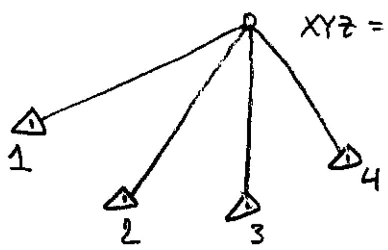
CE 597 Z Adj. of Geospatial Obs — HW2 solution

1. 3D trilateration problem:



$n = 4$
$n_0 = 3$
$r = 1$

use method of indirect observations

$\Longrightarrow u = n_0 = 3$: X, Y, Z of the unknown point

$c = n$, one equation per obs.

initial approximations are not easy here, although it converges over a wide range – suggest 3D CAD display with spheres

$$d_i = \left[(X_i - X)^2 + (Y_i - Y)^2 + (Z_i - Z)^2\right]^{1/2}$$

$$F_i = d_i - \left[(X_i - X)^2 + (Y_i - Y)^2 + (Z_i - Z)^2\right]^{1/2} = 0$$

$$\frac{\partial F}{\partial X} = \frac{X_i - X}{[\cdot]^{1/2}}, \quad \frac{\partial F}{\partial Y} = \frac{Y_i - Y}{[\cdot]^{1/2}}, \quad \frac{\partial F}{\partial Z} = \frac{Z_i - Z}{[\cdot]^{1/2}}$$

$$f_i = -F_i(\ell, x^\circ), \quad \sigma_0^2 = (0.4)^2 \quad w_{ii} = \sigma_0^2 / \sigma_i^2$$

chose $X^0 = 5560$, $Y^0 = 10500$, $Z^0 = 400$

after 5 iterations,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 5654.751 \\ 10510.180 \\ 483.976 \end{bmatrix}$$

$$V = \begin{bmatrix} -1.48 \\ 4.61 \\ -0.06 \\ 1.15 \end{bmatrix}$$

notes: (1) v_i's large compared to a priori σ's. That should worry you.
(2) v_3 much smaller than others consistent with smaller a priori σ

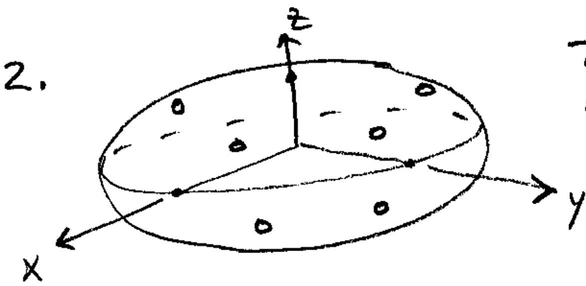
see listing for implementation in MATLAB.

```
% trilat3d.m  10-oct-07
% do ce506 homework problem, indirect obs.
%
% di = sqrt ( (xi - x)^2 + (yi - y)^2 + (zi - z)^2 )
%
% Fi = di - ( (xi - x)^2 + (yi - y)^2 + (zi - z)^2 )^(1/2) = 0
%
% dF/dx = - 0.5*(...)^(-1/2) * 2*(xi-x)*(-1) = (xi-x) / (...)^(1/2)
% dF/dx = (xi-x) / (...)^(1/2)
% dF/dy = (yi-y) / (...)^(1/2)
% dF/dz = (zi-z) / (...)^(1/2)
%
% f = -F
%
% n=4
% no=3;
% r=1
% u=3
% c=r+u  chk

n=4;
no=3;
r=1;
u=3;
c=r+u;

% define observations

d1=698.00;
d2=628.40;
d3=483.05;
d4=430.30;
l=[d1;d2;d3;d4];

% define the sigmas and weights

sigd1=0.40;
sigd2=0.40;
sigd3=0.05;
sigd4=0.40;
% define a priori sigma-naught squared
sig0=0.40;
W=eye(4);
W(1,1)=sig0^2/sigd1^2;
W(2,2)=sig0^2/sigd2^2;
W(3,3)=sig0^2/sigd3^2;
W(4,4)=sig0^2/sigd4^2;

% the points

x1= 5075.00;
y1=10550.00;
z1=  100.00;
x2= 5320.00;
y2=10115.00;
z2=  120.00;
x3= 5560.00;
y3=10190.00;
z3=  135.00;
x4= 5815.00;
y4=10640.00;
z4=  105.00;

x= 5560.0;
y=10500.0;
z=  400.0;

B=zeros(c,u);
f=zeros(c,1);
max_iter=10;
iter=1;
keep_going=1;
% convergence variables
phi=10;
last_phi=20;
threshold=1.0e-06;

while keep_going == 1
  % dF/dx = (xi-x) / (...)^(1/2)
  % dF/dy = (yi-y) / (...)^(1/2)
  % dF/dz = (zi-z) / (...)^(1/2)
  d10=sqrt((x1-x)^2 + (y1-y)^2 + (z1-z)^2);
  d20=sqrt((x2-x)^2 + (y2-y)^2 + (z2-z)^2);
```

```
  d30=sqrt((x3-x)^2 + (y3-y)^2 + (z3-z)^2);
  d40=sqrt((x4-x)^2 + (y4-y)^2 + (z4-z)^2);

  % make coefficients of the condition equations

  B(1,:)=[(x1-x)/d10  (y1-y)/d10 (z1-z)/d10];
  B(2,:)=[(x2-x)/d20  (y2-y)/d20 (z2-z)/d20];
  B(3,:)=[(x3-x)/d30  (y3-y)/d30 (z3-z)/d30];
  B(4,:)=[(x4-x)/d40  (y4-y)/d40 (z4-z)/d40];

  f(1)=-(d1 - sqrt((x1-x)^2 + (y1-y)^2 + (z1-z)^2));
  f(2)=-(d2 - sqrt((x2-x)^2 + (y2-y)^2 + (z2-z)^2));
  f(3)=-(d3 - sqrt((x3-x)^2 + (y3-y)^2 + (z3-z)^2));
  f(4)=-(d4 - sqrt((x4-x)^2 + (y4-y)^2 + (z4-z)^2));

  if iter == 1
    disp('iteration 1 B,f,W');
    B
    f
    W
    end

  % now solve and update and check convergence

  N=B'*W*B;
  t=B'*W*f;
  iter
  del=inv(N)*t
  x=x + del(1);
  y=y + del(2);
  z=z + del(3);
  v=f-B*del;
  phi=v'*W*v;

  if( abs(phi-last_phi)/last_phi < threshold )
    keep_going=0;
    disp('we have converged');
    end
  last_phi=phi;
  if iter > 10
    keep_going=0;
    disp('too many iterations');
    end
  iter=iter+1;
  end;

disp('final coordinates');
[x y z]
disp('residuals');
v=f - B*del
lhat=l + v
```

```
trilat3d
iteration 1 B,f,W
B =
       -0.847201961354498        0.0873404083870616         -0.52404245032237
       -0.450173684486989        -0.722153618864545        -0.525202631901488
                        0        -0.760121344351798        -0.649781149203957
        0.615486138525602         0.337913958406213         -0.71203298378452
f =
        -125.527293227005
         -95.272435527856
        -75.2203787118939
        -15.9933502826681
W =
     1     0     0     0
     0     1     0     0
     0     0    64     0
     0     0     0     1
iter =
     1
del =
         84.2012454507889
         11.2968278498699
         102.452344553807
iter =
     2
del =
         10.7735478429505
        -1.00763919322787
        -18.2131730293733
iter =
     3
del =
       -0.223312424702833
       -0.108807783498614
       -0.262881421569946
iter =
     4
del =
    -0.000229557855063712
   -3.86830203854812e-005
    5.36510426031894e-005
iter =
     5
del =
   -2.64788004749516e-007
   -4.19623422490706e-007
    4.49683936952399e-007
we have converged
final coordinates
ans =
         5654.75125104639          10510.1803417705           483.97634420359
residuals
v =
        -1.48370357950332
         4.61238671828909
      -0.0616568230310241
         1.15784808013185
lhat =
         696.516296420497
         633.012386718289
         482.988343176969
         431.457848080132
diary off
```

2.



$n = 6 \times 3 = 18$

$n_0 = 2 + 6 \times 2 = 14$

$r = 4$

$u = 2 \quad a, b$

$c = r + u = 6$, one equation per point

use general least squares = mixed model

$$\frac{x_i^2 + y_i^2}{a^2} + \frac{z_i^2}{b^2} = 1 \quad , \quad F_i = 1 - \frac{x_i^2 + y_i^2}{a^2} - \frac{z_i^2}{b^2} = 0$$

$$\left.\frac{\partial F_i}{\partial x_i} = -\frac{2 x_i}{a^2} \ , \ \frac{\partial F_i}{\partial y_i} = -\frac{2 y_i}{a^2} \ , \ \frac{\partial F}{\partial z_i} = -\frac{2 z_i}{b^2} \right\} \text{ for } A \text{ matrix}$$

$$\frac{\partial F_i}{\partial a} = \frac{2 (x_i^2 + y_i^2)}{a^3} \quad , \quad \frac{\partial F_i}{\partial b} = \frac{2 z_i^2}{b^3}$$

$$f_i = -F - A(l - l^0)^2$$

use ~WGS-84 parameters as initial approximations :

$a \approx 6378.0$ km , $b \approx 6378.0$ km

converged in 2-3 iterations, with

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 6378.141 \\ 6356.744 \end{pmatrix} \ , \quad V^T = [-.007 \ -.003 \ -.006 \ .002 \ .002 \ -.014 \ -.004 \ .004 \ .006 \ .005 \ .006 \ -.013 \ \underline{.000 \ .000 \ .000} \ -.000 \ .000 \ .004]$$

notes: as expected, small σ for $x_5 y_5 z_5$ yields small residuals. my solution code did not terminate after convergence.

```
% ellfit.m  2-oct-07
% solve homework 2 problem 2

n=6*3;
n0=2+6*2;
r=n-n0;
u=2;
c=r+u;

sig=0.02;
sig5=0.002;
sig0=0.02;
W=eye(n);
for i=1:n
  W(i,i)=sig0^2 / sig^2;
  end
idx=(5-1)*3 + 1;
W(idx,idx)=    sig0^2 / sig5^2;
W(idx+1,idx+1)=sig0^2 / sig5^2;
W(idx+2,idx+2)=sig0^2 / sig5^2;

x=[4600;1000;-3200;-2100;-2700;200];
y=[2200;1000;3400;-2500;-5000;-500];
z=[3818.7;-6198.5;4330.6;5460.75;-2886.95;-6334.05];
a=6378;
b=6378;
l=[x(1);y(1);z(1);x(2);y(2);z(2);x(3);y(3);z(3);x(4);y(4);z(4);x(5);y(5);z(5);x(6);y(6);z(6)];
l0=l;
for iter=1:10
  A=zeros(c,n);
  B=zeros(c,u);
  f=zeros(c,1);
  F=zeros(c,1);
  X=[l0(1);l0(4);l0(7);l0(10);l0(13);l0(16)];
  Y=[l0(2);l0(5);l0(8);l0(11);l0(14);l0(17)];
  Z=[l0(3);l0(6);l0(9);l0(12);l0(15);l0(18)];
  for i=1:6
    col=(i-1)*3+1;
    A(i,col)=-2*X(i)/(a^2);
    A(i,col+1)=-2*Y(i)/(a^2);
    A(i,col+2)=-2*Z(i)/(b^2);
    B(i,1)=2*(X(i)^2 + Y(i)^2)/(a^3);
    B(i,2)=2*(Z(i)^2)/(b^3);
    F(i)=1-(X(i)^2 + Y(i)^2)/(a^2)-(Z(i)^2)/(b^2);
    end
  f=-F - A*(l-l0);
  Q=inv(W);
  Qe=A*Q*A';
  We=inv(Qe);
  del=inv(B'*We*B)*B'*We*f;
  del
  a=a+del(1);
  b=b+del(2);
  k=We*(f-B*del);
  v=Q*A'*k;
  l0=l+v;
  if(iter == 1)
    A
    B
    f
    end
  end
a
b
v
```

```
ellfit
del =
        0.141195453773856
        -21.3630940379217
A =
  Columns 1 through 4
      -0.0002261615633099     -0.000108164225940908     -0.000187748513454794
     0
                        0                         0                         0
-4.91655572458674e-005
                        0                         0                         0
     0
                        0                         0                         0
     0
                        0                         0                         0
     0
                        0                         0                         0
     0
  Columns 5 through 8
                        0                         0                         0
     0
    -4.91655572458674e-005      0.000304752706588509                         0
     0
                        0                         0      0.000157329783186776
-0.000167162894635949
                        0                         0                         0
     0
                        0                         0                         0
     0
                        0                         0                         0
     0
  Columns 9 through 12
                        0                         0                         0
     0
                        0                         0                         0
     0
     -0.000212916362208953                         0                         0
     0
                        0      0.000103247670216322      0.000122913893114669
-0.00026848081673037
                        0                         0                         0
     0
                        0                         0                         0
     0
  Columns 13 through 16
                        0                         0                         0
     0
                        0                         0                         0
     0
                        0                         0                         0
     0
                        0                         0                         0
     0
      0.000132747004563842      0.000245827786229337      0.000141938505490957
     0
                        0                         0                         0
-9.83311144917348e-006
  Columns 17 through 18
                        0                         0
                        0                         0
                        0                         0
                        0                         0
                        0                         0
     2.45827786229337e-005      0.000311417097873186
B =
      0.000200424033927964      0.000112410669227002
     1.54172333790741e-005      0.000296175862619767
      0.000168047843831908      0.000144568140197882
     8.21738539104651e-005      0.000229869335208587
      0.000248911232905152     6.42473139584694e-005
     2.23549883996575e-006        0.0003092711616155
f =
      -0.00237013163881294
      -0.00632961685969546
      -0.00306762692899826
       -0.0048942698993415
      -0.00133739405191152
      -0.00660525980752069
del =
     1.98155609665469e-006
         0.106744093385646
del =
     6.70410926604951e-009
```

```
     2.66645939635962e-006
del =
     2.16864070104461e-013
    -1.22098511856628e-013
del =
     2.29728833612414e-013
    -2.00928249732835e-013
del =
     2.32591127347775e-013
    -2.19027487119439e-013
del =
     2.35017788650232e-013
    -2.30873913736884e-013
del =
     2.35017788650232e-013
    -2.30873913736884e-013
del =
     2.35017788650232e-013
    -2.30873913736884e-013
del =
     2.35017788650232e-013
    -2.30873913736884e-013
a =
          6378.14119744203
b =
          6356.74365272192
v =
      -0.00690299682795318
      -0.00330143326554282
      -0.00576918202102412
       0.00226724710356425
       0.00226724710356425
       -0.0141483022932827
      -0.00467754575515591
       0.00496989236485316
       0.00637286919263785
       0.00506295861504281
       0.00602733168457478
       -0.0132542827964667
    -2.38478079730187e-005
    -4.41626073574421e-005
    -2.56710023586623e-005
     -0.000140126460143534
      0.000350316150358834
       0.00446776686736413
diary off
```