6-1

basis set of vectors
(1) span the VS
(2) L.I.

dimension of VS # of vectors in basis

R^3 $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$

dot product: $\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

inner product $a_x b_x + a_y b_y + a_z b_z$ $\vec{a}\cdot\vec{b} = 0$

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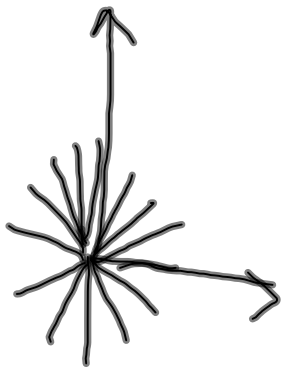
6-2

$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$ a vector

eigenvalues & eigenvectors

$Ax = \lambda x$

λ eigenvalue, x eigenvector



condition # $= \frac{\lambda_{max}}{\lambda_{min}}$ $Ax = t$ $x = A^{-1}t$

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differentiate with respect to vector 6-3

Vector

$\frac{d}{d\vec{x}} A\vec{x} = A$

$\frac{d}{d\vec{x}} x^T A x = 2x^T A$

A: symmetric

$\frac{d}{d\vec{x}} u^T v = u^T \frac{dv}{d\vec{x}} + v^T \frac{du}{d\vec{x}}$

$x^T A = u^T \quad x = v$

$u = Ax \quad v^T = x^T$

Scalar

$\frac{d}{dx} ax = a$

$\frac{d}{dx} ax^2 = 2ax$

$2 \times a$

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Constrained Minimization 6-4

$\Phi = \sum_{i=1}^{h} v_i^2 \quad , \quad \sum w_i v_i^2$

So far, solved by substitution

what if you do not have parameters?

or you don't have n_0 of them?

the we have min. problem in a different

Way

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6-5

Minimize x^2+y^2 ←

Subj. to $y = 0.2x+5$ ←

1. substitution

$$y^2 = 0.04x^2 + 25 + \underbrace{2 \cdot 5 \cdot 0.2x}_{2x}$$

$$\Phi = x + 0.04x^2 + 2x + 25$$

$$1.04x^2 + 2x + 25$$

$$\frac{d\Phi}{dx} = 2.08x + 2 = 0, \quad 2.08x = -2$$

$$x = -2/2.08 = -0.9615$$

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6-6

$y = 4.8077$

2. constrained min: Lagrange Mult.

$$\Phi' = \underbrace{x^2+y^2}_{\text{original } \Phi} + \lambda \underbrace{(y-0.2x-5)}_{\text{constraint eqn } (=0)}$$

L.M.

$$\frac{\partial \Phi'}{\partial x} = 2x - 0.2\lambda = 0$$

$$\frac{\partial \Phi'}{\partial y} = 2y + \lambda = 0$$

$$\frac{\partial \Phi'}{\partial \lambda} = y - 0.2x - 5 = 0$$

} 3 eqn 3 unk

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$2x - 0.2\lambda = 0$

$2y + \lambda = 0$

$-0.2x + y = 5$

$$\begin{bmatrix} 2 & 0 & -0.2 \\ 0 & 2 & 1 \\ -0.2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

6-7

$$\begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 0.9615 \\ 4.8077 \\ -9.6154 \end{bmatrix}$$

$Ax = b$

$x = A^{-1}b$

$\phi' = v_1^2 + v_2^2 + \dots + v_n^2 + \lambda_1(\text{—}) + \lambda_2(\text{—}) \dots$

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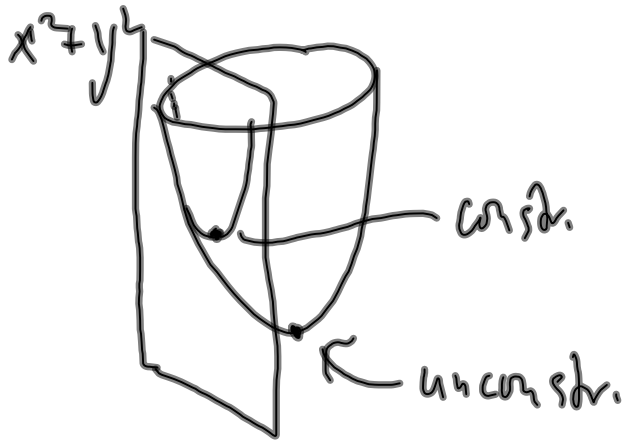
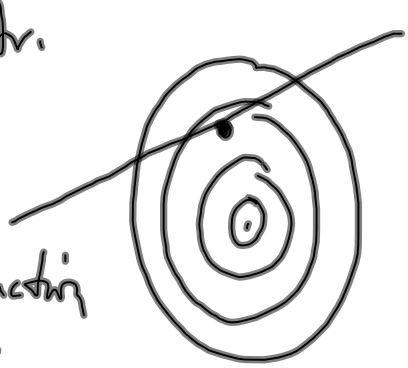


6-8



Constraint line tangent to contour line of obj. function

λ : proportionality factor between 2 equations

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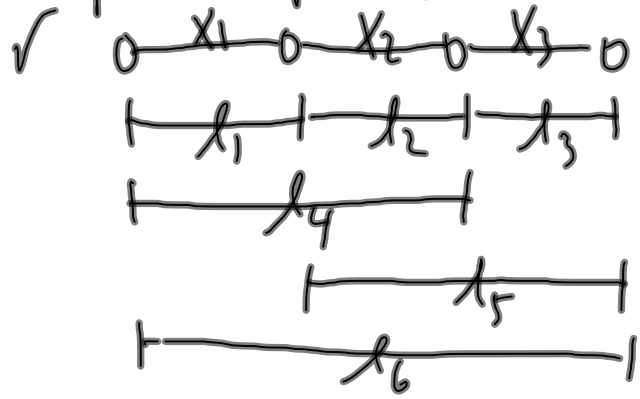
Example LS problems

6-9

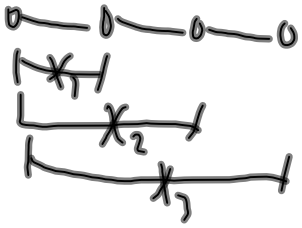


$n = 6$

$n_0 = 3$

$r = 3$

I/O:



$\hat{l}_1 = x_1$ $\quad \hat{l}_4 = x_1 + x_2$

$\hat{l}_2 = x_2$ $\quad \hat{l}_5 = x_2 + x_3$

$\hat{l}_3 = x_3$ $\quad \hat{l}_6 = x_1 + x_2 + x_3$

$v_1 = x_1 - l_1$

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I/O

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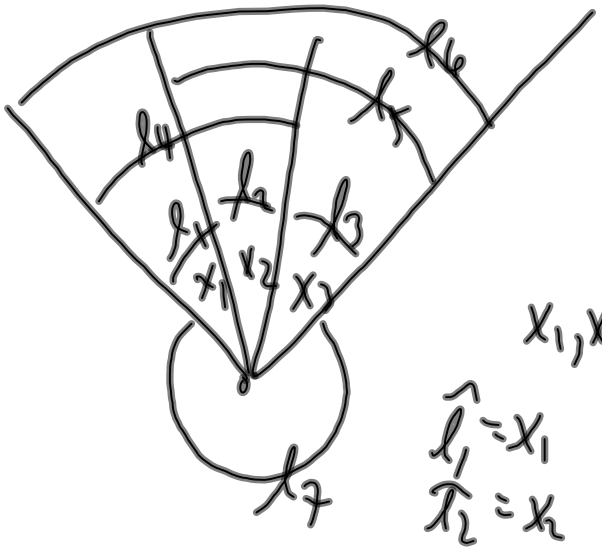

$n = 7$

$n_0 = 3$

$r = 4$

x_1, x_2, x_3

$\hat{l}_1 = x_1$ $\quad \hat{l}_5 = x_2 + x_3$

$\hat{l}_2 = x_2$ $\quad \hat{l}_6 = x_1 + x_2 + x_3$

$\hat{l}_3 = x_3$ $\quad \hat{l}_7 = 360^0 - x_1 - x_2 - x_3$

$\hat{l}_4 = x_1 + x_2$

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