Data 1, HW7 Constraint Problem Solution

(a) $n = 16$, $n_0 = 3$, $r = 13$, $\mu = 3$ (m_1, b_1, b_2), $c = 16$

7 points on first line
9 points on second line
two lines must be perpendicular
part (a) by substitution: $m_2 = -1/m_1$
It is a nonlinear problem
2 approaches (a) & (b) must yield same results

(indirect observations)

(1) $Y_i = m_1 x_i + b_1$, $F_1 = Y_i - m_1 x_i - b_1 = 0$

(2) $Y_i = -1/m_1 x_i + b_2$, $F_2 = Y_i + m_1^{-1} x_i - b_2 = 0$

$$\frac{\partial F_1}{\partial p} = \begin{bmatrix} -x_i & -1 & 0 \end{bmatrix}, \quad f_1 = -F_1$$

$$\frac{\partial F_2}{\partial p} = \begin{bmatrix} -x_i/(m_1)^2 & 0 & -1 \end{bmatrix}, \quad f_2 = -F_2$$

See computer listing for results and code implementation

(b) $n = 16$, $n_0 = 3$, $r = 13$, $\mu = 4$ (m_1, b_1, m_2, b_2), $c = 16$, $s = 1$ (# constraints)

use formal constraint approach

(1) $Y_i = m_1 x_i + b_1$, $F_1 = Y_i - m_1 x_i - b_1$

(2) $Y_i = m_2 x_i + b_2$, $F_2 = Y_i - m_2 x_i - b_2$

(constr) $m_1 = -1/m_2$, $F_c = m_1 + \frac{1}{m_2} = 0$

$c + s = r + \mu$
$16 + 1 = 13 + 4$ ✓

$$\frac{\partial F_1}{\partial p} = \begin{bmatrix} -x_i & -1 & 0 & 0 \end{bmatrix}, \quad f_1 = -F_1$$

$$\frac{\partial F_2}{\partial p} = \begin{bmatrix} 0 & 0 & -x_i & -1 \end{bmatrix}, \quad f_2 = -F_2$$

$$\frac{\partial F_c}{\partial p} = \begin{bmatrix} 1 & 0 & -1/m_2^2 & 0 \end{bmatrix}, \quad g = -F_c$$

$$\begin{bmatrix} N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k_c \end{bmatrix} = \begin{bmatrix} t \\ g \end{bmatrix}$$

$V = f - B\Delta$

See computer listing for results and code implementation

```
hw7a
iter =
     1
del =
    0.0298
   -0.0994
    0.0429
iter =
     2
del =
  1.0e-003 *
    0.1276
   -0.3828
    0.3940
iter =
     3
del =
  1.0e-006 *
    0.3022
   -0.9067
   -0.5254
iter =
     4
del =
  1.0e-008 *
    0.0713
   -0.2140
   -0.1266
iter =
     5
del =
  1.0e-011 *
    0.1684
   -0.5051
   -0.2989
m1 =
    2.5999
b1 =
    2.9002
b2 =
   16.5933
m2 =
   -0.3846
v =
   -0.0998
    0.3001
   -0.1999
    0.2000
   -0.3001
   -0.0001
    0.0998
    0.0548
   -0.1145
    0.1163
    0.0470
   -0.2222
   -0.0915
    0.1393
   -0.0300
    0.1008
diary off
```

```
% hw7a.m  10-dec-09
% two line fit, constrain slopes by substitution

x1=[0;1;2;3;4;5;6];
y1=[3.0;5.2;8.3;10.5;13.6;15.9;18.4];
x2=[4;6;8;10;12;14;16;18;20];
y2=[15.0;14.4;13.4;12.7;12.2;11.3;10.3;9.7;8.8];

W=eye(16);
B=zeros(16,3);
f=zeros(16,1);
m1=2.57;
b1=3;
b2=16.55;

for iter=1:5
  iter
  for i=1:7
    B(i,:)=[-x1(i) -1 0];
    f(i)=-(y1(i) - m1*x1(i) - b1);
    end
  for i=1:9
    ii=i+7;
    B(ii,:)=[-x2(i)/m1^2 0 -1];
    f(ii)=-(y2(i) + x2(i)/m1 - b2);
    end

  N=B'*W*B;
  t=B'*W*f;
  del=inv(N)*t
  m1=m1 + del(1);
  b1=b1 + del(2);
  b2=b2 + del(3);
  end

m1
b1
b2
m2=-1/m1

v=f-B*del
```

```
hw7b
iter =
     1
del =
    0.0299
   -0.0996
    0.0024
    0.0432
iter =
     2
del =
  1.0e-003 *
    0.0537
   -0.1612
   -0.0066
    0.0793
iter =
     3
del =
  1.0e-006 *
   -0.1053
    0.3158
   -0.0157
    0.1882
iter =
     4
del =
  1.0e-009 *
   -0.2503
    0.7508
   -0.0370
    0.4443
iter =
     5
del =
  1.0e-011 *
   -0.0591
    0.1772
   -0.0087
    0.1048
m1 =
    2.5999
b1 =
    2.9002
m2 =
   -0.3846
b2 =
   16.5933
v =
   -0.0998
    0.3001
   -0.1999
    0.2000
   -0.3001
   -0.0001
    0.0998
    0.0548
   -0.1145
    0.1163
    0.0470
   -0.2222
   -0.0915
    0.1393
   -0.0300
    0.1008
diary off
```

```
% hw7b.m  10-dec-09
% two line fit, constrain slopes by substitution
% same problem by formal constraint method
% now 4 parameters plus one constraint

x1=[0;1;2;3;4;5;6];
y1=[3.0;5.2;8.3;10.5;13.6;15.9;18.4];
x2=[4;6;8;10;12;14;16;18;20];
y2=[15.0;14.4;13.4;12.7;12.2;11.3;10.3;9.7;8.8];

W=eye(16);
B=zeros(16,4);
f=zeros(16,1);
C=zeros(1,4);

m1=2.57;
b1=3;
m2=-0.387;
b2=16.55;

for iter=1:5
  iter
  for i=1:7
    B(i,:)=[-x1(i) -1 0 0];
    f(i)=-(y1(i) - m1*x1(i) - b1);
    end
  for i=1:9
    ii=i+7;
    B(ii,:)=[0 0 -x2(i) -1];
    f(ii)=-(y2(i) - m2*x2(i) - b2);
    end
  C=[1 0 -1/m2^2 0];
  Fc=m1 + 1/m2;
  g=-Fc;
  N=B'*W*B;
  t=B'*W*f;
  M=[-N C';C 0];
  s=[-t;g];
  xdel=inv(M)*s;
  del=xdel(1:4)
  m1=m1 + del(1);
  b1=b1 + del(2);
  m2=m2 + del(3);
  b2=b2 + del(4);
  end

m1
b1
m2
b2

v=f-B*del
```