness [32]. Four different edges were selected as the reference stimuli: 2.5, 5.0, 10.0 and 20.0 mm. For each reference stimulus, seven comparison stimuli were selected with equal inter-stimulus spacing. The spacing was 0.5, 1.0, 2.0 and 4.0 mm for the four reference stimuli, respectively. For example, the seven stimulus alternatives for the reference stimulus of 2.5 mm were 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0 mm. On each trial, one of the seven radius values was selected randomly with an equal *a priori* probability. Each radius value was presented 10 times in a random order, resulting in a total of 70 trials per block of trials at a particular reference radius. Each participant conducted 8 blocks of trials (4 reference stimuli × 2 conditions). Half the participants completed the F condition first, while the other half completed the F+CLD condition first. The order of the four reference stimuli within each condition was randomized for each participant.

Training was available at the beginning of each block of trials. The participant could see and feel the reference stimulus and all other comparison stimuli by typing a number on the keyboard. The virtual finger was always visible during the training, which was terminated by the participant when s/he was ready. On each trial, the participant explored the reference stimulus and one randomly-selected comparison stimulus sequentially. No visual information was shown, except between trials (see below). The order of the reference and comparison stimuli was randomized. The participant's task was to indicate which edge was more curved (sharper) by pressing the number key "1" (first edge stimulus) or "2" (second edge stimulus) as the response. To avoid a large collision force upon the presentation of a new stimulus, the participant was asked to raise his/her finger before entering a response. A horizontal line appeared on the screen to indicate how high the finger needed to be lifted, and the virtual finger was shown to indicate the participant's current finger position. After a new stimulus was selected, a green dot at the top of the edge was shown on the screen and the participant was asked to lower his/her finger towards the green dot. When the virtual finger came within 2 mm of the green dot, the horizontal line, the virtual finger and the green dot disappeared. The participant continued to lower the finger to touch the virtual object.

The participant wore a pair of earplugs and headphones with 31dB noise reduction to block any audio cues from the experimental apparatus. The participant's hand and the experimental apparatus were covered by a black curtain to occlude any possible visual cues. After each block of trials, the participant was instructed to take a 5-min break before continuing. It took each participant about 4 hours to complete the experiment.

## 3 Results

For each participant under each condition, the proportion of times that a stimulus was judged to be more curved, P("more curved"), was tabulated for each reference stimulus. The values of 1−P("more curved") were then fit with an ogive function using the probit analysis tool provided by a SAS software package. The discrimination threshold, or the just noticeable difference (JND), was calculated as the average of the upper JND (radius difference between the 50- and 75-percentile points) and lower JND (radius difference between the 25- and 50-percentile points) [32]. Figure 4 shows a representative data plot for one participant.

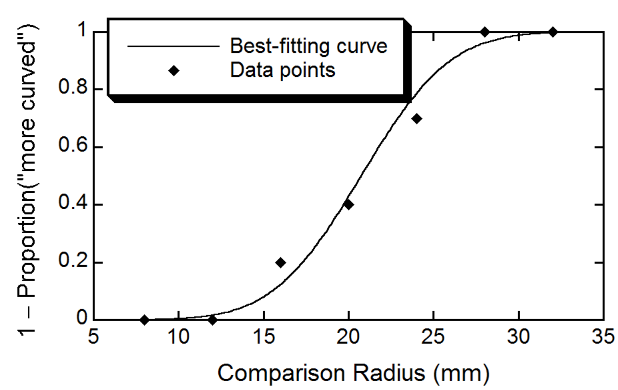



Figure 4. A representative data plot for one participant at a reference radius of 20.0 mm under the F condition.

Eight data plots like the one shown in Figure 4 were obtained for each participant. The JNDs for the same reference stimulus and experimental condition were averaged across the 14 participants. Figure 5 shows the average data from all 14 participants under both experimental conditions as a function of the reference radius. For both conditions, edge sharpness discrimination thresholds increased from about 2 mm to 7 mm monotonically when the reference radius increased from 2.5 mm to 20.0 mm. The JNDs for the F condition were lower than those for the F+CLD condition at reference radii 5 and 10 mm, but the differences were not statistically significant.

A two-way ANOVA with the factors experimental condition and reference radius indicates that the reference radius was a significant factor [$F(3,107)=28.81$, $p<0.0001$], but the experimental condition was not [$F(1,107)=2.13$, $p=0.1478$]. A subsequent Tukey test shows that the JNDs for the F condition and F+CLD conditions belonged to the same group (means: 3.9 and 4.5 mm, respectively). Additionally, a one-way ANOVA with the factor experimental condition was conducted at each reference radius and none of the threshold pairs was significantly different. Therefore, we conclude that edge sharpness discrimination threshold increases with reference radius, and the addition of contact location information does not lead to a significantly different discrimination threshold. This is contrary to our initial expectation that contact location information should enhance edge sharpness perception by lowering the edge sharpness discrimination thresholds. We explore the implications of our findings in the next section.

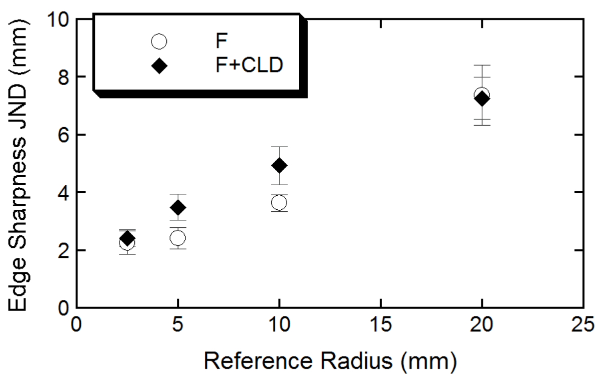



Figure 5. Means and standard errors of edge sharpness discrimination thresholds for all fourteen participants.

## 4 Discussion

The edge sharpness discrimination thresholds obtained in the present study can be compared with the curvature discrimination JNDs from a 2005 study by Provancher et al. [19]. The 2005 study reported curvature discrimination JNDs of 1.35 and 2.25 mm for nominal radii of 10 and 20 mm, respectively. These JNDs are much lower than those found in the present study for either the F or the F+CLD condition. There are several important differences between the present study and the 2005 study that might account for the different results. First, the present study allowed unrestricted range of motion, including finger rotation, along the object surface that included the edge as well as the two adjacent surfaces (see Figure 1). In the 2005 study, the curvature wheel (the apparatus used to present curvature stimuli) could only pivot 15° which corresponded to about ±10 mm of fingertip movement (see Figure 6 in [19]). The relatively small range of motion used in the 2005 study might have helped the participants discriminate curvatures by judging the extent of fingertip motions. Second, the virtual finger was modeled as a circular arc in the present study whereas the 2005 study used a straight line. Third, the present study allowed the user to pitch/rotate the virtual finger model, whereas the 2005 study held the virtual finger in a fixed horizontal orientation. Fourth, the present study used a stationary virtual curved edge, whereas the 2005 study used a cone-shaped curvature wheel that pivoted at its base. Among the differences, the restriction of range of motion in [19] may have had the most effect on the thresholds. To verify this, a follow-up experiment was conducted where three (1 female) of the original fourteen participants repeated the F+CLD condition with a slight change to the virtual object: Two virtual walls were added to the two surfaces at 10 mm from where the edge joined the surfaces (see Figure 6). Feedback force due to collision with the two virtual walls was displayed, but the roller position was calculated without regard to the virtual walls. This paralleled the rendering conditions of [19]. The results are compared in Table 1 for the two common reference radii of 10 and 20 mm, where the "F+CLD" column refers to the corresponding data shown in Figure 5, the "F+CLD+walls" column refers to the additional data collected in the follow-up experiment, and the "IJRR (2005)" column refers to the data from the 2005 study [19]. It is clear that with the additional wall constraints, edge sharpness discrimination thresholds decreased significantly to levels that were similar to the curvature discrimination JNDs reported in the 2005 study [19]. Therefore, the additional data confirmed that the virtual walls served as anchors that provided additional cues for discrimination of the virtual edges used in the present study. Note that the user's ability to rotate his/her finger in our present study (including the F+CLD+walls condition) could also account for some portion of the poorer performance in the present study. Note that the user's ability to rotate his/her finger in our present study (including the F+CLD+walls condition) could also account for some portion of the poorer performance in the present study.

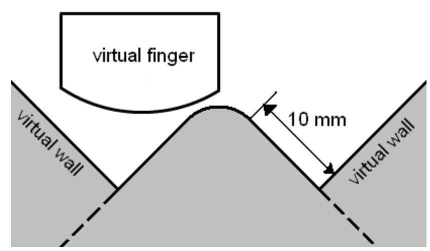



Figure 6. Illustration of the additional walls added to the virtual edges used in the follow-up experiment. The dashed lines show where the two side surfaces used to be, and the gray areas indicate the inside of the virtual object.

Table 1. Comparison of results from the 2005 study, the present study, and the follow-up condition of F+CLD+wall, all in mm.

| Reference R | F+CLD | F+CLD+walls | IJRR (2005) |
|---|---|---|---|
| 10 | 4.92 | 1.84 | 1.35 |
| 20 | 7.25 | 2.27 | 2.25 |

The main goal of the present study was to investigate the extent to which the CLD benefits the exploration and perception of shape primitives in touch. We anticipated that edge sharpness discrimination thresholds would be lower for the F+CLD condition than for the F condition. The results, however, indicate that there are no statistically significant differences between the two conditions. There are at least two possible explanations of the results. On the one hand, it is conceivable that both force and contact location cues contribute to the perception of edge sharpness, but force cues dominate the perception. Alternatively, it is possible that contact location information does not benefit edge-shape perception and therefore the addition of CLD does not lead to lower edge sharpness discrimination thresholds. It thus became important to assess the edge sharpness discrimination threshold with contact location information alone, as a way to differentiate the two explanations. To do this, five (2 females) of the original fourteen participants were randomly selected to take part in a second follow-up experiment. Figure 7 shows a 2D view of the haptic stimuli. Compared to the stimuli used in the main experiment, a flat virtual plane was added to render the resistive force needed in order for the finger to traverse the fore-aft span of the virtual object, yet provide no force information about the shape of the virtual edge that would have occurred if the virtual object's contour was followed. Contact location was rendered through the CLD as described in the main experiment (i.e., roller position was calculated based on the virtual object contour rather than the virtual plane contour). Note that the edge sharpness discrimination thresholds, so obtained, should be viewed as the upper bounds for the CLD alone condition, as the force cues based on a flat surface could potentially confuse the participants and cause the CLD cues to be less effective in conveying edge curvature information.

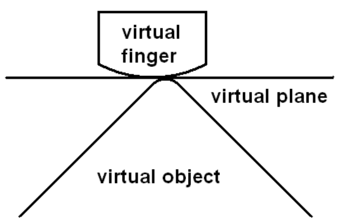



Figure 7. Illustration of virtual objects used in the CLD alone condition. The virtual plane was used in rendering force. The virtual object was used in calculating contact locations.

The results of the second follow-up experiment are plotted in Figure 8 ("CLD" condition), along with the results from the main experiments ("F" and "F+CLD" conditions). Like the thresholds for the F and F+CLD conditions, the thresholds for the CLD alone condition increased with the reference radius. However, the thresholds for the CLD alone condition were much higher than those for the F or F+CLD condition. A two-way ANOVA with the factors experimental condition and reference radius indicates that both factors were significant [Condition: $F(2,126)=54.83$, $p<0.0001$; Radius: $F(3,126)=25.92$, $p<0.0001$]. A subsequent Tukey test indicates two threshold groups: one for the CLD only condition (mean: 15.6 mm) and another for the F and F+CLD conditions (mean: 3.9 and 4.5 mm, respectively). We can conclude that (1) contact location cues do contribute to edge shape perception as indicated by the measurable albeit larger edge sharpness discrimination thresholds for the CLD alone condition; and (2) force information dominates edge perception as indicated by the much smaller thresholds for the F alone condition as compared to the CLD alone condition. Finally, it is observed that the differences between the three test conditions shown in Figure 8 increased as reference radius increased, indicating an increasing trend of force dominance as the edge became flatter. Conversely, one can argue that the relative contributions of force and contact location cues are more similar for the perception of sharper edges or more localized surface shape primitives.

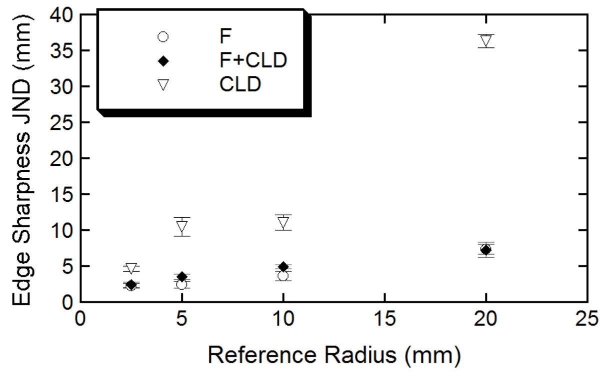



Figure 8. Comparison of edge sharpness JNDs from the F+CLD, F and CLD conditions.

Other perception studies have shown that one type of cue can be dominant for the perception of certain object features even though other cues may be available. For example, in haptic texture perception, spatial-intensive (size-depth of microstructures) and vibrational (temporal variations) cues both contribute to texture roughness perception. Although some studies appear to reach seemingly contradictory conclusions [33-38], a consensus that has emerged is that humans use vibration cues while exploring surface textures via a probe. While the same temporal cues are available during fingerpad exploration, humans prefer to use spatial and/or intensive cues instead. This is similar to our finding that both force and contact location information contribute to edge sharpness discrimination, but force cues apparently dominate the perception of edge sharpness. More studies are needed to discover when and how contact location information contributes to the perception of local shape primitives.

## 5 Concluding Remarks

In the present study, we investigated the effect of contact location information on the perception of virtual edges by comparing human edge sharpness discrimination thresholds under the force-alone and force-plus-contact-location conditions. The results showed similar thresholds for the two conditions, indicating that the addition of contact location information did not significantly improve the participants' ability to discriminate the sharpness of virtual edges. A subsequent experiment was conducted with contact location information alone. The results revealed that contact location cues were effective at resolving edge sharpness, although force cues dominate edge perception when the two are combined. The discrepancy between the force and contact location cues at conveying edge sharpness information increased as the edges became flatter.

Future work will investigate the effect of the roller size on the perception of local shape primitives. The roller used in the present study had a radius of 4.8 mm while circular arcs with radii of 1.0 to 32.00 mm were rendered. Some participants commented that the perception of the shape of the roller itself interfered with their perception of the virtual edges. For example, it was difficult to perceive an edge of 1.0 mm radius when the finger is physically in contact with a roller of 4.8 mm radius. To investigate this issue, rollers of smaller sizes will be fabricated and the same edge sharpness discrimination experiments will be conducted. The results will be compared to those reported in the present study to examine whether smaller roller sizes lead to an improvement of edge sharpness discrimination in the form of decreased thresholds. The findings will help guide the design and evaluation of future generations of the contact location display.

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