## CE 503

## Homework \#1 8-Parameter Transformation Assigned 28-August-02, due one week.

The 8-parameter transformation is used to relate corresponding points in two planes that are related by perspective projection (i.e. a film plane and an object plane). We will assume that the area around the engineering mall is a plane. Download the annotated image file shown below, and measure the image points in a program like Adobe Photoshop. Make sure the measuring units are set to "pixels". To avoid ambiguity call the image coordinates (row,column) or (r,c) rather than photoshop's ( $y, x$ ). For each point you can form 2 matrix equations in 8 unknowns. Thus with 8 points you should have 16 equations. Solve that overdetermined system and submit your results with (a) the parameter values, and (b) the image point residuals.


| Control Points (units meters) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ID | X | Y | Z |  |
| 1 | 14374.19 | 75071.87 | 189.5 |  |
| 2 | 14397.55 | 75057.60 | 189.5 |  |
| 3 | 14427.64 | 75075.23 | 189.5 |  |
| 4 | 14419.03 | 75121.11 | 189.5 |  |
| 5 | 14412.81 | 75198.59 | 189.5 |  |
| 6 | 14384.74 | 75156.97 | 189.5 |  |
| 7 | 14358.01 | 75164.70 | 189.5 |  |
| 8 | 14336.41 | 75138.63 | 189.5 |  |

See equation for one point below. Use MATLAB and build a $16 x 8$ coefficient matrix, A, and a $16 \times 1$ vector $\mathbf{b}$. Then solve for the unknowns by $\mathbf{x}=\boldsymbol{\operatorname { i n v }}\left(\mathbf{A}^{\mathbf{T}} \mathbf{A}\right) * \mathbf{A}^{\mathbf{T}} \mathbf{b}$, etc.
Suggestion: type your commands into a $*$.m file, then if you later want to rerun, you do not have to type everything again. We are using a linear approximation to the true nonlinear 8-parameter model. Obtain the residual by $\mathbf{b}-\mathbf{A} * \mathbf{x}$, where $\mathbf{x}$ is the solution vector that you solved for. This is the first step to performing a simple rectification. Notice that we assume all points are in the object plane so the $\mathbf{z}$ coordinate is not used. The transformation is not valid for any points displaced from that object plane. (notes: (1) for image points $5 \& 7$, use the dark corner nearest Hovde Hall - the one with columns.
(2) to avoid numerical problems, subtract from each $r$, the mean of all the $r$ 's, likewise the c's, the x's, and the y's)

$$
\begin{aligned}
& r=\frac{a_{0}+a_{1} x+a_{2} y}{1+c_{1} x+c_{2} y} \\
& c=\frac{b_{0}+b_{1} x+b_{2} y}{1+c_{1} x+c_{2} y} \\
& r+r c_{1} x+r c_{2} y=a_{0}+a_{1} x+a_{2} y \\
& c+c c_{1} x+c c_{2} y=b_{0}+b_{1} x+b_{2} y \\
& r=a_{0}+a_{1} x+a_{2} y-r c_{1} x-r c_{2} y \\
& c=b_{0}+b_{1} x+b_{2} y-c c_{1} x-c c_{2} y
\end{aligned}
$$

$$
\left[\begin{array}{l}
r \\
c
\end{array}\right]=\left[\begin{array}{llllllll}
1 & x & y & 0 & 0 & 0 & -r x & -r y \\
0 & 0 & 0 & 1 & x & y & -c x & -c y
\end{array}\right]\left[\begin{array}{l}
a_{0} \\
a_{1} \\
a_{2} \\
b_{0} \\
b_{1} \\
b_{2} \\
c_{1} \\
c_{2}
\end{array}\right]
$$

