Documentation for Pate's code correlating transition on flat plates and circular cones in conventional wind tunnels using parameters relating to tunnel noise. S.P. Schneider, 765-494-3343, 2 April 2003

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

An experimental investigation was conducted to determine the effects of radiated aerodynamic-noise on boundary-layer transition in supersonic-hypersonic wind tunnels. It is conclusively shown that the aerodynamic noise (pressure fluctuations associated with sound waves), which radiate from the tunnel wall, turbulent boundary layer, will dominate the transition process on sharp flat places and sharp slender cones at zero incidence. Transition data measured

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20. ABSTRACT (Continued)

in supersonic tunnels ($M_\infty \gtrsim 3$) varying in test section heights from 1 to 16 ft have demonstrated a significant and monatonic increase in transition Reynolds numbers with increasing tunnel size. It has also been shown that the measured root-mean-square pressure fluctuations in the tunnel test section decrease with increasing tunnel size. A unique set of "shroud" experiments enabled the wall boundary layer to be directly controlled (either laminar or turbulent) and allowed transition Reynolds numbers to be correlated with the root-mean-square of the pressure fluctuations. Correlations of transition Reynolds numbers as a function of the radiated noise parameters [tunnel wall C_F and δ^* values and tunnel test section circumference (c)] have been developed. These correlations were based on sharp-flat-plate transition Reynolds number data from 13 wind tunnels having test section heights ranging from 7.9 in. to 16 ft, for Mach numbers from 3 to 8, and a unit Reynolds number per inch range from 0.1 x 10^6 to 1.9 x 10^6, and sharp-slender-cone transition Reynolds number data from 17 wind tunnels varying in size from 9 to 54 in. for a Mach number range from 3 to 14 and a unit Reynolds number range from 0.1 x 10^6 to 2.75 x 10^6. A FORTRAN IV computer code has been developed using the aerodynamic-noise-transition correlations. This code will accurately predict transition locations on sharp flat plates and sharp slender cones in all sizes of conventional supersonic-hypersonic wind tunnels for the Mach number range $3 \lesssim M_\infty \lesssim 15$. The effect of aerodynamic noise on transition Reynolds numbers must be considered when supersonic-hypersonic wind tunnel data are used to (a) develop transition correlations, (b) evaluate theoretical stability-transition math models, and (c) analyze transition-sensitive aerodynamic data. The radiated aerodynamic-noise transition dominance theory as presented in this research provides an explanation for the unit Reynolds number effect in conventional supersonic-hypersonic wind tunnels. If a true Mach number effect exists, it is doubtful that it can be determined from data obtained in conventional supersonic-hypersonic wind tunnels because of the adverse effect of radiated noise. It has been shown that the ratio of cone transition Reynolds numbers to flat-plate values does not have a constant value of three, as often assumed. The ratio will vary from a value of near three at $M_\infty = 3$ to near one at $M_\infty = 8$. The exact value is unit Reynolds number and tunnel size dependent. The aerodynamic-noise-transition empirical equations developed in this research correctly predict this trend. The boundary-layer trip correlation developed by van Driest and Blumer has been shown to be valid for different sizes of wind tunnels and not dependent on the free-stream radiated noise levels. The trip correlation developed by Potter and Whitfield remains valid if the effect of tunnel size on the smooth body transition location is taken into account. Wind tunnel transition Reynolds numbers have also been shown to be significantly higher than ballistic range values.

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APPENDIX C

DEVELOPMENT OF FORTRAN IV COMPUTER PROGRAM FOR PREDICTING TRANSITION LOCATIONS USING THE AERODYNAMIC-NOISE-TRANSITION CORRELATION

I. METHOD OF APPROACH

The algorithm developed to solve the aerodynamic-noise-transition empirical equations for a sharp flat plate, Eq. (10), page 248, and sharp slender cone, Eq. (11), page 252, at zero angle of attack are presented and discussed in this section. At first glance, Eqs. (10) and (11) appear fairly simple;

Sharp Flat Plate at Zero Angle of Attack (Eq. (10), page 248)

$$(Re_t)_{FP} = \frac{0.0126\ (C_{F_{II}})^{-2.55}\ (\bar{c})}{\sqrt{\frac{\delta^*}{C}}} \tag{C-1}$$

Sharp Slender Cone at Zero Angle of Attack (Eq. (11), page 252)

$$(Re_t)_{cone} = \frac{48.5\ (C_{F_{II}})^{-1.40}\ (\bar{c})}{\sqrt{\frac{\delta^*}{C}}} \tag{C-2}$$

however, computation of the tunnel wall turbulent-boundary-layer displacement thickness (δ^*), tunnel wall skin-friction coefficient (C_F), along with the tunnel free-stream unit Reynolds number and flow properties at the surface of an inviscid cone, become a fairly involved and lengthy process. In order to provide a systematic approach and to aide persons who might be interested in the details of the computer program, the equations required to execute each individual program option are presented in this section. A special effort has been made to include many comment statements in the FORTRAN Program so that the program will be essentially self explanatory. A program listing is provided in Section IV.

There are four program options (KOPT). These options were developed in a manner considered most beneficial to potential users and are described in detail in the program listing. Only a brief description will be given here.

<u>KOPT = 1 and 3</u>

Calculations of transition Reynolds numbers and locations on sharp flat plates at zero angle of attack are provided.

KOPT = 1 $3 \leq M_\infty \leq 10$ (ideal gas flow)

KOPT = 3 $M_\infty \gtrsim 10$ (real gas flow)

<u>KOPT = 2 and 4</u>

Calculations of transition Reynolds numbers and locations on sharp slender cones at zero angle of attack are provided.

KOPT = 2 $3 \leq M_\infty \leq 10$ (ideal gas flow)

KOPT = 4 $M_\infty \gtrsim 10$ (real gas flow)

Required input data for all programs options are defined in the program listing (Section IV) and illustrated in the included check problems (Section V).

II. BASIC EQUATIONS

The following equations were used in the development of KOPT = 1 and KOPT = 2.

Tunnel Test Section Conditions (Free Stream)

The tunnel test section static temperature (T_∞) and pressure (P_∞) are computed using perfect gas, one-dimensional isentropic flow relationships (155) with $\gamma = 1.4$.

$$T_\infty = \frac{T_0}{1 + \frac{\gamma - 1}{2} M_\infty^2} = \frac{T_0}{1 + 0.2\, M_\infty^2} \qquad \text{(C-3)}$$

$$P_\infty = \frac{P_0}{\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}} = \frac{P_0}{\left(1 + 0.2\, M_\infty^2\right)^{3.5}} \qquad \text{(C-4)}$$

and

$$U_\infty = M_\infty A_\infty = M_\infty \sqrt{\gamma R T_\infty} = (M_\infty)(49)\sqrt{T_\infty} \qquad \text{(C-5)}$$

The value of the absolute viscosity (μ) is temperature dependent, and consequently two different viscosity laws (linear and Sutherland) are used to compute this parameter. A recent discussion of viscosity laws is given by Fiore in Reference (159).

For temperature below 216°R, the viscosity varies linearly with temperature:

Linear Viscosity Law for Air

$$\mu = (0.0805)(T)(10^{-8}) \text{ lb-sec/ft}^2 \qquad \text{(C-6)}$$

For temperature above 216°R and up to about 5,000°K, the Sutherland equation provides the best estimate:

Sutherland Viscosity Law for Air

$$\mu = \frac{(2.270)(T)^{1.5}}{198.6 + T} \times 10^{-8} \text{ lb-sec/ft}^2 \qquad \text{(C-7)}$$

The free-stream Reynolds number is computed using Eq. (C-8):

$$\mathrm{Re}_\infty = \frac{\rho_\infty U_\infty}{\mu_\infty} = \frac{144\ P_\infty U_\infty}{T_\infty \mu_\infty R} \qquad \text{(C-8)}$$

$P_\infty \sim$ lb/in.^{2}

$U_\infty \sim$ ft/sec

$\mu_\infty \sim$ lb-sec/ft^2; Eqs. (C-6) or (C-7)

$T_W \sim {}^{\circ}$R

$R \sim$ 1,716 ft^2/sec^2-$^{\circ}$R

Length Reynolds Number

The Reynolds number based on tunnel free-stream conditions and the nozzle length is required in the computation of C_F and δ^* and is computed using the free-stream unit Reynolds number and the tunnel length (ℓ):

$$\mathrm{Re}_\ell = \mathrm{Re}_{\infty,\ell} = (\mathrm{Re}_\infty)(\ell)/(12) \qquad \text{(C-9)}$$

where

$\mathrm{Re}_\infty \sim$ ft^{-1}

$\ell \sim$ in.

NOTE: $\ell = \ell_m$

where ℓ_m = model leading-edge location

Boundary-Layer Displacement Thickness

The turbulent boundary-layer displacement thickness used in Eqs. (C-1) and (C-2) is the tunnel wall value computed at the model leading-edge position (ℓ_m).

Four separate analytical expressions are required to provide adequate calculation of δ^* for the Mach number range from 3 to 20 for two-dimensional, axisymmetric contoured nozzles and conical nozzles. The development of these correlations were discussed in Appendix B (see Figures B-7, page 339, and B-8, page 341).

For two-dimensional nozzles and $2 \leq M_\infty \leq 5$, the following empirical equation developed by Maxwell and Jacocks is applicable:

$$\delta^* = 0.0131 \left(\frac{\mu_o}{\rho_o a_o} \right)^{1/7} (\ell)^{6/7} (\overline{\delta^*}); \text{ ft} \qquad \text{(C-10)}$$

where

$\mu_o \sim$ computed from Eq. (C-7)

$\ell \sim$ nozzle length, ft

$$\rho_o = \frac{144\ P_o}{RT_o},\ \frac{\text{lb-sec}^2}{\text{ft}^2} \qquad \text{(C-10a)}$$

$a_o = \sqrt{1.4\ RT_o} = 49\sqrt{T_o}$, ft/sec

$P_o \sim$ lb/in.^{2}

$T_o \sim {}^oR$

$R = 1{,}717\ \text{ft}^2/\text{sec}^2\text{-}{}^oR$

The value $\overline{\delta^*}$ is computed from a third-degree polynomial curve fit of the theoretical curve in Figure B-7, page 339:

$$\begin{matrix}\text{Two-Dimensional} \\ \text{Adiabatic} \\ \text{Nozzles}\end{matrix} \left\{ \overline{\delta^*} = 1.1408\ M_\infty + 0.08813\ M_\infty^2 + 0.02698\ M_\infty^3;\ 2 \leq M_\infty \leq 5 \right\} \qquad \text{(C-11)}$$

Equation (C-11) matches the curve in Figure B-7, page 339, to within ±1.8% for $2 \leq M_\infty \leq 6$.

For two-dimensional and axisymmetric nozzles and $5 < M_\infty \leq 10$ (see Figure B-7, page 339) the following linear equations are used to compute $\overline{\delta^*}$:

$5 < M_\infty \leq 10$
Cool Wall
Non-Adiabatic
Nozzles

$$\left\{\begin{array}{ll} \overline{\delta^*} = 2.0 + 1.8333\, M_\infty : & \text{two-dimensional nozzle} \\ \overline{\delta^*} = 0.167 + 1.833\, M_\infty : & \text{axisymmetric nozzles} \end{array}\right\} \qquad \begin{array}{l} \text{(C-12a)} \\ \text{(C-12b)} \end{array}$$

For the following special conditions

1. Contoured or conical nozzle with $10 < M_\infty \lesssim 15$
2. Very high Reynolds number flow, $Re_\infty \geq 2.0 \times 10^6$, and $7 < M_\infty < 10$

δ^* is computed using Whitfield's empirical formula. A discussion of this empirical equation is given in Reference (105) and compared with experimental data in Figure B-8, page 341. Whitfield's empirical equation is

$$\delta^* = \frac{(0.22)(\ell)(M_\infty)^{0.5}}{(Re_{\infty,\ell})^{0.25}} \qquad \text{(C-13)}$$

Turbulent Skin-Friction Coefficient

The method of van Driest-II (see Appendix B) was used to compute the turbulent flow mean-skin friction coefficient for the tunnel wall at the model location in the test section.

The van Driest-II mean skin-friction formula is

$$\boxed{\begin{aligned} \frac{0.242}{A\sqrt{C_F}\,\sqrt{T_w/T_\infty}}\left(\sin^{-1}\alpha + \sin^{-1}\beta\right) &= \log_{10}(Re_\ell C_F) + 1.5\log_{10}(T_e/T_w) \\ &\quad + \log_{10}\frac{198.6 + T_w}{198.6 + T_e} \end{aligned}} \qquad \text{(C-14)}$$

T_w = tunnel wall temperature

$Re_\ell \sim$ determined by Eq. (C-9)

$$A = \sqrt{\left(\frac{\gamma - 1}{2} r M_\infty^2\right)\frac{T_\infty}{T_w}} \; ; \; r = \text{recovery factor} = 0.9 \qquad \text{(C-14a)}$$

$$B = \left(1 + \frac{\gamma - 1}{2} r M_\infty^2\right)\left(T_\infty/T_w\right) - 1 \qquad \text{(C-14b)}$$

$$\alpha = (2A^2 - B)/\sqrt{B^2 + 4A^2} \qquad \text{(C-14c)}$$

$$\beta = B/\sqrt{B^2 + 4A^2} \qquad \text{(C-14d)}$$

$$\gamma = 1.4$$

Since Eq. (C-14) is implicit in C_F, the Newton-Raphson method was used to solve for C_F when M_∞, Re_ℓ, T_w, and an initial value of C_F are specified. Equation (C-14) can be rewritten as

$$FCF = \sqrt{C_F}\,(C_2 + C_3 + \log_{10} C_F) - C_4 \qquad \text{(C-15)}$$

where

$$C_4 = \frac{0.242\,(\sin^{-1}\alpha + \sin^{-1}\beta)}{A\sqrt{T_w/T_\infty}} \qquad \text{(C-15a)}$$

$$C_2 = \log_{10} Re_\ell \qquad \text{(C-15b)}$$

$$C_3 = 1.5 \log_{10}\left(\frac{T_e}{T_w}\right) + \log_{10}\left(\frac{198.6 + T_w}{198.6 + T_e}\right) \qquad \text{(C-15c)}$$

Application of the Newtonian-Raphson method gives

$$C_{F_i} = C_{F_{i-1}} - FCF/\frac{dFCF}{dC_F} \qquad \text{(C-16)}$$

$\frac{dFCF}{dC_F}$ is the derivative of FCF with respect to C_F.

$$\frac{dFCF}{dC_F} = 0.5\,(C_F)^{-\frac{1}{2}}\left[C_2 + C_3 + \log_{10} C_F + 0.8686\right] \qquad \text{(C-17)}$$

By computing an initial value of $C_{F_{II}}$ using $C_{F_{II}} = 0.0050/M_\infty$, Eqs. (C-15) and (C-17) are solved and a new value of the C_F term (C_{F_i}) is computed from Eq. (C-16)

This process is repeated using the newly computed C_F term in the right side of Eq. (C-16) until the difference in successive calculations of C_F are within the specified limit of 0.0000010:

$$(|C_{F_i} - C_{F_{i-1}}| \leq 0.0000010)$$

This value of C_F is then used along with the computed δ^* value [Eqs. (C-12) or (C-13)] to calculate the transition Reynolds number from Eqs. (C-1) or (C-2).

Program Option 1 (Sharp Flat Plate at $\alpha = 0$)

The location of transition on a flat plate is determined by

$$X_t = (Re_t/Re_\infty)(12);\ \text{in.} \tag{C-18}$$

where Re_t is computed from Eq. (C-1) and Re_∞ from Eq. (C-8).

Program Option 2 (Sharp Slender Cone at $\alpha = 0$)

The analytical expressions required to determine the transition location on a sharp cone are presented in this section.

The values for $C_{F_{II}}$ and δ^* are computed exactly as in Program Option 1 [Eqs. (C-9) through (C-17)]. With known values of $C_{F_{II}}$, δ^*, and the tunnel coordinates C, ℓ, then the transition Reynolds number is computed using Eq. (C-2). However, what one usually wants to know is the location of transition on the cone surface and this requires knowing the cone surface inviscid Reynolds number, i.e., the local free-stream Reynolds number at the edge of the boundary layer on the cone surface.

For a flat plate at zero angle of attack, the plate surface inviscid parameters are assumed equal to the free-stream parameters, e.g., plate boundary-layer edge conditions equal M_∞, Re_∞, T_∞, p_∞, etc. However, in order to determine the inviscid surface parameters for a sharp cone, relationships between the surface values and free-stream values are required.

Cone Surface Static Pressure

The static pressure on the surface of a sharp cone at zero angle of attack is computed by the "approximate" analytical expression developed by Rasmussen (160). The cone surface pressure coefficient from Reference (160) is

$$\frac{C_{p_s}}{\sin^2 \delta} = 1 + \left\{ \left[\frac{(\gamma + 1)K^2 + 2}{(\gamma - 1)K^2 + 2} \right] \ln \left(\frac{\gamma + 1}{2} + \frac{1}{K^2} \right) \right\} \qquad \text{(C-19)}$$

where

$K = M_\infty \sin \delta_c$

δ_c = ½ the cone included angle

γ = ratio of specific heats = 1.4

and C_{p_s} is the pressure coefficient defined as

$$C_{p_s} = \frac{p_s - p_\infty}{q_\infty} = \frac{2(p_s - p_\infty)}{p_\infty M_\infty^2} \qquad \text{(C-20)}$$

and

$$q_\infty = \tfrac{1}{2} \rho_\infty U_\infty^2 = \frac{1}{2} \frac{p_\infty}{RT_\infty} M_\infty^2 \, RT_\infty = \tfrac{1}{2} p_\infty \gamma M_\infty^2 \qquad \text{(C-21)}$$

is defined as the free-stream dynamic pressure.

Using Eqs. (C-19), (C-20), and (C-21) the cone surface static pressure (p_s) can be expressed as

$$\frac{p_s}{p_\infty} = 1 + \sin^2\left(\frac{\gamma M_\infty^2}{2}\right)\left\{1 + \left[\frac{(\gamma + 1) M_\infty^2 \sin^2 \delta + 2}{(\gamma - 1) M_\infty^2 \sin^2 \delta + 2}\right] \ln\left(\frac{\gamma + 1}{2} + \frac{1}{M_\infty^2 \sin^2 \delta}\right)\right\} \qquad \text{(C-22)}$$

Cone Shock Wave Angle

A formula for the bow shock angle (ϕ) as developed by Rasmussen (160) is

$$\frac{\sin \phi}{\sin \delta_c} = \left[\frac{\gamma + 1}{2} + \frac{1}{K^2}\right]^{\frac{1}{2}} \qquad \text{(C-23)}$$

Cone Surface Velocities and Static Temperature

The velocity and temperature at the cone surface is found by using oblique shock wave theory and isentropic flow theory in conjunction with the known cone surface pressure and shock wave angle computed from Eqs. (C-22) and (C-23), respectively.

Oblique Shock Wave Theory

The static pressure (p_2) and temperature (T_2) are computed using oblique shock wave equations (155). For $\gamma = 1.4$, these equations are

$$\frac{P_2}{P_\infty} = \frac{7\, M_\infty^2 \sin^2 \phi - 1}{6} \qquad \text{(C-24)}$$

$$\frac{T_2}{T_\infty} = \frac{(7\, M_\infty^2 \sin^2 \phi - 1)(M_\infty^2 \sin^2 \phi + 5)}{36\, M_\infty^2 \sin^2 \phi} \qquad \text{(C-25)}$$

where P_2 and T_2 are the static pressure and temperature immediately downstream of the oblique shock wave.

Isentropic flow theory is valid between downstream of the bow shock wave and the cone surface since the flow field in this region is free from shock waves and consists of an isentropic compression process.

Using the isentropic flow relationship

$$\frac{p}{\rho^{\gamma}} = \text{constant} \qquad \text{(C-26)}$$

and the ideal gas equation of state

$$p = \rho RT \qquad \text{(C-27)}$$

one obtains

$$\frac{T_s}{T_2} = \left(\frac{p_2}{p_s}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{(C-28)}$$

T_s can be computed directly from Eq. (C-28) since p_s, p_2, and T_2 are known quantities determined from Eqs. (C-20), (C-24), and (C-25), respectively, and $\gamma = 1.4$ for air.

Cone Surface Velocities

The velocity at the cone surface is computed using the equation for total enthalpy and the basic physical law that energy (total enthalpy) is conserved.

since

$$H_{o_\infty} = H_{o2} = H_{os} = h_s + \frac{U_s^2}{2} \qquad \text{(C-29)}$$

and

$$H_{o_\infty} = C_p\ T_o \qquad \text{(C-30)}$$

$$h_s = C_p\ T_s \qquad \text{(C-31)}$$

where for an ideal gas, C_p = constant = 6,006 ft^2/sec^2-°R. Then from Eqs. (C-28), (C-29), (C-30, and (C-31), one gets

$$U_s = (2C_p)^{\frac{1}{2}} \left[T_o - T_2 \left(\frac{p_s}{p_2} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{2}} \qquad \text{(C-32)}$$

and

$$M_s = \frac{U_s}{\sqrt{\gamma R T_s}} \qquad \text{(C-33)}$$

Cone Surface Reynolds Number

The unit Reynolds number at the surface of an inviscid cone is computed from

$$Re_s = \frac{\rho_s \ U_s}{\mu_s} = \frac{p_s}{RT_s} \ \frac{U_s}{\mu_s} \qquad \text{(C-34)}$$

using Eqs. (C-22), (C-25), (C-28), and (C-32) with μ_s computed from either Eqs. (C-6) or (C-7), depending on the value of T_s.

The location of transition on the cone surface is computed from

$$(X_t)_c = \frac{(Re_t)_c (12)}{Re_s}, \text{ in.} \qquad \text{(C-35)}$$

using Eqs. (C-2) and (C-34).

Cone Surface Reynolds Number Ratios

Results obtained using the methods developed in the preceding section for computing inviscid cone surface flow properties and surface Reynolds number are compared in Figure C-1 with the "exact" numerical technique as developed by Jones (161) and used by Sims (162) to compute extensive tables of cone properties and by applying the appropriate

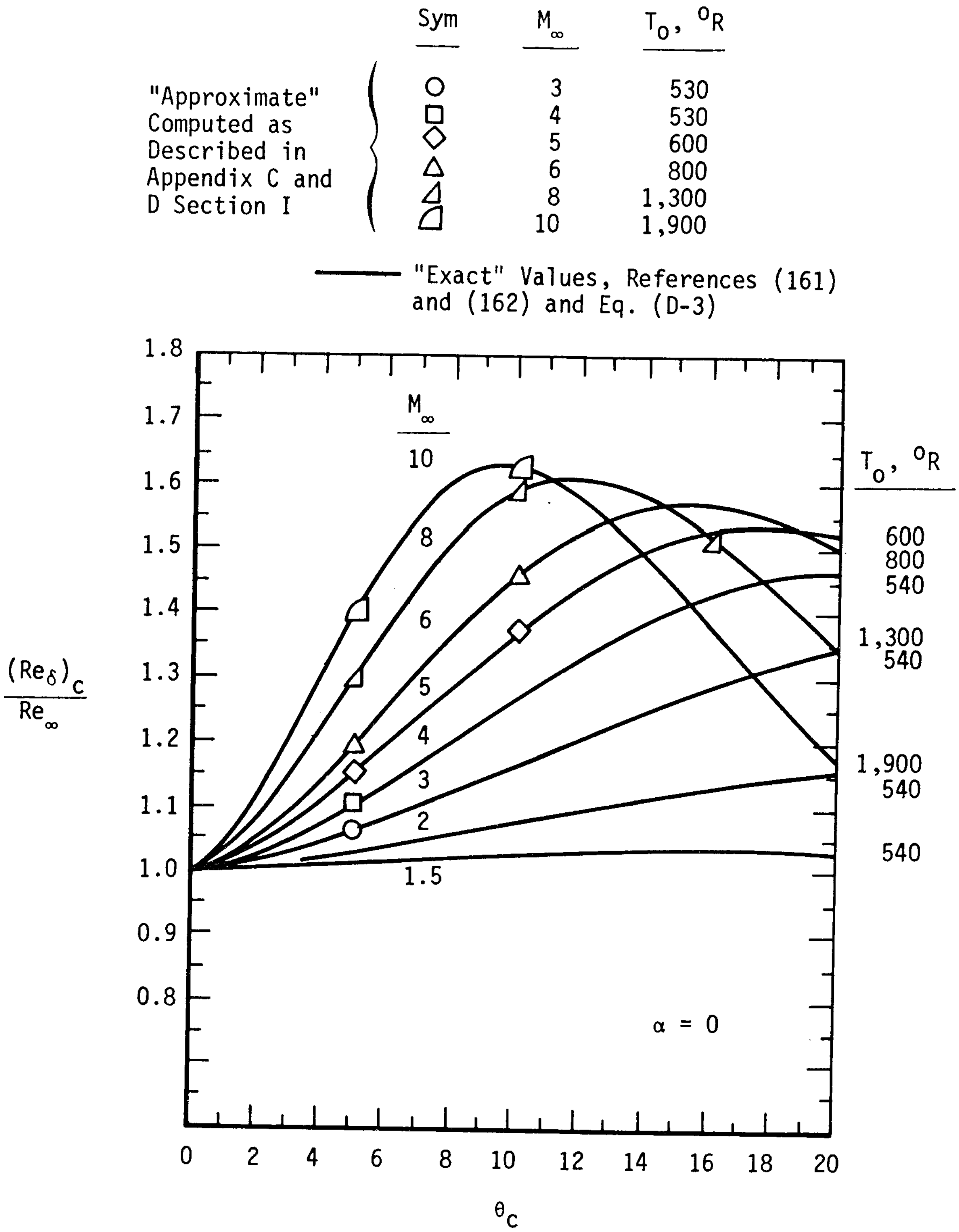


Figure C-1. Comparisons of approximate and exact cone surface Reynolds number ratios.

viscosity law [Eqs. (C-6) or (C-7)]. The results presented in Figure C-1 show that the approximate theories of Rasmussen (160) in conjunction with constant total enthalpy and isentropic flow theory as developed in the previous section adequately predicts the Reynolds number on the surface of inviscid cones for $M_\infty \geq 3$ and $\delta \geq 5$ deg.

Program Options 3 and 4

Transition Reynolds numbers are computed using Eqs. (C-1) or (C-2), page 326, in conjunction with Eqs. (C-13) and (C-14). The free-stream unit Reynolds number ($Re_\infty = \rho_\infty U_\infty/\mu_\infty$) is a required manual input and not computed using the ideal gas equations [Eqs. (C-3) through (C-8)]. KOPTS 3 and 4 are designed to accommodate wind tunnels having a real gas nozzle expansion. Sharp cone surface Reynolds numbers are also a required manual input and can be obtained from Figure D-2, page 358.

Section IV provides a detailed description of Program Options 1, 2, 3, and 4 and specifies all required input data.

III. COMPUTER CODE NOMENCLATURE

Symbols			
Computer	Conventional	Definition	Units
A	---	Variable in Skin-Friction Formula, Eq. (C-14a)	---
AO	a_O	Tunnel Stilling Chamber Speed of Sound	ft/sec
ALPHA	α	Variable in Skin-Friction Formula, Eq. (C-14c)	---
B	---	Variable in Skin-Friction Formula, Eq. (C-14b)	---

Symbols			
Computer	Conventional	Definition	Units
BC	$\overline{C}$	Tunnel Size Parameter for Cones, Eq. (11)	---
BFP	$\overline{c}$	Tunnel Size Parameter for Flat Plates, Eq. (10)	---
BARDEL	$\overline{\delta}$	Tunnel Wall Boundary Displacement Thickness Parameter, Eq. (C-11)	---
BETA	β	Variable in Skin-Friction Formula, Eq. (C-14b)	---
C	C	Tunnel Test Section Circumference	in.
CC CFP	---	Aerodynamic-Noise-Transition Correlation Parameter, Eq. (10), $CC = \sqrt{\delta^*/C}$	---
CF	C_F	Mean Turbulent Skin-Friction Coefficient, Eq. (C-14), ($C_F = C_{F_{II}}$)	---
CP	C_p	Specific Heat of Air at Constant Pressure, $C_p = 6{,}006\ ft^2/sec^2\text{-}{}^oR$	$\frac{ft^2}{sec^2\text{-}{}^oR}$
C1	C_1	Test Section Circumference of 12- by 12-in. Tunnel, $C_1 = 48$ in.	in.
C2,C3, C4	C_2, C_3, C_4	Variables in Skin-Friction Formula, Eq. (C-15)	
DELS	δ^*	Tunnel Wall Boundary-Layer Displacement Thickness	in.
DELTA	δ_c, θ_c	Cone Half-Angle	deg
FCF	FC_F	See Eq. (C-15)	---
FPRIME	$\frac{dFCF}{dC_F}$	See Eq. (C-17)	---

Symbols

Computer	Conventional	Definition	Units
GAMMA	γ	Ratio of Specific Heats ($\gamma = 1.4$ for Air)	---
K	---	Counter in Number of Loops Used to Satisfy C_F Convergence Criteria. If $K \geq 100$ Program Will Be Terminated.	---
KOPT	Option	Program Option	---
KGEOM	Geometry	Wind Tunnel Nozzle Geometry	---
		$K_{geom} = 1$, Two-Dimensional Nozzle	---
		$K_{geom} = 2$, Axisymmetric Nozzle	---
		$K_{geom} = 3$, Conical Nozzle	---
OK	OK	Variable in δ^* Equation, Eq. (C-10), $OK = 0.0131\ (\mu_\infty/\rho_0\ a_0)^{1/7}$	$(ft)^{1/7}$
PHI	ϕ	Sharp Cone Bow Shock Angle, Eq. (C-23)	radians
PO	p_0	Tunnel Stilling Chamber Pressure	psia
PS1,PS2, PS3,PS4, PS5	--- --- ---	Variables in Cone Surface Static Pressure Equation, Eq. (C-22)	--- --- ---
P1	p_∞	Free-Stream Static Pressure in Wind Tunnel Test Section, Eq. (C-4)	psia
R		Gas Constant, $R = 1{,}716\ ft^2/sec^2\text{-}{}^oR$	$ft^2/sec^2\text{-}{}^oR$
RT	r, η_r	Temperature Recovery Factor $r = (T_{aw} - T_\delta)/(T_0 - T_\delta)$	---
REL	Re_{ℓ_m}	Reynolds Number Based on Distance to Model Leading-Edge Location, $Re_{\ell_m} = \rho_\infty U_\infty \ell_m/\mu_\infty$, Eq. (C-9)	---

Symbols			
Computer	Conventional	Definition	Units
RES,RESC	$(Re_{\delta})_c$	Cone Surface Unit Reynolds Number $Re_s = \rho_s U_s/\mu_s$, Eq. (C-34)	$(ft)^{-1}$
RHO	ρ_0	Tunnel Stilling Chamber Density, Eq. (C-10a)	$\frac{\text{lb-sec}^2}{\text{ft}^2}$
RC1C	$\frac{C_1}{C}$	Ratio of Tunnel Circumference to Reference Value	---
RP21	$\frac{P_2}{P_\infty}$	Ratio of Static Pressure Immediately Behind Cone Bow Shock Wave to Free-Stream Static Pressure, Eq. (C-24)	---
RTS1	$\frac{T_s}{T_\infty}$	Ratio of Cone Surface Static Temperature to Free-Stream Value	---
RT21	$\frac{T_2}{T_\infty}$	Ratio of Static Temperature Immediately Downstream of Cone Bow Shock Wave to Free-Stream Value, Eq. (C-25)	---
REINF	Re_∞	Tunnel Test Section Free-Stream Unit Reynolds Number, Eq. (C-8)	$(ft)^{-1}$
RETFP	Re_t $(Re_t)_{FP}$	Flat-Plate Transition Reynolds Number $Re_t = \rho_\infty U_\infty x_t/\mu_\infty$, Eq. (C-1)	---
RETSC	$(Re_t)_c, (Re_t)_\delta$	Cone Transition Reynolds Number $(Re_t)_\delta = \rho_\delta U_\delta x_t/\mu_\delta$, Eq. (C-2)	---
RMACH	M_∞	Tunnel Test Section Free-Stream Mach Number	---
RMACHS	M_δ	Cone Surface Reynolds Number, Eq. (C-33)	---

Symbols			
Computer	Conventional	Definition	Units
RPSI	$\frac{p_s}{p_\infty}$	Ratio of Cone Surface Static Pressure to Tunnel Free-Stream Value, Eq. (C-22)	---
RRHOSI	$\frac{\rho_s}{\rho_\infty}$	Ratio of Cone Surface Static Density to Tunnel Test Section Free-Stream Value	---
RRESC RRES1	$\frac{Re_s}{Re_\infty}$	Ratio of Cone Surface Unit Reynolds Number to Tunnel-Stream Value	---
RTWTAW	$\frac{T_w}{T_{aw}}$	Ratio of Wall Temperature to Adiabatic Wall Temperature	---
TO	T_o	Tunnel Stilling Chamber Temperature	^oR
TS	T_s	Cone Surface Static Temperature, Eq. (28)	^oR
TW	T_w	Wall Temperature	^oR
T1	T_∞	Tunnel Test Section Free-Stream Static Temperature, Eq. (C-3)	^oR
TAW	T_{aw}	Adiabatic Wall Temperature	
US	U_s	Flow Velocity at Cone Surface, Eq. (C-32)	ft/sec
U1	U_∞	Tunnel Test Section Free-Stream Velocity, Eq. (C-5)	ft/sec
VISO	μ_o	Tunnel Stilling Chamber Absolute Viscosity, Eq. (C-7)	$\frac{lb\text{-}sec}{ft^2}$
VISS		Cone Surface Absolute Viscosity Based on Linear Law, Eq. (C-6)	$\frac{lb\text{-}sec}{ft^2}$
VIS1		Tunnel Test Section Free-Stream Viscosity, Eqs. (C-6) or (C-7)	$\frac{lb\text{-}sec}{ft^2}$
XL	X_ℓ	Distance from Tunnel Throat to Model Leading Edge	in.

Symbols

Computer	Conventional	Definition	Units
XTFP	x_t $(x_t)_{FP}$	Transition Location on Flat Plate, Eq. (C-18)	in.
XTSC	$(x_t)_c$ $(x_t)_\delta$	Transition Location on Cone Surface, Eq. (C-35)	in.

IV. COMPUTER PROGRAM LISTING

```
C      *****************************************************************
C      *********PREDICTION OF BOUNDARY LAYER TRANSITION*****************
C      *********ON SHARP FLAT PLATES AND SHARP CONES USING *************
C      *********PATE'S AERODYNAMIC-NOISE-TRANSITION CORRELATION*********
C      *****************************************************************
C
C     ******************** GENERAL COMMENTS ***************************
C          **THIS COMPUTER CODE ALLOWS REASONABLY ACCURATE
C            PREDICTIONS OF TRANSITION REYNOLDS NUMBERS AND
C            LOCATIONS TO BE MADE ON SHARP FLAT PLATES AND
C            SLENDER CONES AT ZERO INCIDENCE FOR ALL SIZE
C            WIND TUNNELS AND 3 <N< 20.
C
C          **THIS COMPUTER CODE IS VALID FOR AIR OR NITROGEN
C
C          **THE AERODYNAMIC-NOISE-TRANSITION CORRELATION AND
C            CONSEQUENTLY ,THIS COMPUTER PROGRAM  IS APPLICABLE
C            ONLY TO CONVENTIONAL SUPERSONIC-HYPERSONIC WIND
C            TUNNELS HAVING  TURBULENT BOUNDARY LAYERS ON THE
C            NOZZLE WALLS AND  3 <M <20 .
C
C          **THE PREDICTED LOCATION OF TRANSITION CORRESPONDS TO
C            THE  END OF TRANSITION AS DEFINED BY THE PEAK IN
C            A SURFACE PITOT PROBE PRESSURE TRACE
C
C          **THE VALUE OF THE TUNNEL WALL MEAN TURBULENT SKIN
C            FRICTION COEFFICINT USED IN THE AERODYNAMIC-NOISE
C            TRANSITION CORRELATION IS COMPUTED USING THE METHOD
C            OF VAN DRIEST-II ,INCLUDING NON-ADIABATIC WALL EFFECTS
C
C         ** THE  TUNNEL TEST SECTION WALL TURBULENT BOUNDARY
C            LAYER DISPLACEMENT THICKNESS USED IN THE AERODYNAMIC-
C            NOISE-TRANSITION CORRELATION IS COMPUTED USING
C            CORRELATION  DEVELOPED FROM  2-D AND 3-D NOZZLE DATA
C
C         **FOUR PROGRAM OPTIONS ARE AVAILABLE
C          *KOPT = 1 AND 2 ARE FOR IDEAL GASES,GAMMA=1.4
C          *KOPT = 3 AND 4 ARE FOR REAL GAS NOZZLE EXPANSIONS
C
C     *****************************************************************
C                   PROGRAM OPTIONS
C     *****************************************************************
C
C         KOPT = 1
C             **TRANSITION REYNOLDS NUMBERS AND LOCATIONS ON SHARP
C               FLAT PLATE OR HOLLOW CYLINDER AT ZERO INCIDENCE
C             **IDEAL GAS, GAMMA = 1.4
C             **3 < RMACH < 10
C             **** REQUIRED INPUT DATA ****
C                * KOPT = 1
C                * KGEOM = 1 OR 2
C                * C=TUNNEL TEST SECTION CIRCUMFERENCE
C                * XL= AXIAL DISTANCE FROM TUNNEL THROAT
C                      TO MODEL LEADING EDGE LOCATION,INCHES
```

```
C                * PO= TUNNEL STILLING CHAMBER PRESSURE,PSIA
C                * TO= TUNNEL STILLING CHAMBER TEMPERATURE ,DEG R
C                * RMACH= TUNNEL TEST SECTION MACH NUMBER
C                * TW=TUNNEL WALL TEMPERATURE, DEG R
C
C         KOPT = 2
C             **TRANSITION REYNOLDS NUMBERS AND LOCATIONS ON
C               SHARP SLENDER CONES AT ZERO INCIDENCE
C             **IDEAL GAS, GAMMA =1.4
C             **3 < RMACH < 10
C             **** REQUIRED INPUT DATA ****
C                * KOPT = 2
C                * KGEOM = 1 OR 2
C                * C=TUNNEL TEST SECTION CIRCUMFERENCE
C                * XL= AXIAL DISTANCE FROM TUNNEL THROAT
C                      TO MODEL LEADING EDGE LOCATION,INCHES
C                * PO= TUNNEL STILLING CHAMBER PRESSURE,PSIA
C                * TO= TUNNEL STILLING CHAMBER TEMPERATURE ,DEG R
C                * RMACH= TUNNEL TEST SECTION MACH NUMBER
C                * DELTA = CONE HALF ANGLE ,DEG
C                * TW=TUNNEL WALL TEMPERATURE, DEG R
C         KOPT = 3
C             ** TRANSITION REYNOLDS NUMBERS AND LOCATIONS
C                ON SHARP FLAT PLATES  AT ZERO INCIDENCE
C             ** REAL GAS EFFECTS CONSIDERED IN NOZZLE EXPANSION
C                PROCESS
C             ** 7 < RMACH < 20  , PO > 5000 PSIA
C             ** FREE-STREAM STATIC TEMP. SET EQUAL TO 90 DEG R
C              **** REQUIRED INPUT DATA ****
C                 * KOPT = 3
C                 * KGEOM = 3
C                 * C= TUNNEL TEST SECTION CIRCUMFERENCE
C                 * XL=AXIAL DISTANCE FROM TUNNEL THROAT TO
C                      MODEL LEADING EDGE LOCATION,INCHES
C                  * RMACH
C                 * REINF = TUNNEL TEST SECTION UNIT REYNOLDS
C                           NUMBER ( OBTAIN FROM FIG       )
C                 * TW = TUNNEL WALL TEMPERATURE,DEG R
C         KOPT = 4
C             ** TRANSITION REYNOLDS NUMBERS AND LOCATIONS
C                ON SHARP SLENDER CONES AT ZERO INCIDENCE
C             ** REAL GAS EFFECTS CONSIDERED IN NOZZLE
C                EXPANSION PROCESS
C             ** 7 < RMACH < 20  , PP > 5000 PSIA
C             ** FREE-STREAM STATIC TEMPERATURE SET EQUAL
C                TO 90 DEG R
C              **** REQUIRED INPUT DATA ****
C                 *KOPT = 4
C                 *KGEOM = 3
C                 * C= TUNNEL TEST SECTION CIRCUMFERENCE
C                 * XL=AXIAL DISTANCE FROM TUNNEL THROAT TO
C                      MODEL LEADING EDGE LOCATION,INCHES
C                  * RMACH
C                 * REINF = TUNNEL TEST SECTION UNIT REYNOLDS
C                           NUMBER ( OBTAIN FROM FIG       )
C                 *RRESC = RATIO OF CONE SURFACE UNIT REYNOLDS
```

```
C                            NUMBER TO FREE-STREAM VALUE
C                            (USE FIG     )
C                   * TW = TUNNEL WALL TEMPERATURE,DEG R
C                   *DELTA= CONE HALF ANGLE DEG
C     ****************************************************************
C          TUNNEL NOZZLE GEOMETRIES
C            KGEOM = 1
C                   * FOR TWO-DIMENSIONAL CONTOURED NOZZLES
C                   * 1.5< RMACH<5  ADIABATIC  WALLS
C                   *  5 < RMACH * 10  NON-ADIABATIC WALLS
C                   * METHOD OF MAXWELL IS USED TO COMPUTE
C                     BOUNDARY LAYER DISPLACEMENT THICKNESS ON
C                     TUNNEL TEST SECTION WALL
C                   * IDEAL GAS ,GAMMA = 1.4
C                   * USE WITH KOPT = 1 OR 2
C           KGEOM = 2
C                   * FOR AXISYMMETRIC CONTOURED NOZZLES
C                   * 5 < RMACH < 10
C                   * NON-ADIABATIC WALL
C                   *
C           KGEOM = 3
C                   *FOR CONICAL AND CONTOURED AXISYMMETRIC NOZZLES
C                  *NON-ADIABATIC WALL
C                   * 7 < RMACH < 20
C                   * USE WITH KOPT = 3 OR  4
C                   *METHOD OF WHITFIELS USED TO COMPUTE BOUNDARY
C                    LAYER DISPLACEMENT THICKNESS ON TUNNEL TEST
C                    SECTION WALL
C     ****************************************************************
C          **FOR CONTINUOUS FLOW AND INTERMITTENT WIND-TUNNELS-
C            WITHOUT WALL COOLING  ASSUME  TW = TAW For M∞ < 5
C            TW = 1.0 + 0.9*(GAMMA- 1.0 )*(RMACH**2.0)/ 2.0
C            TW = 1.0 + 0.18*RMACH**2.0
C          **FOR FACILITIES WITH COMPLETE WATER COOLED NOZZLES
C            ASSUME  TW = WATER TEMP. = 530 DEG R
C          **FOR IMPULSE FACILITIES ASSUME  TW= AIR TEMP.=530 DEG R
C     ****************************************************************
C
    1 READ (5,2,END=8000)  KOPT, KGEOM
    2 FORMAT(I1,I1)
    3 IF(KOPT-2)  4, 100, 200
C     ****************************************************************
C     KOPT = 1
C     WRITE COLUMN HEADINGS
    4 WRITE (6,12) KOPT , KGEOM
   12 FORMAT ( '1' , ' ESTIMATION OF BOUNDARY-LAYER TRANSITION ON SHARP
     1FLAT PLATES' //
     2,2X,'IN CONVENTIONAL SUPERSONIC-HYPERSONIC WIND TUNNELS USING'//
     3,1X, ' PATES AERODYNAMIC NOISE TRANSITION CORRELATION ' //
     4,2X,'KOPT=',I2,3X,'KGEOM=',I2,//
     5,4X,'C,IN.',2X,'XL,IN.',2X,'RMACH',2X,'PO,PSIA',3X,'TO,R',4X,'TW,R
     6',3X,'REINF/FT',6X,'REL',8X,'CF',5X,'DELS,IN.',3X,'RETFP',4X,
     7'XTFP,IN.',2X,'TW/TAW')
   14 READ (5,16,END= 1 )   C, XL,PO,TO,RMACH,TW
   16 FORMAT ( 6F10.0)
   18 C1 = 48.0
```

```
      GAMMA = 1.4
      R = 1716
C     COMPUTE FREE-STREAM TEMPERATURE
      T1=TO/(1.0+0.2*(RMACH**2.0))
C     COMPUTE FREE-STREAM VELOCITY
      U1=RMACH*SQRT(GAMMA*(R*T1))
C     COMPUTE FREE-STREAM ABSOLUTE VISCOSITY
C     USE SUTHERLANDS  VISCOSITY LAW (T17216 DEG R)
      IF(T1 .LE. 216) GO TO 30
      VIS1= 2.270*(T1**1.5)*(10.0**(-8.0))/(198.6+T1)
      GO TO 32
C     USE LINEAR VISCOSITY LAW (T1<216 DEG R)
   30 VIS1=(0.0805*T1)*(10.0**(-8.0))
   32 CONTINUE
C     COMPUTE FREE-STREAM STATIC PRESSURE
      P1=PO/((1.0+0.2*(RMACH**2.0))**3.5)
C     COMPUTE FREE-STREAM UNIT REYNOLDS NUMBER
      REINF=((144.0*P1)*U1)/((R*T1)*VIS1)
C     COMPUTE  REYNOLDS NUMBER BASED ON TUNNEL NOZZLE LENGTH(XL)
   34 REL = REINF * (XL/12.0)
      IF ( KGEOM-3) 35,90,8000
   35 CONTINUE
C     COMPUTE TUNNEL TEST SECTION WALL TURBULENT BOUNDARY LAYER
C     DISPLACEMENT THICKNESS COMPUTED USING MAXWELL'S CORRELATION
C     ABSOLUTE VISCOSTY (VIS) COMPUTED USING SUTHERLAND'S LAW
C     VALID FOR   216<T< 5000 ,DEG R
   36 VISO=(2.270)*(TO**1.5)*(10.0**(-8.0))/(198.6+TO)
C     RHO IS STILLING CHAMBER DENSITY
   38 RHO =(144 * PO)/(R* TO )
C     AO IS STILLING CHAMBER SPEED OF SOUND
   40 AO = SQRT((GAMMA*R)* TO)
C     OK IS THE VARIABLE IN MAXWELL'S EQ.
   42 OK = (0.0131)*((VISO/(RHO* AO))**(0.14286))
C     BARDEL IS MAXWELL CORRELATION PARAMETER
C    1BARDEL IS COMPUTED USING A THIRD DEGREE POLYNOMINAL CURVE FIT OF
C    1MAXWELL'S  ORIGINAL CORRELATION FOR RMACH  .LE. 6.0
      IF (RMACH - 5.0 ) 46, 46, 50
   46 IF (KGEOM .GT. 1) GO TO 8000
   48 BARDEL =   1.140824 * RMACH  + 0.088132*((RMACH)**2.0)
     1+ 0.026978 *((RMACH)**3.0)
      GO TO 56
   50 IF ( RMACH.GT. 10.0 ) GO TO 8000
      IF (KGEOM - 2 ) 52, 54, 8000
C     IF 5< RMACH<10 AND KGEOM = 1 THEN BARDEL = 2.0 + 1.8333*RMACH
   52 BARDEL  = 2.0 +  1.8333 * RMACH
      GO TO 56
   54 BARDEL = 0.167 + 1.833*RMACH
C     DELS  IS TUNNEL WALL DISPLACEMENT THICKNESS IN IN.
   56 DELS=(OK)*(BARDEL)*((XL/12.0)**0.857143)*12.0
C     COMPUTE  MEAN TURBULENT SKIN FRICTION(CF) USING VAN DRIEST-II WITH
C     TW IS INPUT DATA , DEG. RANKINE
C     RT IS THE RECOVERY FACTOR FOR A TURBULENT BOUNDARY LAYER
   57  RT= 0.90
   58 A=SQRT((((GAMMA-1.0)/2.0)*RT*(RMACH**2.0))*(T1/TW))
      B = (T1/TW) + (A**2.0)-1.0
      B1=B**2
```

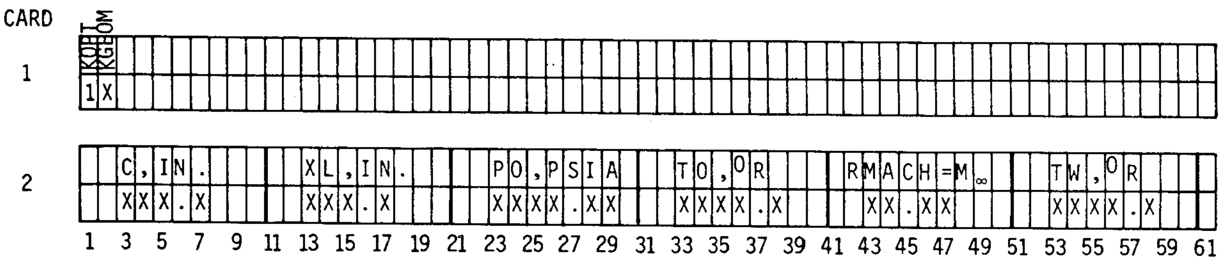
```
      B=SQRT(B1)
      ALPHA=(2.0*(A**2.0)-B)/SQRT(B**2.0+4.0*(A**2.0))
      BETA = B/SQRT ((B**2.0)+4.0*(A**2.0))
      C4 = (0.242)*(ARSIN(ALPHA) + ARSIN(BETA))/(A * SQRT(TW/T1))
      C2 = ALOG10(REL)
      W = 0.76
      C3= 1.5*ALOG10(T1/TW) + ALOG10((198.6+TW)/(198.6+T1))
C     STARTING VALUE OF CF IS 0.0050/RMACH
      CF=(0.0050)/RMACH
      K=0
   70 FCF =  SQRT(CF)*(C2+C3  + ALOG10(CF)) - C4
      K=K+1
      IF (K .GT. 100) GO TO 8000
C     FPRIME IS THE DERIVATIVE OF FCF WITH RESPECT TO CF
      FPRIME = (0.50 / SQRT(CF))*( C2+C3 + ALOG10(CF)+0.8686)
C     THE NEWTON-RAPHSON METHOD IS
      CF = CF - (FCF / FPRIME)
C     INTERATE UNTIL SUCCESSIVE APPROXIMATIONS OF ROOT IS LESS THAN
C    10.0000010
      ROOT = ABS( FCF/FPRIME)
      IF (ROOT .GT. 0.0000010) GO TO 70
      C1=48.0
      IF (KOPT .EQ. 2) GO TO 152
      IF (KOPT .EQ. 4) GO TO 222
      AFP=(CF)**(-2.55)
      RC1C = C1/C
      IF (RC1C - 1.0 ) 71, 71, 72
   71 BFP = 0.56 + (0.44)* C1/C
      GO TO 73
   72 BFP = 1.00
   73 CFP = SQRT (DELS/ C )
      DFP=(0.0126)*AFP
      RETFP=(BFP*DFP)/CFP
      XTFP=(RETFP/REINF)*12.0
C     TAW= TUNNEL WALL RECOVERY TEMPERATURE
      TAW= T1*(1.0+ 0.18*(RMACH**2.0))
      RTWTAW= TW/TAW
      IF ( KOPT .EQ. 3) GO TO 210
   80 WRITE (6,82)  C,XL,RMACH,PO,TO,TW,REINF,REL,CF,DELS,RETFP,XTFP,
     1RTWTAW
   82 FORMAT(1H,F8.1,F7.1,F8.1,F9.2,2F8.1,1PE11.4,1PE11.4,0PF10.6,
     1F9.4,1PE11.4,0PF8.2,F10.3)
      GO TO 14
C     COMPUTE DELS USING WHITFIELDS CORRELATION FOR CONICAL NOZZLES OR
C     REAL GAS AXISYMMETRIC NOZZLES
C     WHEN   7< RMACH <20
   90 DELS=(0.22)*SQRT(RMACH)/SQRT(REL)
      GO TO 58
C     ************************************************************
C     KOPT = 2
  100 WRITE (6,101) KOPT, KGEOM
  101 FORMAT ( '1', 'ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP SL
     1ENDER CONES IN CONVENTIONAL SUPERSONIC-HYPERSONIC',//
     1,1X,'WIND TUNNELS USING THE AERODYNAMIC NOISE CORRELATIONS BY PATE
     1',//
     1,1X,'KOPT =',I2,3X,'KGEOM =',I2,//
```

```
     1,2X,'C,IN.',2X,'XL,IN.',2X,'RMACH',3X,'PO,PSIA',1X,'TO,R',4X,
     2'TW,R',3X,'REINF/FT',3X,'DELTA,DEG',1X,'RTS',4X,'RPS',3X,'RMACHS'
     3,1X,'RRESC',4X,'RETSC',3X,'XTSC,IN',2X,'TW/TAW')
  102 READ(5,104,END= 1 )  C,XL, PO, TO, RMACH, DELTA, TW
  104 FORMAT ( 7F10.0)
C     COMPUTE CONE SURFACE STATIC PRESSURE RATIO (RPS1 = PS/P1) USING
      DELTA = DELTA/ 57.296
C    1RASMUSSEN'S .EQ.
      PS1 = (SIN(DELTA)**2.0)*(1.4*(RMACH**2.0))/2.0
      PS2= 2.4*(RMACH**2.0)*(SIN(DELTA)**2.0) + 2.0
      PS3 = 0.4*(RMACH**2.0)*(SIN(DELTA)**2.0) + 2.0
      PS4= ALOG(1.20 + 1.0/((RMACH**2.0)*(SIN(DELTA)**2.0)))
      PS5=(PS2/PS3)*PS4
      RPS1=1.0+PS1*(1.0+PS5)
      GAMMA = 1.40
      R=1716.0
C     COMPUTE CONE BOW SHOCK ANGLE(PHI)
  110 PHI = ARSIN( SIN(DELTA)*((1.20 + 1.0/((RMACH*SIN(DELTA))**2.0))**
     10.50))
C     COMPUTE STATIC PRESSURE AND TEMPERATURE BEHIND BOW SHOCK USING
C     2-D  OBLIQUE SHOCK WAVE THEORY
C     RT21 =  T2/ T1
  112 RT21 = (7.0* ((RMACH*SIN(PHI))**2.0) -1.0)*(((RMACH*SIN(PHI))**2.0
     1)+ 5.0)/(36.0*((RMACH*SIN(PHI))**2.0))
C     RP21 =  P2/P1
  114 RP21 =  (7.0*((RMACH * SIN(PHI))**2.0)-1.0 ) /6.0
C     COMPUTE PRESSURE AND TEMPERATURE ON CONE SURFACE BY USING CONDITIO
C     BEHIND SHOCK WAVE AND ISENTROPIC COMPRESSION PROCESS
C     RTS1 = TS/T1
  116 RTS1 = RT21 *( (RPS1 / RP21)**0.285714 )
C     COMPUTE FREE-STREAM TEMPERATURE(T1)
  118 T1= TO/(1.0 + 0.2*(RMACH)**2.0)
  120 TS=T1*RTS1
C     COMPUTE VELOCITY AT CONE SURFACE (US)
C     RATIO OF SPECIFIC HEAT IS CP=6006 FT.SQ./ SEC. SQ.-DEGR
      CP = 6006
  122 US =(SQRT(2.0 * CP))*SQRT( TO - (T1*RT21)*((RPS1/RP21)**0.285714))
C     COMPUTE MACH NUMBER AT CONE SURFACE (RMACHS)
  124 RMACHS = US /(49.0 * SQRT(TS))
C     COMPUTE DENSITY AT CONE SURFACE (RHOS)
      R=1716.0
C     COMPUTE FREE-STREAM STATIC PRESSURE (P1)
  126 P1 = PO/ ((1.0 + 0.2*(RMACH**2.0))**3.5)
C     COMPUTE DENSITY RATIO AT CONE SURFACE (RRHOS1 = RHOS/RHO1)
  128 RRHOS1=(RPS1/RTS1)
C     COMPUTE REYNOLDS NUMBER AT CONE SURFACE (RRES1 = RES/RE1)
C     COMPUTE FREE-STREAM VELOCITY (U1)
  130 U1 = (RMACH * 49.0)* SQRT(T1)
  132 IF ( T1  .GE. 216) GO TO 138
C     COMPUTE FREE-STREAM ABSOLUTE VISCOSITY(VIS1) USING LINEAR LAW
      VIS1 = (0.0805 * T1)*(10.0)**(-8.0)
  136 GO TO 140
C     COMPUTE FREE-STREAM ABSOLUTE VISCOSITY USING SUTHERLANDS LAW
  138 VIS1 = (2.270*(T1**1.50))*(10.0**(-8.0))/(198.6 + T1)
C     RE1 = REINF = RHO1 * U1/VIS1
C     COMPUTE FREE-STREAM-UNIT REYNOLDS NUMBER( RE1=REINF)
```

```
  140 REINF=(U1*P1*144.0) / (R*T1 *VIS1)
  142 IF (TS .GE. 216) GO TO 146
C       COMPUTE CONE SURFACE VISCOSITY VISS USING LINEAR LAW (TS) 216 DEGR
  144 VISS = (0.0805 * TS)*(10.0**(-8.0))
      GO TO 148
C       COMPUTE CONE SURFACE VISCOSITY USING SUTHERLANDS LAW (TS>216 DEGR)
  146 VISS = (2.270*(TS**1.50))*(10.0**(-8.0))/(198.6 + TS)
C       COMPUTE CONE SURFACE REYNOLDS NUMBER (RES)
  148 RES=(RPS1*P1*144.0*US)/(R*TS*VISS)
C       COMPUTE REYNOLDS NUMBER RATIO(RRES1 = RES/REINF)
  149 RRES1 = RES/REINF
  150 GO TO 34
  152 CONTINUE
      DELTA = DELTA * 57.296
C       COMPUTE TRANSITION REYNOLDS NUMBER ON CONE SURFACE (RETSC)
  153 AC  = (CF)**(-1.40)
      RC1C = C1/ C
      IF (RC1C - 1.0 ) 154,154, 155
  154 BC  = 0.80 + 0.20*(C1/C)
      GO TO 156
  155 BC  = 1.00
  156 CC  = SQRT (DELS/C)
      RETSC=(48.5)*(AC  * BC  ) / CC
      XTSC =(RETSC / RES) * 12.0
C       TAW= TUNNEL WALL RECOVERY TEMPERATURE
      TAW= T1*(1.0+ 0.18*(RMACH**2.0))
      RTWTAW= TW/TAW
      WRITE (6,158) C,XL,RMACH,PO ,TO,TW ,REINF,DELTA,RTS1,RPS1,
     2RMACHS,RRES1,RETSC,XTSC,RTWTAW
  158 FORMAT ( 1H,F6.1,F7.1,F8.1,F9.1,F7.1,F7.1,1PE12.4,0PF8.2
     2,F8.3 ,F7.3,F7.2,F7.3,1PE12.4,0PF7.2,F8.3)
      GO TO 102
C     ******************************************************************
C     KOPT = 3
C
  200 IF(KOPT-4)    201, 214, 8000
  201 IF  (KGEOM - 3 ) 8000 , 202 , 8000
  202 WRITE ( 6, 204) KOPT, KGEOM
  204 FORMAT ( '1', 'PREDICTION OF BOUNDARY LAYER TRANSITION ON SHARP
     1FLAT PLATES'//
     1,2X,'FOR NON IDEAL GAS NOZZLE EXPANSION PROCESS'//
     1,2X,'RMACH > 10 OR RMACH > 8 , REINF > 15*10**6 ,SEE FIG   '//
     1,2X,'USING PATES   AERODYNAMIC NOISE TRANSITION CORRELATION'//
     1,2X, 'KOPT = ',I2,4X,'KGEOM= ',I2,//
     1,4X, 'C,IN.',5X, 'XL,IN.',4X, 'RMACH',5X, 'TW',5X,'REINF',8X, 'CF'
     1,3X,'DELS,IN.',8X, 'RETFP',5X, 'XTFP,IN.',5X,'TW/TAW')
  206 READ(5, 208, END= 1 )  C, XL, RMACH, REINF, TW
  208 FORMAT (5F10.0)
      REL = REINF*(XL/12.0)
C     COMPUTE DELS USING WHITFIELDS  FORMULAS
      DELS = XL*(0.22)*SQRT(RMACH)/(REL**0.25)
C     COMPUTE TUNNEL WALL SKIN FRICTION USING VAN DRIEST-II
      GAMMA=1.40
C     SET FREE-STREAM STATIN TEMPERATURE EAUAL TO 90 DEG R
      T1 = 90.0
C     TAW= TUNNEL WALL RECOVERY TEMPERATURE
```

```
      TAW= T1*(1.0+ 0.18*(RMACH**2.0))
      RTWTAW= TW/TAW
      GO TO 57
  210 CONTINUE
      WRITE (6,211) C,XL,RMACH,TW,REINF,CF,DELS,RETFP,XTFP,RTWTAW
  211 FORMAT (1H,1F8.1,1F10.1,1F9.1,1F8.1,1PE12.4,0PF10.6,1F8.4,1PE16.4,
     10PF10.2,F8.2)
  212 GO TO 206
C     ****************************************************************
C     KOPT= 4
C
  214 IF (KGEOM - 3 ) 8000 , 215 , 8000
  215 WRITE ( 6, 216) KOPT, KGEOM
  216 FORMAT ( '1', 'ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP
     1SLENDER CONES'//
     1,2X, 'FOR MACH NUMBERS GREATER THAN 10 ON VERY HIGH REYNOLDS'//
     1,2X,'AT MACH NUMBER EQUAL APPROX 8(REINF>10**7)'//
     1,2X,'USING PATES   AERODYNAMIC NOISE TRANSITION CORRELATION'//
     1,2X, 'KOPT = ',I2,4X, 'KGEOM= ',I2,//
     1,1X,'C,IN.',5X,'XL,IN.',4X,'RMACH',3X,'TW,DEGR',4X,'REINF/FT'
     2,3X,'RESC/REINF',2X,'DELS,IN.',7X,'RETSC',5X,'XTSC,IN.',3X.
     3'DELTA,DEG',5X,'TW/TAW' )
  218 READ(5,220,END= 1  ) C,XL,RMACH,REINF,RRESC,TW,DELTA
  220 FORMAT (7F10.0)
      REL = REINF*(XL/12.0)
C     COMPUTE DELS USING WHITFIELD'S FORMULS
      DELS = XL*(0.22)*SQRT(RMACH)/(REL**0.25)
C     COMPUTE TUNNEL WALL SKIN FRICTION USING VAN DRIEST-II
      GAMMA=1.40
C     SET FREE-STREAM STATIC TEMPERATURE EQUAL TO 90 DEG R
      T1 = 90.0
C     TAW= TUNNEL WALL RECOVERY TEMPERATURE
      TAW= T1*(1.0+ 0.18*(RMACH**2.0))
      RTWTAW= TW/TAW
      GO TO 57
  222 CONTINUE
C     COMPUTE TRANSITION REYNOLDS NUMBER  ON CONE
      AC  =  (CF)**(-1.40)
      RC1C = C1 /C
      IF (RC1C - 1.0 ) 224, 224 , 225
  224 BC  = 0.80 + 0.20*(C1/C )
      GO TO 226
  225 BC  = 1.00
  226 CC  = SQRT (DELS/ C )
      RETSC =(48.5)*(AC * BC ) /CC
      RESC=RRESC*REINF
      XTSC=(RETSC/RESC)*12.0
      WRITE(6,230)C,XL,RMACH,TW,REINF,RRESC,DELS,RETSC,XTSC,DELTA,RTWTAW
  230 FORMAT(1H,F5.1,2F10.2,F10.2,1PE13.4,0PF9.3,F12.3,1PE16.4,0PF10.3,
     1F10.3,F13.2 )
      GO TO 218
 8000 CONTINUE
      STOP
      END
```

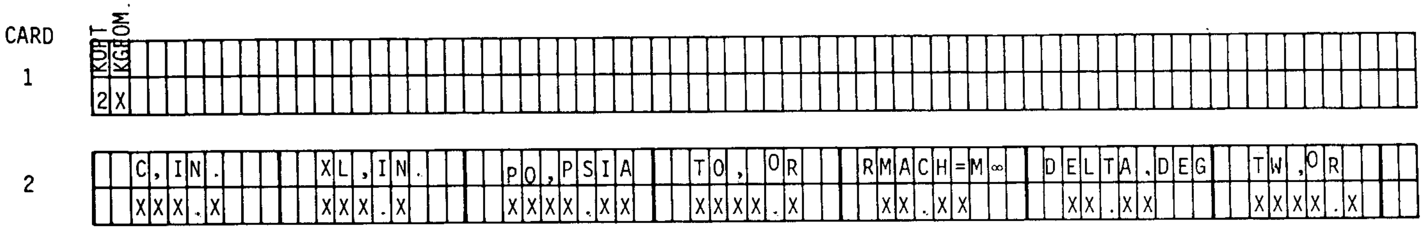
V. INPUT DATA FORMAT AND CHECK PROBLEMS



```
ESTIMATION OF BOUNDARY-LAYER TRANSITION ON SHARP FLAT PLATES
IN CONVENTIONAL SUPERSONIC-HYPERSONIC WIND TUNNELS USING
PATES AERODYNAMIC NOISE TRANSITION CORRELATION
KOPT = 1   KGEOM = 1
C, IN.   XL, IN.   RMACH    PO,PSIA   TO,R     TW,R     REINF/FT       REL          CF        DELS,IN.      RETFP      XTFP,IN.   TW/TAW
160.0    213.0      4.0      40.00    540.0    495.0    3.6932E 06   6.5554E 07   0.001172   1.3061     2.8744E 06     9.34      0.992
```

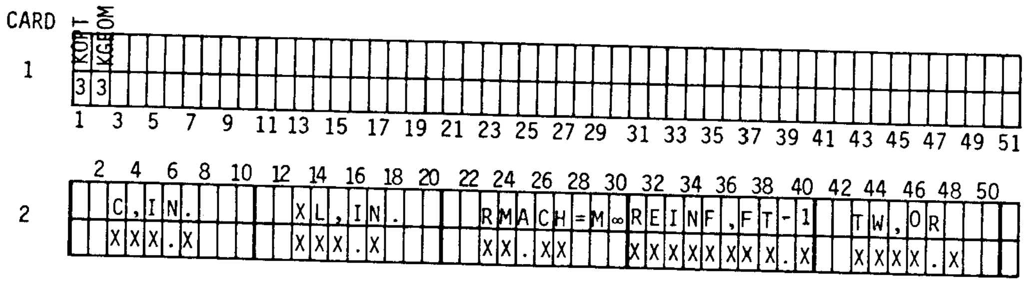
```
ESTIMATION OF BOUNDARY-LAYER TRANSITION ON SHARP FLAT PLATES
IN CONVENTIONAL SUPERSONIC-HYPERSONIC WIND TUNNELS USING
PATES AERODYNAMIC NOISE TRANSITION CORRELATION
KOPT = 1   KGEOM = 1
C, IN.   XL, IN.   RMACH    PO,PSIA   TO, R    TW,R     REINF/FT       REL          CF       DELS, IN.      RETFP      XTFP,IN.   TW/TAW
157.0    300.0      8.0     450.00   1400.0   530.0   1.8438E 06   4.6096E 07   0.000857   3.1547      4.0874E 06    26.60      0.417
```

```
ESTIMATION OF BOUNDARY-LAYER TRANSITION ON SHARP FLAT PLATES
IN CONVENTIONAL SUPERSONIC-HYPERSONIC WIND TUNNELS USING
PATES AERODYNAMIC NOISE TRANSITION CORRELATION
KOPT = 1   KGEOM = 2
C, IN.   XL, IN.   RMACH    PO,PSIA   TO, R    TW,R     REINF/FT       REL          CF       DELS, IN.      RETFP      XTFP,IN.   TW/TAW
157.0    300.0      8.0     450.00   1400.0   530.0   1.8438E 06   4.6096E 07   0.000857   2.8073      4.3329E 06    28.20      0.417
```



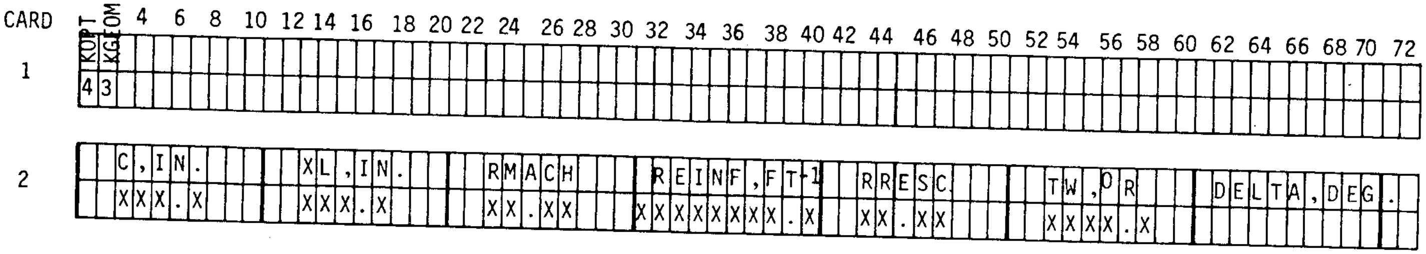
```
ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP SLENDER CONES IN CONVENTIONAL SUPERSONIC-HYPERSONIC
WIND TUNNELS USING THE AERODYNAMIC NOISE CORRELATIONS BY PATE
KOPT = 2    KGEOM = 1
C,IN.  XL,IN.  RMACH  PO,PSIA  TO,R    TW,R    REINF/FT    DELTA,DEG    RTS   RPS    RMACHS  RRESC   RETSC        XISC,IN.  TW/TAW
160.0  213.0    4.0    40.0    540.0   495.0  3.6921E 06    5.00      1.078 1.299    3.81   1.105  5.8567E 06    17.23     0.992


ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP SLENDER CONES IN CONVENTIONAL SUPERSONIC-HYPERSONIC
WIND TUNNELS USING THE AERODYNAMIC NOISE CORRELATIONS BY PATE
KOPT = 2    KGEOM = 1
C,IN.  XL,IN.  RMACH  PO,PSIA   TO,R    TW,R    REINF/FT    DELTA,DEG    RTS    RPS   RMACHS  RRESC   RETSC        XTSC,IN.  TW/TAW
157.0  300.0    8.0    450.0   1400.0  530.0  1.8433E 06    5.00      1.210  1.920    7.22   1.302  5.7956E 06    28.99    0.417


ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP SLENDER CONES IN CONVENTIONAL SUPERSONIC-HYPERSONIC
WIND TUNNELS USING THE AERODYNAMIC NOISE CORRELATIONS BY PATE
KOPT = 2    KGEOM = 2
C,IN.  XL,IN.  RMACH  PO,PSIA   TO,R    TW,R    REINF/FT    DELTA,DEG    RTS    RPS   RMACHS  RRESC   RETSC        XTSC,IN.  TW/TAW
157.0  300.0    8.0    450.0   1400.0  530.0  1.8433E 06    5.00      1.210  1.920    7.22   1.302  6.1437E 06    30.73     0.417
```

```
PREDICTION OF BOUNDARY LAYER TRANSITION ON SHARP FLAT PLATES
FOR NON IDEAL GAS NOZZLE EXPANSION PROCESS
RMACH > 10 OR RMACH > 8 . REINF > 15*10**6 *SEE FIG.
USING PATES AERODYNAMIC NOISE TRANSITION CORRELATION
KOPT = 3    KGEOM = 3
C, IN.   XL,IN.  RMACH   TW      REINF        CF         DELS,IN.   RETFP        XTFP,IN.  TW/TAW
157.0    300.0   8.0     530.0   3.0000E 06   0.000771   2.0060     6.7141E 06   26.86     0.47
```



```
ESTIMATION OF BOUNDARY LAYER TRANSITION ON SHARP SLENDER CONES
FOR MACH NUMBERS GREATER THAN 10 ON VERY HIGH REYNOLDS
AT MACH NUMBER EQUAL APPROX 8(REINF>10**7)
USING PATES AERODYNAMIC NOISE TRANSITION CORRELATION
KOPT = 4    KGEOM = 3
C, IN.   XL,IN.   RMACH   TW,DEGR   REINF/FT     RESC/REINF   DELS,IN.   REISC        XTSC,IN.   DELTA,DEG   TW/TAW
157.0    300.00   8.00    530.00    3.0000E 06   1.500        2.006      8.4295E 06   21.341     10.000      0.47
```

22.479?



S.P. Schneider reran all these test cases and got agreement, 4-2-2003, except for the last case above, which agrees except for getting 22.479 instead of 21.341 in XTSC. Not clear why this is so, a hand check of the formula and the other numbers printed out suggests that 22.479 is correct.

VI. Flow Chart for Fortran Computer Program