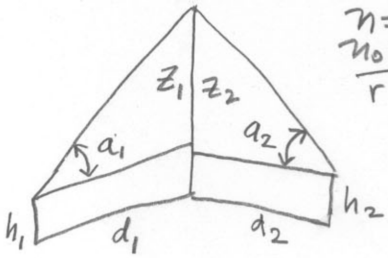
CE 506 Homework 5 Solution 14-Oct-03

1. (a) Observations only

$$F = h_1 + d_1 \tan a_1 - h_2 - d_2 \tan a_2 = 0$$



$n = 6$
$n_0 = 5$
$r = 1$

$$\frac{\partial F}{\partial d_1} = \tan a_1 \qquad \frac{\partial F}{\partial d_2} = -\tan a_2$$

$$\frac{\partial F}{\partial h_1} = 1 \qquad \frac{\partial F}{\partial h_2} = -1$$

$$\frac{\partial F}{\partial a_1} = \frac{d_1}{\cos^2 a_1} \qquad \frac{\partial F}{\partial a_2} = \frac{-d_2}{\cos^2 a_2}$$

$$A = \left[\frac{\partial F}{\partial d_1}\ \frac{\partial F}{\partial h_1}\ \frac{\partial F}{\partial a_1}\ \frac{\partial F}{\partial d_2}\ \frac{\partial F}{\partial h_2}\ \frac{\partial F}{\partial a_2}\right] = \left[\tan a_1 \quad 1 \quad \frac{d_1}{\cos^2 a_1} \quad -\tan a_2 \quad -1 \quad \frac{-d_2}{\cos^2 a_2}\right]$$

$$f = -F(l^0) - A(l - l^0)$$

$$W = \begin{bmatrix} 1.96 & & & & & \\ & 1 & & & \emptyset & \\ & & 579 & & & \\ & & & 5.4 & & \\ & \emptyset & & & 1.3 & \\ & & & & & 402 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.0282 \\ -0.0326 \\ -0.0026 \\ 0.0076 \\ 0.0239 \\ 0.0034 \end{bmatrix} \qquad \hat{l} = \begin{bmatrix} 11.992 \\ 1.6474 \\ 1.0388\ R \\ 15.958 \\ 1.634 \\ 0.90661\ R \end{bmatrix}$$

(See MATLAB program listing hw51a.m, hw51a.lst)

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1. (b) Indirect Observations

choose 5 parameters D_1, H_1, Z_1, D_2, H_2

$F_1 = d_1 - D_1 = 0$ $\qquad f_1 = -F_1$

$F_2 = h_1 - H_1 = 0$ $\qquad f_2 = -F_2$

$F_3 = a_1 - \tan^{-1}(Z_1/D_1) = 0$ $\qquad f_3 = -F_3$

$F_4 = d_2 - D_2 = 0$ $\qquad f_4 = -F_4$

$F_5 = h_2 - H_2 = 0$ $\qquad f_5 = -F_5$

$F_6 = a_2 - \tan^{-1}\left(\frac{Z_1 + H_1 - H_2}{D_2}\right) = 0$ $\qquad f_6 = -F_6$

W = same as part (a)

$$\frac{\partial F_3}{\partial Z_1} = -\left[\frac{1}{1 + Z_1^2/D_1^2} \cdot \frac{1}{D_1}\right]$$

$$\frac{\partial F_3}{\partial D_1} = -\left[\frac{1}{1 + Z_1^2/D_1^2} \cdot \frac{(-1)Z_1}{D_1^2}\right]$$

$$\frac{\partial F_6}{\partial H_1} = -\left[\frac{1}{1 + \frac{(Z_1 + H_1 - H_2)^2}{D_2^2}}\right]\left[\frac{1}{D_2}\right]$$

$$\frac{\partial F_6}{\partial Z_1} = -\left[\text{same} \uparrow\right]\left[\frac{1}{D_2}\right]$$

$$\frac{\partial F_6}{\partial D_2} = -\left[\text{same} \uparrow\right]\left[\frac{-(Z_1 + H_1 - H_2)}{D_2^2}\right]$$

$$\frac{\partial F_6}{\partial H_2} = -\left[\text{same}\right]\left[\frac{-1}{D_2}\right]$$

other elements of B matrix by inspection

$\Delta = (B^T W B)^{-1} B^T W f$

$V = f - B\Delta$

$\hat{l} = l + V$

$$V = \begin{bmatrix} -.0282 \\ -.0326 \\ -.0026 \\ .0076 \\ .0239 \\ .0034 \end{bmatrix} \qquad \hat{l} = \begin{bmatrix} 11.992 \\ 1.6474 \\ 1.0388 \\ 15.958 \\ 1.634 \\ 0.90661 \end{bmatrix} \quad \text{same as part (a)}$$

(See MATLAB solution hw51b.m, hw51b.lst)

```
% hw51a.m  14-oct-03
% flagpole problem observations only

n=6;
n0=5;
r=1;

d1=12.02;
h1=1.68;
a1_d=59+40/60;
d2=15.95;
h2=1.61;
a2_d=51+45/60;
degrad=180/pi;
a1=a1_d/degrad;
a2=a2_d/degrad;

l=[d1;h1;a1;d2;h2;a2];
l0=l;

s1=0.05;
s2=0.07;
s3=(10/60)/degrad;
s4=0.03;
s5=0.06;
s6=(12/60)/degrad;

s0=s2;
wts=s0^2*[1/s1^2; 1/s2^2; 1/s3^2; 1/s4^2; 1/s5^2; 1/s6^2];
W=diag(wts)

small_vec=[1.0e-06; 1.0e-06; 1.0e-08; 1.0e-06; 1.0e-06; 1.0e-08];
big=1.0e+05;
delta_l=[big; big; big; big; big; big];
iter=1;

while((any(abs(delta_l) > small_vec)) & (iter < 10))
  dFdh1=1.0;
  dFdd1=tan(a1);
  dFda1=d1/(cos(a1)*cos(a1));
  dFdh2=-1.0;
  dFdd2=-tan(a2);
  dFda2=-d2/(cos(a2)*cos(a2));
  A=[dFdd1 dFdh1 dFda1 dFdd2 dFdh2 dFda2];

  F=h1 + d1*tan(a1) - h2 - d2*tan(a2);
  f=-F - A*(l-l0);

  z=0;
  zv=[0;0;0;0;0;0];
  mx=[-W A';A z];
  vc=[zv; f];
```

```
  sol=inv(mx)*vc;

  v=sol(1:6);
  old_l0=l0;
  l0=l + v;
  delta_l=l0 - old_l0;
  iter
  delta_l
  d1=l0(1);
  h1=l0(2);
  a1=l0(3);
  d2=l0(4);
  h2=l0(5);
  a2=l0(6);
  iter=iter+1;
  end

lhat=l0
v
disp('a1');
[d,m]=adm(a1)
disp('a2');
[d,m]=adm(a2)
disp('v-a1');
[d,m]=adm(v(3))
disp('v-a2');
[d,m]=adm(v(6))

disp('height of the flagpole');
flagpole1=h1 + d1*tan(a1)
flagpole2=h2 + d2*tan(a2)
```

```
hw51a
W =
        1.96            0            0            0            0            0
           0            1            0            0            0            0
           0            0       579.09            0            0            0
           0            0            0       5.4444            0            0
           0            0            0            0       1.3611            0
           0            0            0            0            0       402.14
iter =
     1
delta_l =
     -0.0284
   -0.032571
  -0.0026507
   0.0075886
    0.023929
   0.0033705
iter =
     2
delta_l =
  0.00013293
 -4.4985e-005
  2.6421e-005
  6.3388e-005
   3.305e-005
   3.536e-005
iter =
     3
delta_l =
   4.834e-007
  2.5288e-006
 -6.1179e-008
 -3.5868e-008
 -1.8579e-006
   5.712e-008
iter =
     4
delta_l =
  3.5458e-009
  -4.729e-010
  4.0164e-010
  1.0114e-009
  3.4744e-010
  5.3876e-010
lhat =
      11.992
      1.6474
      1.0388
      15.958
       1.634
     0.90661
v =
```

```
     -0.028266
     -0.032613
    -0.0026243
     0.0076519
      0.023961
     0.0034059
a1
d =
    59
m =
       30.978
a2
d =
    51
m =
       56.709
v-a1
d =
     0
m =
      -9.0217
v-a2
d =
     0
m =
       11.709
height of the flagpole
flagpole1 =
       22.019
flagpole2 =
       22.019
diary off
```

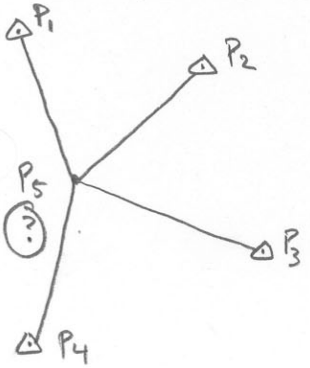
```
% hw51b.m  14-oct-03
% flagpole problem indirect observations

n=6;
n0=5;
r=1;
c=n;
u=n0;

% parameters: D1,H1,Z1,D2,H2
% Z1=difference between flagpole height and H1

d1=12.02;
h1=1.68;
a1_d=59+40/60;
d2=15.95;
h2=1.61;
a2_d=51+45/60;
degrad=180/pi;
a1=a1_d/degrad;
a2=a2_d/degrad;

l=[d1;h1;a1;d2;h2;a2];

s1=0.05;
s2=0.07;
s3=(10/60)/degrad;
s4=0.03;
s5=0.06;
s6=(12/60)/degrad;

s0=s2;
wts=s0^2*[1/s1^2; 1/s2^2; 1/s3^2; 1/s4^2; 1/s5^2; 1/s6^2];
W=diag(wts)

small_vec=[1.0e-06; 1.0e-06; 1.0e-06; 1.0e-06; 1.0e-06];
big=1.0e+05;
delta=[big; big; big; big; big];
iter=1;
% initial approximations
D1=d1;
H1=h1;
Z1=d1*tan(a1);
D2=d2;
H2=h2;

while((any(abs(delta) > small_vec)) & (iter < 10))
  iter
  dF1dD1=-1;
  dF2dH1=-1;
  base=1/(1 + (Z1*Z1)/(D1*D1));
  dF3dZ1=-base*(1/D1);
```

```
  dF3dD1=-base*((-1)*Z1/(D1*D1));
  base=1/(1 + ((Z1+H1-H2)^2)/(D2^2));
  dF4dD2=-1;
  dF5dH2=-1;
  dF6dZ1=-base*(1/D2);
  dF6dH1=-base*(1/D2);
  dF6dH2=-base*(-1/D2);
  dF6dD2=-base*(-(Z1+H1-H2)/(D2^2));
  B=zeros(c,u);
  B(1,1)=dF1dD1;
  B(2,2)=dF2dH1;
  B(3,3)=dF3dZ1;
  B(3,1)=dF3dD1;
  B(4,4)=dF4dD2;
  B(5,5)=dF5dH2;
  B(6,3)=dF6dZ1;
  B(6,2)=dF6dH1;
  B(6,5)=dF6dH2;
  B(6,4)=dF6dD2;
  f=zeros(c,1);
  f(1)= -(d1-D1);
  f(2)= -(h1-H1);
  f(3)= -(a1 - atan(Z1/D1));
  f(4)= -(d2-D2);
  f(5)= -(h2-H2);
  f(6)= -(a2 - atan((Z1+H1-H2)/D2));

  delta=inv(B'*W*B)*B'*W*f;
  delta
  D1=D1 + delta(1);
  H1=H1 + delta(2);
  Z1=Z1 + delta(3);
  D2=D2 + delta(4);
  H2=H2 + delta(5);
  iter=iter+1;
  end

v=f - B*delta
lhat=l+v

disp('a1');
[d,m]=adm(lhat(3))
disp('a2');
[d,m]=adm(lhat(6))
disp('v-a1');
[d,m]=adm(v(3))
disp('v-a2');
[d,m]=adm(v(6))

disp('height of the flagpole');
flagpole1=H1 + Z1
```

```
hw51b
W =
        1.96             0             0             0             0             0
           0             1             0             0             0             0
           0             0        579.09             0             0             0
           0             0             0        5.4444             0             0
           0             0             0             0        1.3611             0
           0             0             0             0             0        402.14
iter =
     1
delta =
    -0.02821
   -0.032353
     -0.1723
   0.0076795
     0.02377
iter =
     2
delta =
  -5.572e-005
 -0.00026137
   0.0011596
 -2.7433e-005
  0.00019203
iter =
     3
delta =
 -2.4444e-007
   1.726e-006
 -4.5068e-006
 -1.2672e-007
 -1.2681e-006
iter =
     4
delta =
  1.0501e-009
 -5.3388e-009
   2.074e-008
  7.4558e-010
  3.9224e-009
v =
   -0.028266
   -0.032613
  -0.0026243
   0.0076519
    0.023961
   0.0034059
lhat =
      11.992
      1.6474
      1.0388
      15.958
```

```
        1.634
      0.90661
a1
d =
    59
m =
       30.978
a2
d =
    51
m =
       56.709
v-a1
d =
     0
m =
      -9.0217
v-a2
d =
     0
m =
       11.709
height of the flagpole
flagpole1 =
       22.019
diary off
```

2. (a) Trilateration Problem by Indirect observations



$n = 4$
$n_0 = 2$
$r = 2$

Choose parameters
X, Y of point 5

$$F_i = d_i - \left[(X_i - X_5)^2 + (Y_i - Y_5)^2\right]^{1/2} = 0$$

$$\frac{\partial F_i}{\partial X_5} = \frac{(X_i - X_5)}{\left[(X_i - X_5)^2 + (Y_i - Y_5)^2\right]^{1/2}}$$

$$\frac{\partial F_i}{\partial Y_5} = \frac{(Y_i - Y_5)}{\left[(X_i - X_5)^2 + (Y_i - Y_5)^2\right]^{1/2}}$$

$$f_i = -F_i(\ell, x^\circ)$$

$W = I_4$

$$V = \begin{bmatrix} .576 \\ -.149 \\ .412 \\ .317 \end{bmatrix} \quad \hat{\ell} = \begin{bmatrix} 80.876 \\ 66.851 \\ 80.113 \\ 60.918 \end{bmatrix}$$

(See MATLAB program + listing hw52a.m, hw52a.lst)

(b) same problem but constrain $Y_5 = 0.5 X_5 + 60$

$n = 4$
$n_0 = 1$
$r = 3$

choose parameter X_5, eliminate Y_5 by substitution

$$F_i = d_i - \left[(X_i - X_5)^2 + (Y_i - 0.5X_5 - 60)^2\right]^{1/2} = 0$$

$$\frac{\partial F_i}{\partial X_5} = -\tfrac{1}{2}\left[\nearrow\right]^{-1/2}\left(2(X_i - X_5)(-1) + 2(Y_i - 0.5X_5 - 60)(-0.5)\right)$$

$$= \frac{(X_i - X_5) + \tfrac{1}{2}(Y_i - 0.5X_5 - 60)}{\left[(X_i - X_5)^2 + (Y_i - 0.5X_5 - 60)^2\right]^{1/2}}$$

$$\underset{4,1}{B} = \begin{bmatrix} \partial F_1/\partial X_5 \\ \partial F_2/\partial X_5 \\ \partial F_3/\partial X_5 \\ \partial F_4/\partial X_5 \end{bmatrix} \quad f = \begin{bmatrix} -F_1(\ell, x^\circ) \\ -F_2(\ell, x^\circ) \\ -F_3(\ell, x^\circ) \\ -F_4(\ell, x^\circ) \end{bmatrix} \quad W = I_4 \quad V = \begin{bmatrix} .345 \\ -.239 \\ .587 \\ .465 \end{bmatrix} \quad \hat{\ell} = \begin{bmatrix} 80.646 \\ 66.760 \\ 80.288 \\ 61.065 \end{bmatrix}$$

(See MATLAB solution in hw52b.m & hw52b.lst)

```
% hw52a.m  14-oct-03
% do ce506 homework problem, indirect obs.

n=4;
n0=2;
r=2;
c=n;
u=n0;

% choose x,y of point 5 tobe the 2 parameters

% define observations

d1=80.3;
d2=67.0;
d3=79.7;
d4=60.6;
l=[d1;d2;d3;d4];

% define the sigmas and weights

sigd1=0.3;
sigd2=0.3
sigd3=0.3;
sigd4=0.3;
% define a priori sigma-naught squared
sig0=0.3;
W=eye(4);
W(1,1)=sig0^2/sigd1^2;
W(2,2)=sig0^2/sigd2^2;
W(3,3)=sig0^2/sigd3^2;
W(4,4)=sig0^2/sigd4^2;
W

% the points

x1=20;
y1=160;
x2=90;
y2=150;
x3=140;
y3=80;
x4=50;
y4=30;

x5=60;
y5=90;

B=zeros(c,u);
f=zeros(c,1);
max_iter=10;
iter=1;
```

```
keep_going=1;
% convergence variables
phi=10;
last_phi=20;
threshold=1.0e-06;

while keep_going == 1
  d_15=sqrt((x1-x5)^2 + (y1-y5)^2);
  d_25=sqrt((x2-x5)^2 + (y2-y5)^2);
  d_35=sqrt((x3-x5)^2 + (y3-y5)^2);
  d_45=sqrt((x4-x5)^2 + (y4-y5)^2);

  % make coefficients of the condition equations

  B(1,:)=[(x1-x5)/d_15  (y1-y5)/d_15];
  B(2,:)=[(x2-x5)/d_25  (y2-y5)/d_25];
  B(3,:)=[(x3-x5)/d_35  (y3-y5)/d_35];
  B(4,:)=[(x4-x5)/d_45  (y4-y5)/d_45];

  f(1)=-(d1 - d_15);
  f(2)=-(d2 - d_25);
  f(3)=-(d3 - d_35);
  f(4)=-(d4 - d_45);

  if iter == 1
    disp('iteration 1 B,f,W');
    B
    f
    W
    end

  % now solve and update and check convergence

  N=B'*W*B;
  t=B'*W*f;
  iter
  del=inv(N)*t
  x5=x5 + del(1);
  y5=y5 + del(2);
  v=f-B*del;
  phi=v'*W*v;

  % use fractional change in the quadratic form vTWv as
  % the convergence criterion, units-free alternative to
  % checking magnitude of delta

  if( abs(phi-last_phi)/last_phi < threshold )
    keep_going=0;
    disp('we have converged');
    end
  last_phi=phi;
  if iter > 10
```

```
    keep_going=0;
    disp('too many iterations');
    end
  iter=iter+1;
  end;

disp('final coordinates');
[x5 y5]
disp('residuals');
v=f - B*del
lhat=l + v
```

```
hw52a
sigd2 =
          0.3
W =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
iteration 1 B,f,W
B =
     -0.49614      0.86824
      0.44721      0.89443
      0.99228     -0.12403
      -0.1644     -0.98639
f =
      0.32258
     0.082039
      0.92258
      0.22763
W =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
iter =
     1
del =
      0.51766
     0.004008
iter =
     2
del =
   -0.0032618
  -0.00060267
iter =
     3
del =
  2.0874e-005
  6.2824e-006
we have converged
final coordinates
ans =
       60.514       90.003
residuals
v =
      0.57608
     -0.14949
      0.41258
      0.31767
lhat =
       80.876
       66.851
```

```
       80.113
       60.918
diary off
```

```
% hw52b.m  14-oct-03
% do ce506 homework problem, indirect obs.
% constrain solution to a line y=0.5x + 60

n=4;
n0=1;
r=3;
c=n;
u=n0;

% choose x of point 5 to be the 1 parameter
% substitute the expression for y to eliminate y

% define observations

d1=80.3;
d2=67.0;
d3=79.7;
d4=60.6;
l=[d1;d2;d3;d4];

% define the sigmas and weights

sigd1=0.3;
sigd2=0.3
sigd3=0.3;
sigd4=0.3;
% define a priori sigma-naught squared
sig0=0.3;
W=eye(4);
W(1,1)=sig0^2/sigd1^2;
W(2,2)=sig0^2/sigd2^2;
W(3,3)=sig0^2/sigd3^2;
W(4,4)=sig0^2/sigd4^2;
W

% the points

x1=20;
y1=160;
x2=90;
y2=150;
x3=140;
y3=80;
x4=50;
y4=30;

x5=60;

B=zeros(c,u);
f=zeros(c,1);
max_iter=10;
```

```
iter=1;
keep_going=1;
% convergence variables
phi=10;
last_phi=20;
threshold=1.0e-06;

while keep_going == 1
  % make coefficients of the condition equations

  B(1)=[((x1-x5)+0.5*(y1-0.5*x5-60))/sqrt((x1-x5)^2+(y1-0.5*x5-60)^2)];
  B(2)=[((x2-x5)+0.5*(y2-0.5*x5-60))/sqrt((x2-x5)^2+(y2-0.5*x5-60)^2)];
  B(3)=[((x3-x5)+0.5*(y3-0.5*x5-60))/sqrt((x3-x5)^2+(y3-0.5*x5-60)^2)];
  B(4)=[((x4-x5)+0.5*(y4-0.5*x5-60))/sqrt((x4-x5)^2+(y4-0.5*x5-60)^2)];

  f(1)=-(d1 - sqrt((x1-x5)^2 + (y1-0.5*x5-60)^2));
  f(2)=-(d2 - sqrt((x2-x5)^2 + (y2-0.5*x5-60)^2));
  f(3)=-(d3 - sqrt((x3-x5)^2 + (y3-0.5*x5-60)^2));
  f(4)=-(d4 - sqrt((x4-x5)^2 + (y4-0.5*x5-60)^2));

  if iter == 1
    disp('iteration 1 B,f,W');
    B
    f
    W
    end

  % now solve and update and check convergence

  N=B'*W*B;
  t=B'*W*f;
  iter
  del=inv(N)*t
  x5=x5 + del(1);
  v=f-B*del;
  phi=v'*W*v;

  % use fractional change in the quadratic form vTWv as
  % the convergence criterion, units-free alternative to
  % checking magnitude of delta

  if( abs(phi-last_phi)/last_phi < threshold )
    keep_going=0;
    disp('we have converged');
    end
  last_phi=phi;
  if iter > 10
    keep_going=0;
    disp('too many iterations');
    end
  iter=iter+1;
  end;
```

```
y5=0.5*x5 + 60.0;
disp('final coordinates');
[x5 y5]
disp('residuals');
v=f - B*del
lhat=l + v
```

```
hw52b
sigd2 =
          0.3
W =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
iteration 1 B,f,W
B =
    -0.062017
      0.89443
      0.93026
      -0.6576
f =
      0.32258
     0.082039
      0.92258
      0.22763
W =
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
iter =
     1
del =
      0.36253
iter =
     2
del =
   -0.0021796
iter =
     3
del =
  1.3208e-005
we have converged
final coordinates
ans =
        60.36        90.18
residuals
v =
      0.34593
     -0.23985
      0.58765
      0.46547
lhat =
       80.646
        66.76
       80.288
       61.065
diary off
```