CE506 HW#2 Fall '03

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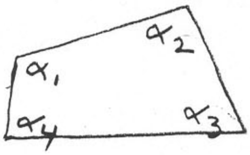

α_1 110 00 20
α_2 90 02 15
α_3 80 05 25
α_4 79 52 40

$n = 4$
$n_0 = 3$
$r = 1$

360 - 00 - 40 $\Sigma \alpha_i$

1 condition equation: $\hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 + \hat{\alpha}_4 = 360°$

$$\alpha_1 + V_1 + \alpha_2 + V_2 + \alpha_3 + V_3 + \alpha_4 + V_4 = 360$$

$$V_1 + V_2 + V_3 + V_4 = 360 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 = -40''$$

$$\boxed{V_1 + V_2 + V_3 + V_4 = -40''}$$

solve for V_1: $\boxed{V_1 = -40'' - V_2 - V_3 - V_4}$ using substitution method

plug into objective function

$$\phi = V_1^2 + V_2^2 + V_3^2 + V_4^2 = (-40'' - V_2 - V_3 - V_4)^2 + V_2^2 + V_3^2 + V_4^2$$

$$\partial\phi/\partial V_2 = \cancel{2}(-40 - V_2 - V_3 - V_4)(-1) + \cancel{2}V_2 = 0$$
$$\partial\phi/\partial V_3 = \cancel{2}(-40 - V_2 - V_3 - V_4)(-1) + \cancel{2}V_3 = 0$$
$$\partial\phi/\partial V_4 = \cancel{2}(-40 - V_2 - V_3 - V_4)(-1) + \cancel{2}V_4 = 0$$

$$2V_2 + V_3 + V_4 = -40$$
$$V_2 + 2V_3 + V_4 = -40$$
$$V_2 + V_3 + 2V_4 = -40$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -40 \\ -40 \\ -40 \end{bmatrix}$$

solution via matlab

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -10'' \\ -10'' \\ -10'' \end{bmatrix}$$

plug back into equation that eliminated V_1

$$V_1 = -40 - V_2 - V_3 - V_4 = -10''$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -10'' \\ -10'' \\ -10'' \\ -10'' \end{bmatrix}$$

$\hat{\alpha}_1 =$ 110° 00 10"
$\hat{\alpha}_2 =$ 90 02 05"
$\hat{\alpha}_3 =$ 80 05 15"
$\hat{\alpha}_4 =$ 79 52 30"

satisfies cond. equation ✓

Equivalent simple method is to distribute the angle misclosure equally among 4 observations: excess is 40" so each obs. gets $-\frac{40''}{4} = -10''$ correction

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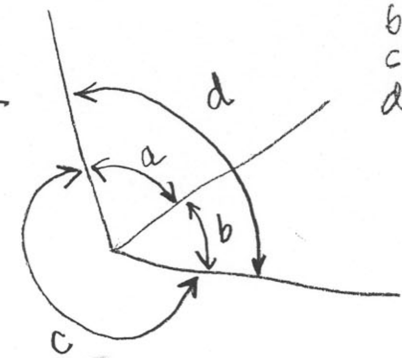

a = 60-00-00
b = 60-00-00
c = 240-00-25
d = 120-00-05

$n = 4$
$n_0 = 2$
$r = 2$

write r = 2 cond. equs

$\hat{a} + \hat{b} = \hat{d}$

$\hat{d} + \hat{c} = 360°$

$a + v_a + b + v_b = d + v_d$

$d + v_d + c + v_c = 360°$

$$\boxed{\begin{aligned} V_a + V_b - V_d &= +5'' \\ V_c + V_d &= -30'' \end{aligned}}$$

$V_a + V_b - V_d = -a - b + d = +5''$

$V_c + V_d = 360° - c - d = -30''$

solve for & eliminate V_a, V_c ; retain V_b, V_d

$$\left.\begin{aligned} V_a &= -V_b + V_d + 5'' \\ V_c &= -V_d - 30'' \end{aligned}\right\}$$

$\phi = V_a^2 + V_b^2 + V_c^2 + V_d^2$, now substitute

$\phi = (-V_b + V_d + 5'')^2 + V_b^2 + (-V_d - 30'')^2 + V_d^2$

$\partial\phi / \partial V_b = \cancel{2}(-V_b + V_d + 5'')(-1) + \cancel{2}V_b = 0$

$\partial\phi / \partial V_d = \cancel{2}(-V_b + V_d + 5'') + \cancel{2}(-V_d - 30'')(-1) + \cancel{2}V_d = 0$

$$\left.\begin{aligned} 2V_b - V_d &= +5'' \\ -V_b + 3V_d &= -35'' \end{aligned}\right\}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} V_b \\ V_d \end{bmatrix} = \begin{bmatrix} +5'' \\ -35'' \end{bmatrix}, \quad \begin{bmatrix} V_b \\ V_d \end{bmatrix} = \begin{bmatrix} -4'' \\ -13'' \end{bmatrix}$$

from elimination equation

$V_a = \cancel{+4}\ -13 + 5 = -4$

$V_c = +13 - 30 = -17$

$\hat{a} = 59° - 59' - 56''$

$\hat{b} = 59° - 59' - 56''$

$\hat{c} = 240° - 00 - 08''$

$\hat{d} = 119 - 59 - 52''$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} -4'' \\ -4'' \\ -17'' \\ -13'' \end{bmatrix}$$

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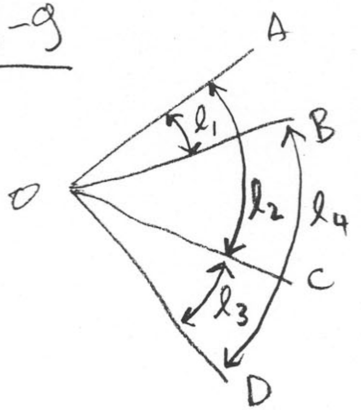


l_1 30-00-20
l_2 50-00-00
l_3 20-00-00
l_4 40-00-20

1 condition equation

$$\hat{l}_1 + \hat{l}_4 = \hat{l}_2 + \hat{l}_3$$

$n = 4$
$n_0 = 3$
$r = 1$

$$l_1 + v_1 - l_2 - v_2 - l_3 - v_3 + l_4 + v_4 = 0$$

$$v_1 - v_2 - v_3 + v_4 = -l_1 + l_2 + l_3 - l_4$$

$$\boxed{v_1 - v_2 - v_3 + v_4 = -40''}$$

solve for v_1 : $v_1 = v_2 + v_3 - v_4 - 40''$, $\phi = v_1^2 + v_2^2 + v_3^2 + v_4^2$

$$\phi = (v_2 + v_3 - v_4 - 40'')^2 + v_2^2 + v_3^2 + v_4^2$$

$$\frac{\partial\phi}{\partial v_2} = \cancel{2}(v_2 + v_3 - v_4 - 40) + \cancel{2}v_2 = 0$$

$$\frac{\partial\phi}{\partial v_3} = \cancel{2}(v_2 + v_3 - v_4 - 40) + \cancel{2}v_3 = 0$$

$$\frac{\partial\phi}{\partial v_4} = \cancel{2}(v_2 + v_3 - v_4 - 40)(-1) + \cancel{2}v_4 = 0$$

$$2v_2 + v_3 - v_4 = 40$$
$$v_2 + 2v_3 - v_4 = 40$$
$$-v_2 - v_3 + 2v_4 = -40$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 40 \\ 40 \\ -40 \end{bmatrix} \qquad \begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -10 \end{bmatrix}$$

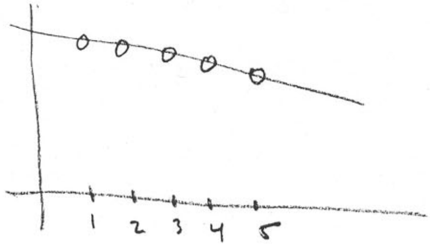
$$v_1 = 10 + 10 + 10 - 40 = -10$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} -10 \\ +10 \\ +10 \\ -10 \end{bmatrix}$$

$\hat{l}_1 =$ 30-00-10
$\hat{l}_2 =$ 50-00-10
$\hat{l}_3 =$ 20-00-10
$\hat{l}_4 =$ 40-00-10

3–10)

x	y
1	9.60
2	8.85
3	8.05
4	7.50
5	7.15



x: constant
y: observation

$n = 5$
$n_0 = 2$
$r = 3$

must write 3 condition equations among the adjusted observations

1. $\dfrac{\hat{y}_2-\hat{y}_1}{x_2-x_1} = \dfrac{\hat{y}_3-\hat{y}_1}{x_3-x_1} \qquad \dfrac{\hat{y}_2-\hat{y}_1}{1} = \dfrac{\hat{y}_3-\hat{y}_1}{2} \qquad 2\hat{y}_2 - 2\hat{y}_1 = \hat{y}_3 - \hat{y}_1$

2. $\dfrac{\hat{y}_2-\hat{y}_1}{x_2-x_1} = \dfrac{\hat{y}_4-\hat{y}_1}{x_4-x_1} \qquad \dfrac{\hat{y}_2-\hat{y}_1}{1} = \dfrac{\hat{y}_4-\hat{y}_1}{3} \qquad 3\hat{y}_2 - 3\hat{y}_1 = \hat{y}_4 - \hat{y}_1$

3. $\dfrac{\hat{y}_2-\hat{y}_1}{x_2-x_1} = \dfrac{\hat{y}_5-\hat{y}_1}{x_5-x_1} \qquad \dfrac{\hat{y}_2-\hat{y}_1}{1} = \dfrac{\hat{y}_5-\hat{y}_1}{4} \qquad 4\hat{y}_2 - 4\hat{y}_1 = \hat{y}_5 - \hat{y}_1$

$$\left.\begin{aligned} 2(y_2+v_2) - \cancel{2}^{\,1}(y_1+v_1) - (y_3+v_3) + \cancel{(y_1+v_1)} &= 0 \\ 3(y_2+v_2) - \cancel{3}^{\,2}(y_1+v_1) - (y_4+v_4) + \cancel{(y_1+v_1)} &= 0 \\ 4(y_2+v_2) - \cancel{4}^{\,3}(y_1+v_1) - (y_5+v_5) + \cancel{(y_1+v_1)} &= 0 \end{aligned}\right\} \begin{aligned} -v_1 + 2v_2 - v_3 &= y_1 - 2y_2 + y_3 \\ -2v_1 + 3v_2 - v_4 &= 2y_1 - 3y_2 + y_4 \\ -3v_1 + 4v_2 - v_5 &= 3y_1 - 4y_2 + y_5 \end{aligned}$$

$$\begin{aligned} -v_1 + 2v_2 - v_3 &= -.05 \\ -2v_1 + 3v_2 - v_4 &= .15 \\ -3v_1 + 4v_2 - v_5 &= .55 \end{aligned}$$

eliminate v_3, v_4, v_5

$$\begin{aligned} v_3 &= -v_1 + 2v_2 + .05 \\ v_4 &= -2v_1 + 3v_2 - .15 \\ v_5 &= -3v_1 + 4v_2 - .55 \end{aligned}$$

$$\phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 = v_1^2 + v_2^2 + (-v_1 + 2v_2 + .05)^2 + (-2v_1 + 3v_2 - .15)^2 + (-3v_1 + 4v_2 - .55)^2$$

$$\partial\phi/\partial v_1 = \cancel{2}v_1 + \cancel{2}(-v_1 + 2v_2 + .05)(-1) + \cancel{2}(-2v_1 + 3v_2 - .15)(-2) + \cancel{2}(-3v_1 + 4v_2 - .55)(-3) = 0$$

$$\partial\phi/\partial v_2 = \cancel{2}v_2 + \cancel{2}(-v_1 + 2v_2 + .05)(2) + \cancel{2}(-2v_1 + 3v_2 - .15)(3) + \cancel{2}(-3v_1 + 4v_2 - .55)(4) = 0$$

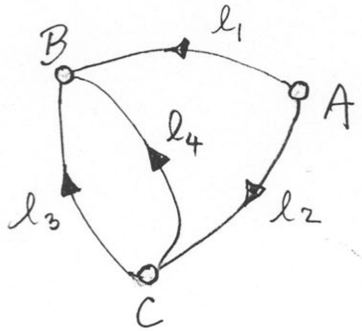
$$v_1 + v_1 + 4v_1 + 9v_1 + -2v_2 - 6v_2 - 12v_2 = .05 - .30 - 1.65$$

$$-2v_1 - 6v_1 - 12v_1 + v_2 + 4v_2 + 9v_2 + 16v_2 = -.10 + .45 + 2.20$$

$$\left.\begin{aligned} 15v_1 - 20v_2 &= -1.9 \\ -20v_1 + 30v_2 &= 2.55 \end{aligned}\right\} \begin{bmatrix} 15 & -20 \\ -20 & 30 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -1.9 \\ 2.55 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -.120 \\ .005 \end{bmatrix}, \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} .180 \\ .105 \\ -.170 \end{bmatrix}$$

$$\hat{y}_1, \hat{y}_2, \hat{y}_3, \hat{y}_4, \hat{y}_5 = 9.480,\ 8.855,\ 8.230,\ 7.605,\ 6.980$$

3-11



l_1	20.410
l_2	10.100
l_3	10.300
l_4	10.315

$n = 4$
$n_0 = 2$
$r = 2$

write 2 condition eqn's.

$$\hat{l}_1 - \hat{l}_4 - \hat{l}_2 = 0$$
$$\hat{l}_3 = \hat{l}_4$$

$$\left.\begin{aligned} l_1 + V_1 - l_4 - V_4 - l_2 - V_2 &= 0 \\ l_3 + V_3 - l_4 - V_4 &= 0 \end{aligned}\right\} \quad \begin{aligned} V_1 - V_2 - V_4 &= -l_1 + l_2 + l_4 \\ V_3 - V_4 &= -l_3 + l_4 \end{aligned}$$

$$\left.\begin{aligned} V_1 - V_2 - V_4 &= .005 \\ V_3 - V_4 &= .015 \end{aligned}\right\}$$

eliminate V_1, V_3

$$V_1 = V_2 + V_4 + .005$$
$$V_3 = V_4 + .015$$

$$\phi = V_1^2 + V_2^2 + V_3^2 + V_4^2, \quad \phi = (V_2 + V_4 + .005)^2 + V_2^2 + (V_4 + .015)^2 + V_4^2$$

$$\left.\begin{aligned} \frac{\partial \phi}{\partial V_2} &= \cancel{2}(V_2 + V_4 + .005) + \cancel{2}V_2 = 0 \\ \frac{\partial \phi}{\partial V_4} &= \cancel{2}(V_2 + V_4 + .005) + \cancel{2}(V_4 + .015) + \cancel{2}V_4 = 0 \end{aligned}\right\} \quad \begin{aligned} 2V_2 + V_4 &= -.005 \\ V_2 + 3V_4 &= -.020 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} V_2 \\ V_4 \end{bmatrix} = \begin{bmatrix} -.005 \\ -.020 \end{bmatrix}, \quad \begin{pmatrix} V_2 \\ V_4 \end{pmatrix} = \begin{bmatrix} 0.001 \\ -0.007 \end{bmatrix}$$

$$V_1 = -0.001$$
$$V_3 = 0.008$$

$$\hat{l}_1 = 20.409$$
$$\hat{l}_2 = 10.101$$
$$\hat{l}_3 = 10.308$$
$$\hat{l}_4 = 10.308$$