8-1

$n = 100$ tech. size of NE

$n_0 = 2$ o/o subs. 2×2 ←

$r = 98$ o/o LM full 198×198

o/o LM elim 98×98

I/O 2×2 ←

I/O $l_i + v_i = q_0 + q_1 x_i + q_2 x_i^2$ $C = n$

$v_i - a_0 - q_1 x_i - q_2 x_i^2 = -l_i$ $M = n_0$

$$v_i - \begin{bmatrix} -1 & -x_i & -x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = -l_i$$

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$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} -1 & -x_1 & -x_1^2 \\ -1 & -x_2 & -x_2^2 \\ -1 & -x_3 & -x_3^2 \\ -1 & -x_4 & -x_4^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix}$$

$v + B \Delta = \underbrace{d - l}_{f}$

$v + B\delta = f$ other fields

$v = f - B\delta$

$A x = l + v$

$\Phi = \sum v_i^2$ or $\sum w_i v_i^2$

$A x = l - v$ ✓

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$\Phi = V^T W V$ $\quad [v_1\ v_2\ v_3]\begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} =$ 8-3

$v = f - B\delta$

$w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2$

$\Phi = (f - B\delta)^T W (f - B\delta)$

$(f^T - \delta^T B^T) W (f - B\delta)$

$(f^T W - \delta^T B^T W)(f - B\delta)$

$\Phi = f^T W f + \delta^T B^T W B\delta - f^T W B\delta - \delta^T B^T W f$

$f^T W'' B\delta$

$-2 f^T W B\delta$

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$\Phi = f^T W f + \delta^T B^T W B\delta - 2 f^T W B\delta$ $\quad \min$ $\quad$ 8-4

$\frac{d}{dx} Ax = A \qquad \frac{d}{dx} x^T A x = 2x^T A$

A : sym.

$\frac{d\Phi}{d\delta} = 2\delta^T B^T W B - 2 f^T W B = 0$ (row vector)

$B^T W B\delta - B^T W f = 0$ (col vector)

$\boxed{B^T W B\delta = B^T W f}$ normal equations

$B^T W B = N \qquad N\delta = t, \quad \delta = N^{-1} t$

$B^T W f = t$

$v = f - B\delta, \quad l + v = \hat{l}$

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Example level network

$n = 5$
$n_0 = 3$
$r = 2$

I/O

relative parameters

absolute

A, B, C

define datum

write $h = 5$ equations

$\ell = \begin{bmatrix} 10 \\ 8 \\ 3 \\ 4 \\ 1 \end{bmatrix}$

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$\hat{\ell}_1 = C - 100$
$\hat{\ell}_2 = A - 100$
$\hat{\ell}_3 = C - A$
$\hat{\ell}_4 = B - A$
$\hat{\ell}_5 = B - C$

$V_1 - C = -100 - 10$ 8-6
$V_2 - A = -100 - 8$
$V_3 - C + A = -3$
$V_4 - B + A = -4$
$V_5 - B + \ell = -1$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 100 \\ -100 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 10 \\ 8 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$V + B\delta = d - \ell$ (d-ℓ = f)

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$$\Delta = (B^T W B)^{-1} B^T W f \Longleftarrow \quad 8\text{-}7$$

$$\Delta = \begin{bmatrix} 107.625 \\ 111.500 \\ 110.375 \end{bmatrix}$$

$$\hat{\ell} = \ell + v$$

$$\hat{\ell} = \begin{bmatrix} 10.375 \\ 7.625 \\ 2.750 \\ 3.875 \\ 1.125 \end{bmatrix}$$

$$v = f - B\Delta$$

$$v = \begin{bmatrix} .375 \\ -.375 \\ -.25 \\ -.125 \\ .125 \end{bmatrix}$$

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