Show justification for LS by maximum likelihood 3-1

example : n obs. of single quantity

all obs same normal distr., independent

iid : independent, identically distr.

$X_1, X_2, \ldots, X_n$ $\quad f(x) = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left[-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right]$

If RVs are independent → joint distr. just product of indiv. distr.

$f(x_1 \ldots x_n) = \left[f(x)\right]^n$

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$$= \frac{1}{(2\pi)^{n/2}\sigma^n} \exp\left[-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2 - \cdots - \frac{1}{2}\left(\frac{x_n-\mu}{\sigma}\right)^2\right]$$ 3-2

$$= \frac{1}{(2\pi)^{n/2}\sigma^n} \exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

likelihood function

maximize $f = ke^A$, maximizing A

$\max -\frac{1}{2\sigma^2}\sum(x_i-\mu)^2$

$\min \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$, $\quad \boxed{\min \sum(x_i-\mu)^2} \quad \sum v^2$

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3-3

$$\sum_{i=1}^{n}(x_i-\mu)^2 , \quad \frac{d}{d\mu}\sum(x_i-\mu)^2 = \sum 2(x_i-\mu)(-1) = 0$$

$$\sum(x_i-\mu) = 0$$

$$\left.\sum x_i - \underbrace{\sum \mu}_{n\mu} = 0 \right\} \quad \sum x_i - n\mu = 0$$

$$\sum x_i = n\mu , \quad \boxed{\mu = \frac{\sum_{i=1}^{n} x_i}{n}}$$

sample mean

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3-4

$\left.\begin{array}{l}\text{normally distr. obs} \\ + \\ \text{Max. Likelihood}\end{array}\right\}$ <u>LS</u>

we did with W_i equal

if $w_1 \neq w_2 \neq 3$ ... $\Longrightarrow \Phi = \sum w_i v_i^2 \to \min$

Corollary: if data not normally distr.

$\Rightarrow$ LS wrong technique

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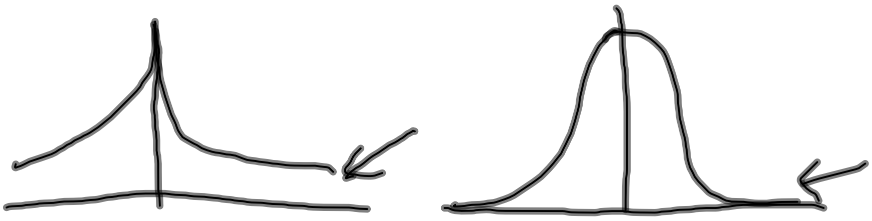
3–5



exponential normal

$\sum |v| \to \min$ $\sum v^2 \to \min$

L_1 L_2

maximum likelihood : Remote Sensing

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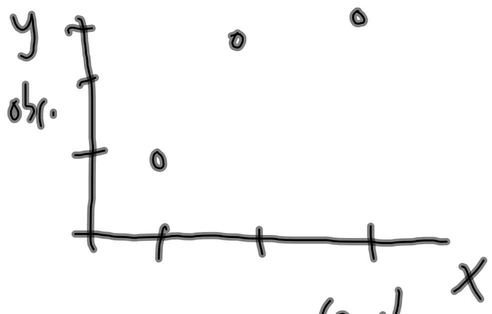
3–6



const.

$n = 3$ equal prec.

$n_0 = 2$

$r = 1$

x	y
1	1.0
2	2.2
3	3.0

indirect observations : choose n_0 unknowns, parameters
write n condition eqns.
1 obs. in terms of unk.

slope, intercept m, b

$$\left.\begin{aligned} \hat{y}_1 &= y_1 + v_1 = m x_1 + b \\ \hat{y}_2 &= y_2 + v_2 = m x_2 + b \\ \hat{y}_3 &= y_3 + v_3 = m x_3 + b \end{aligned}\right\} \text{n cond. eqns}$$

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3-7

$$\left.\begin{aligned} v_1 &= mx_1 + b - y_1 \\ v_2 &= mx_2 + b - y_2 \\ v_3 &= mx_3 + b - y_3 \end{aligned}\right\} \quad \begin{aligned} v_1 &= m + b - 1.0 \\ v_2 &= 2m + b - 2.2 \\ v_3 &= 3m + b - 3.0 \end{aligned}$$

$$\Phi = v_1^2 + v_2^2 + v_3^2 \rightarrow \min$$

$$= (m+b-1.0)^2 + (2m+b-2.2)^2 + (3m+b-3.0)^2$$

$$\frac{\partial \Phi}{\partial m} = \cancel{2}(m+b-1.0) + \cancel{2}(2m+b-2.2)(2) + \cancel{2}(3m+b-3.0)(3) = 0$$

$$\frac{\partial \Phi}{\partial b} = \cancel{2}(m+b-1.0) + \cancel{2}(2m+b-2.2) + \cancel{2}(3m+b-3.0) = 0$$

$$m + 4m + 9m + b + 2b + 3b = 1.0 + 4.4 + 9.0$$

$$m + 2m + 3m + b + b + b = 1.0 + 2.2 + 3.0$$

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3-8

$$\left.\begin{aligned} 14m + 6b &= 14.4 \\ 6m + 3b &= 6.2 \end{aligned}\right\}$$

unique solution

<u>NORMAL EQUATIONS</u>

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 14.4 \\ 6.2 \end{bmatrix}$$

$$Nx = t$$

$$x = N^{-1} t$$

$$x = \begin{bmatrix} 1.0000 \\ 0.0667 \end{bmatrix} \begin{matrix} \text{slope} \\ \text{intercept} \end{matrix}$$

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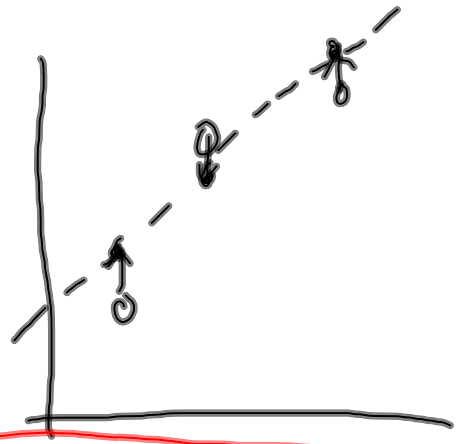
$y_i + v_i = m x_i + b$, $v_i = m x_i + b - y_i$ 3-9

$v_1 = m x_1 + b - y_1 = 0.0667$

$v_2 = m x_2 + b - y_2 = -1.1333$

$v_3 = m x_3 + b - y_3 = 0.0667$



$\hat{y}_1 = y_1 + v_1 = 1.0667$

$\hat{y}_2 = y_2 + v_2 = 2.0667$

$\hat{y}_3 = y_3 + v_3 = 3.0667$

ALWAYS COMPUTE RESIDUALS AFTER LS Adj.

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