1. An instrument is setup to read x,y and a distance (z) to the surface of a parabolic antenna. Fit the equally weighted data, via least squares (indirect observations) to a paraboloid with model shown below. x,y are constants, z is observation. The axis of the paraboloid is parallel to the z-axis. *Extra Credit:* Your customer would like to place his detector at the focus of your best fit paraboloid. What are the coordinates of the focus point in the measurement system?

$$z = a_0 + a_1 x + a_2 y + a_3 \left(x^2 + y^2 \right)$$
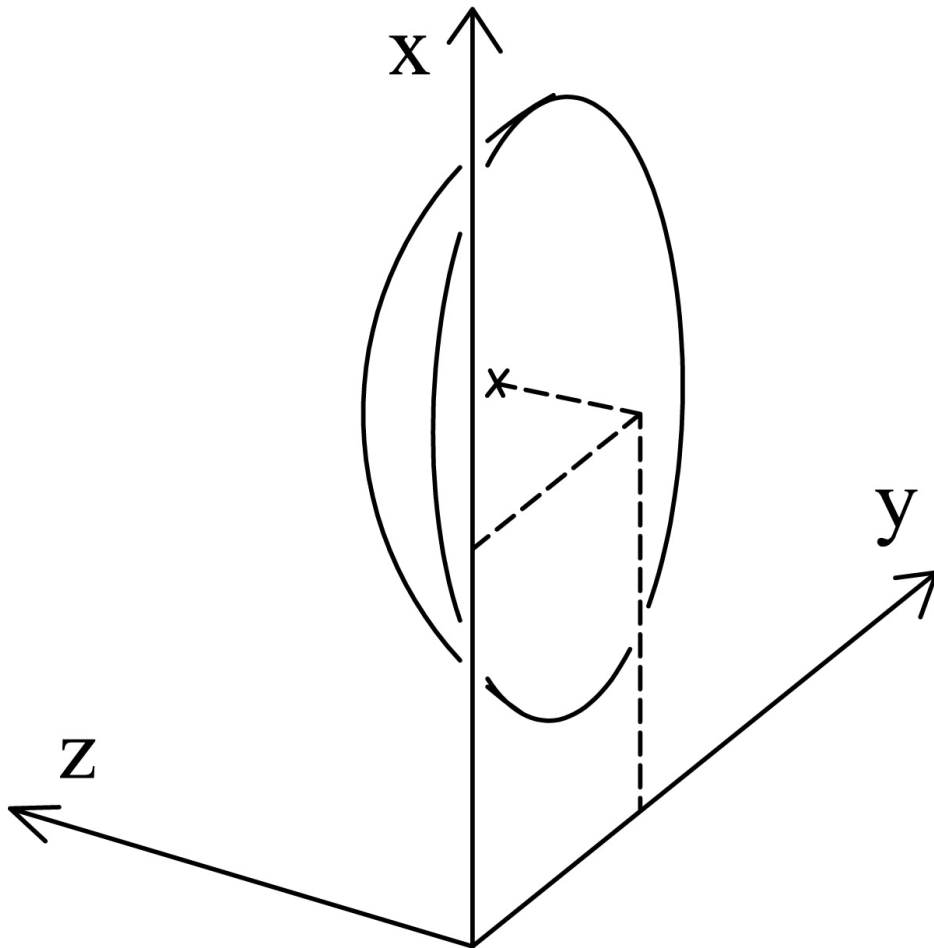
Num	X	Y	Z
1.0000	6.0000	11.0000	1.4387
2.0000	4.0000	8.0000	1.7487
3.0000	6.0000	7.0000	1.9383
4.0000	8.0000	5.0000	1.1855
5.0000	3.0000	10.0000	1.1875
6.0000	5.0000	10.0000	1.6874
7.0000	7.0000	10.0000	1.6875
8.0000	4.0000	9.0000	1.6869
9.0000	8.0000	9.0000	1.6897
10.0000	7.0000	8.0000	1.9338
11.0000	3.0000	7.0000	1.3759
12.0000	7.0000	7.0000	1.8768
13.0000	9.0000	7.0000	1.3765
14.0000	4.0000	6.0000	1.5012
15.0000	5.0000	6.0000	1.6876
16.0000	6.0000	5.0000	1.4389
17.0000	9.0000	10.0000	1.1886
18.0000	4.0000	5.0000	1.1870

1(b). Solve the previous problem again, but this time solve conventionally for the first 4 observations, then add each subsequent observation by the sequential technique shown in class.

For the initial batch solution show $\mathbf{N}, \mathbf{t}, \Delta$

For each sequential step show $\tilde{\mathbf{N}}_{\mathbf{i}}, \mathbf{N}_{\mathbf{i}}, \tilde{\mathbf{t}}_{\mathbf{i}}, \mathbf{t}_{\mathbf{i}}, \Delta_{\mathbf{i}}$

Note that the final result here should be identical to your previous solution. You should definitely get matlab to do the hard work here.

2. We have 100 samples from each of 2 noisy signals which we know was generated from a function of the form:

$$x(t) = a_0 + a_1 \sin(\boldsymbol{w}t) + a_2 \sin(2\boldsymbol{w}t) + a_3 \sin(3\boldsymbol{w}t) + \ldots + a_{10} \sin(10\boldsymbol{w}t)$$

t is time in seconds

$\boldsymbol{w}$ is frequency in radians/sec, $2\boldsymbol{w}$ is twice frequency of $\boldsymbol{w}$, etc.

f is frequency in hertz or cycles per second

$\boldsymbol{w} = 2\boldsymbol{p}f$

for period, p, $p = 1/f$

for this problem, $f = 0.1$, $p = 10$. These represent the

fundamental frequency, i.e. the term with a_1

We wish to estimate by least squares, indirect observations, the coefficients a_0 through a_{10}. The time, t, and all of the frequency variables are constants, x(t) is the observation. Observations have equal precision. Get the data by *mapping network drive:*

\\geomatics.ecn.purdue.edu\data

then go to folder: *share\bethel\ce506* and get hw3_2.mat. Use matlab "load" to retrieve t, f1, and f2. For each one make the least squares estimation. Plot a histogram of the residuals. Make a bar graph of the 11 coefficients. Use the relative magnitudes of the coefficients in the bar graph to determine the most significant frequency component. (you can also do this by inspection from looking at the data). This problem is related to Fourier analysis.

3. Use least squares estimation to obtain the 4-parameter transformation using the given data and the given model. Determine the scale and the rotation between the two systems. Note the figure is only a schematic drawing for a single data point

$$x' = ax + by + c$$

$$y' = -bx + ay + d$$
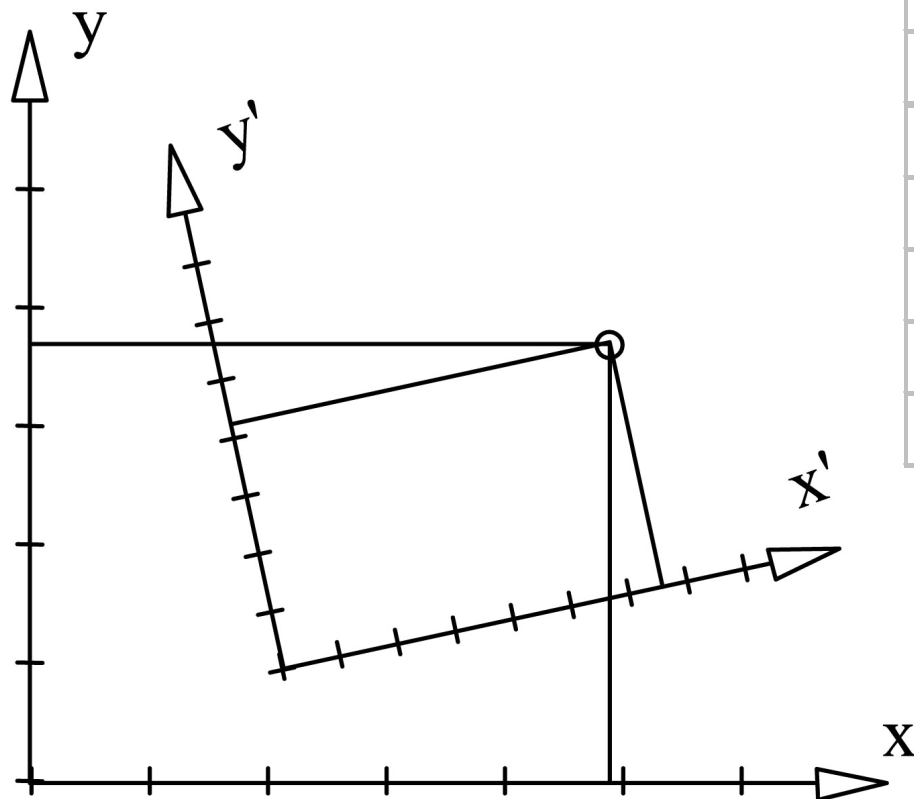
#	x'	y'	x	y
1	0.3502	2.1924	1.0000	1.0000
2	-4.2047	2.9376	3.0000	1.0000
3	-8.4573	3.8103	5.0000	1.0000
4	-0.3592	-2.3156	1.0000	3.0000
5	-4.9342	-1.257	3.0000	3.0000
6	-9.1321	-0.7075	5.0000	3.0000
7	-1.0504	-6.677	1.0000	5.0000
8	-5.6048	-5.8003	3.0000	5.0000
9	-9.9999	-5.1196	5.0000	5.0000

4. There are 9 points in a network surveyed by GPS. The results of the survey are coordinate differences between the network points. Point 1 is fixed at $(X,Y,Z) = (0,0,0)$. Use least squares to determine the best estimate of the remaining 8 points. Get the data from the file hw3_4.mat in the previously mentioned class folder. There are 5 arrays in the data file: from, to, dx, dy, dz. The standard deviation of each observation is 5 cm. There are 18 entries in each array, thus there are 18_x_3 = 54 total observations.

```
7              8              9



4                             6
               5



1              2              3
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