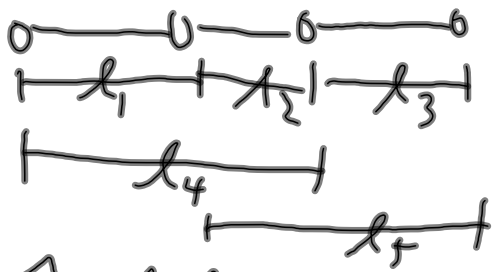
7-1

Observations Only n, n_0

$r = c$ = number of condition eq'ns

(Scalar approach)



l_1	20.0	equal wts.
l_2	18.0	
l_3	21.0	$n = 5$
l_4	37.6	$n_0 = 3$
l_5	39.2	$r = 2$

- $\hat{l}_4 = \hat{l}_1 + \hat{l}_2$
- $\hat{l}_5 = \hat{l}_2 + \hat{l}_3$

$\hat{l}_4 + \hat{l}_5 = \hat{l}_1 + 2\hat{l}_2 + \hat{l}_3$ x

$l_4 + v_4 = l_1 + v_1 + l_2 + v_2$

$l_5 + v_5 = l_2 + v_2 + l_3 + v_3$

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Constrained Minimization

$v_4 - v_1 - v_2 = 0.4$

$v_5 - v_2 - v_3 = -0.2$

Substitution, use each eq to express 1 v's in terms of the others,

$v_4 = v_1 + v_2 + 0.4$

$v_5 = v_2 + v_3 - 0.2$

n_0 | r

$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2$

$\Phi = v_1^2 + v_2^2 + v_3^2 + (v_1 + v_2 + 0.4)^2 + (v_2 + v_3 - 0.2)^2$

$\partial\Phi/\partial v_1 = \frac{1}{2} v_1 + \frac{1}{2}(v_1 + v_2 + 0.4) = 0$ min

$\partial\Phi/\partial v_2 = \frac{1}{2} v_2 + \frac{1}{2}(v_1 + v_2 + 0.4) + \frac{1}{2}(v_2 + v_3 - 0.2) = 0$

$\partial\Phi/\partial v_3 = \frac{1}{2} v_3 + \frac{1}{2}(v_2 + v_3 - 0.2) = 0$

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$$\begin{aligned} 2V_1 + V_2 &= -0.4 \\ V_1 + 3V_2 + V_3 &= -0.2 \\ V_2 + 2V_3 &= 0.2 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0.4 \\ -0.2 \\ 0.2 \end{bmatrix} \quad 7\text{-}3$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -0.175 \\ -0.050 \\ 0.125 \end{bmatrix}$$

$$V_4 = V_1 + V_2 + 0.4$$

$$V_5 = V_2 + V_3 - 0.2$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} -.175 \\ -.050 \\ .125 \\ .175 \\ -.125 \end{bmatrix}$$

$$l + V = \hat{l} = \begin{bmatrix} 19.825 \\ 17.95 \\ 21.125 \\ 37.775 \\ 39.075 \end{bmatrix}$$

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Lagrange Mult:

$$\phi' = V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 + 2\lambda_1(V_4 - V_1 - V_2 - 0.4) + 2\lambda_2(V_5 - V_2 - V_3 + 0.2) \quad 7\text{-}4$$

$$\partial\phi'/\partial V_1 = 2V_1 - 2\lambda_1 = 0$$

$$\partial\phi'/\partial V_2 = 2V_2 - 2\lambda_1 - 2\lambda_2 = 0$$

$$\partial\phi'/\partial V_3 = 2V_3 - 2\lambda_2 = 0$$

$$\partial\phi'/\partial V_4 = 2V_4 + 2\lambda_1 = 0$$

$$\partial\phi'/\partial V_5 = 2V_5 + 2\lambda_2 = 0$$

$$\partial\phi'/\partial\lambda_1 = 2(V_4 - V_1 - V_2 - 0.4) = 0$$

$$\partial\phi'/\partial\lambda_2 = 2(V_5 - V_2 - V_3 + 0.2) = 0$$

$$\begin{aligned} V_1 &= \lambda_1 \\ V_2 &= \lambda_1 + \lambda_2 \\ V_3 &= \lambda_2 \\ V_4 &= -\lambda_1 \\ V_5 &= -\lambda_2 \end{aligned}$$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.4 \\ -0.2 \end{bmatrix}$$

solve matrix eqns: $\begin{bmatrix} -.175 \\ -.050 \\ .125 \\ .175 \\ .125 \\ -.175 \\ .125 \end{bmatrix}$ — same as before; — λ_1' s

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$-\lambda_1 - \lambda_1 - \lambda_1 - \lambda_2 = 0.4$ $\quad -3\lambda_1 - \lambda_2 = 0.4$

$-\lambda_2 - \lambda_1 - \lambda_2 - \lambda_2 = -0.2$ $\quad -\lambda_1 - 3\lambda_2 = -0.2$

$$\begin{bmatrix} -3 & -1 \\ -1 & -3 \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix}$$

Language Mult w/ eliminators

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{bmatrix} .175 \\ .125 \end{bmatrix}$$

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O/O	substitution	$n_0 \times n_0$
	LM-full	$n+r \times n+r$
	LM-elim	$r \times r$
I/O		$n_0 \times n_0$

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$n = 100$
$n_0 = 98$
$r = 2$

O/O sub	98×98
O/O LM-f	102×102
O/O LM-el	2×2
I/O	98×98

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