5-1

Matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = A$ $(2,3)$

vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \vec{b}$ $(3,1)$

scalar mult: kA, $k\vec{b}$

matrix mult: $A \times B = C$

(m,n) (n,p) (m,p)

same — result

Aug 31-9:12 AM

5-2

$m \times n \times p = \square$

$V^T W V = \underline{\Phi}$

$(1,n)$ (n,n) $(n,1)$ $(1,1)$

addition

$\boxed{A} + \boxed{B} = \boxed{C}$

transpose: interchange rows + cols

$A : A^T$

(m,n) (n,m)

Aug 31-9:13 AM

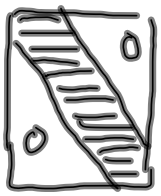
Special matrices 5-3

square : $m = n$

diagonal : square $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

identity : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

band :



symmetric : square $a_{ij} = a_{ji}$

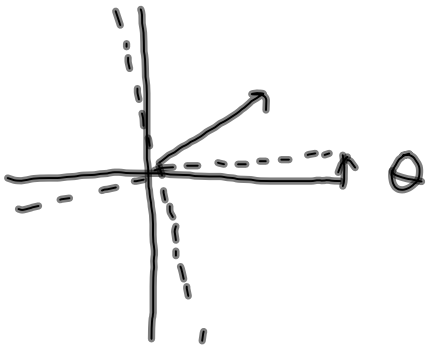
$A = A^T$

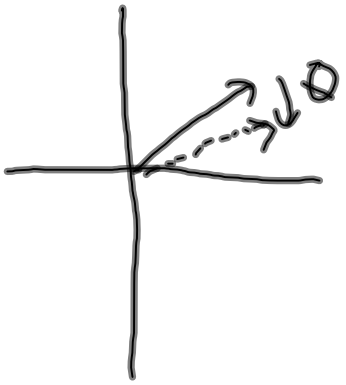
Aug 31-9:13 AM

orthogonal — reflection $A^TA = I$, $A^T = A^{-1}$ 5-4

rotations $AA^T = I$

Rx





inverse

scalar $a \times \frac{1}{a} = 1$

matrices $A \times A^{-1} = I$

(if A invertible)

Aug 31-9:13 AM

$A^{-1} = \frac{C^T}{\det(A)}$ $C_{ij} = (-1)^{i+j} \det(M_{ij})$ 5-5

minor

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $\det(A) = |A| = a_{11}a_{22} - a_{21}a_{12}$

$A^{-1} = \frac{\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}}{a_{11}a_{22} - a_{21}a_{12}}$

$Ax = b$

$A^{-1}Ax = A^{-1}b$

$x = A^{-1}b$

Aug 31-9:13 AM

Identities $(AB)^T = B^T A^T$ 5-6

$(ABC)^T = C^T B^T A^T$

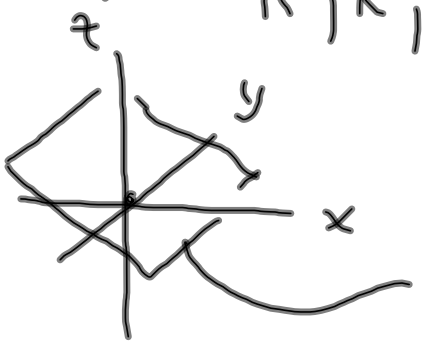
assume inverses exist $(AB)^{-1} = B^{-1}A^{-1}$

$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Vector space : collections of vectors + rules for addition + scalar must obey 8 rules

Aug 31-9:13 AM

usual VS: $R^1, R^2, R^3, \dots, R^n$ 5-7



subspace order 3 dim 2

Matrix 4 fund. subspace

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

col space: all v's reachable by LC of col vectors

$$c_1\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} + c_2\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} + c_3\begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix}$$

Aug 31-9:13 AM

row space: all v's reachable by LC of rows 5-8

$$c_1\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} + c_2\begin{pmatrix} a_{21} \\ a_{22} \\ a_{23} \end{pmatrix}$$

null space: all x, $Ax = 0$

null space of A^T: all x, $A^T x = 0$

Set of vectors v_1, v_2, v_3 is dependent (L.D.) if

$$c_1v_1 + c_2v_2 + c_3v_3 = 0$$

Aug 31-9:13 AM

independent if cannot find $c_1, c_2, c_3 \neq 5\text{-}9$ make expression $= 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \begin{matrix} v_1 \\ v_2 \end{matrix} \qquad 2v_1 - v_2 = 0$$

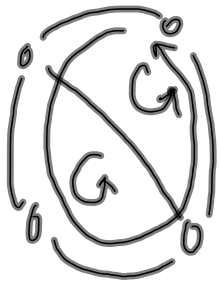
$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 3 & 3 \end{bmatrix} \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \qquad v_1 + v_2 - v_3 = 0$$

rank of matrix : # of L.I rows, or # of L.I. cols,

Aug 31-9:13 AM

I/O ▯ : full column rank 5-10

O/O ▭ : full row rank



Span : VS spanned by v_1, v_2, v_3 if all v_i in VS are of form $c_1v_1 + c_2v_2 + c_3v_3$

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5-11

basis of VS, set vectors

(1) span

(2) are L.I.

dimension : # vectors in basis

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Aug 31-9:13 AM