$w_i \sim \frac{1}{\sigma_i^2}$, $w_i = \frac{\sigma_0^2}{\sigma_i^2}$ σ_0^2 : prop. constant   9-1

value arbitrary

(1) σ^2 of commonly occurring obs

(2) 1.0

$$W = \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & & 0 \\ & \sigma_0^2/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_0^2/\sigma_n^2 \end{bmatrix}$$

$$W = \sigma_0^2 \begin{bmatrix} 1/\sigma_1^2 & & & \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ & & & 1/\sigma_n^2 \end{bmatrix}, \quad W = \sigma_0^2 \Sigma^{-1}$$

$Q = \frac{1}{\sigma_0^2}\Sigma$, $\Sigma = \sigma_0^2 Q$

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W: original observations: diagonal   9-2

derivied observation: usually <u>not</u> diagonal

How to get cond. eqns O/O from cond eqns I/O, if not obvious

$$\begin{array}{r} n \\ -\,n_0 \\ \hline r \end{array}$$

I/O $r + n_0 = n$

r	n_0	n
base # eqn	1 per parameter	total cond. eqn

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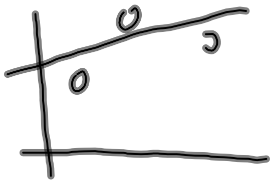
Example of : parameter eqns → no parameters 9-3



$y = mx + b$ $n = 3$

$\frac{n_0 = 2}{r = 1}$

$y_1 = mx_1 + b$

$y_2 = mx_2 + b$

$y_3 = mx_3 + b$

$b = y_1 - mx_1$

$y_2 = mx_2 + y_1 - mx_1 = m(x_2 - x_1) + y_1$

$y_3 = mx_3 + y_1 - mx_1 = m(x_3 - x_1) + y_1$

$m = \frac{y_2 - y_1}{x_2 - x_1}$, $y_3 = \frac{x_2 - y_1}{x_2 - x_1} \cdot x_3 - x_1 + y_1$

$$\boxed{\frac{y_3 - y_1}{x_3 - x_1} = \frac{x_2 - y_1}{x_2 - x_1}}$$

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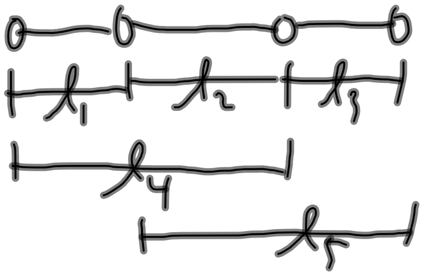
Derive matrix approach to LS using 0/0 method 9-4



$n = 5$

$\frac{n_0 = 3}{r = 2}$

$\hat{l}_4 = \hat{l}_1 + \hat{l}_2$

$\hat{l}_5 = \hat{l}_2 + \hat{l}_3$

$\hat{l}_4 - \hat{l}_1 - \hat{l}_2 = 0$

$\hat{l}_5 - \hat{l}_2 - \hat{l}_3 = 0$

$$\underset{A}{\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \hline -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{array}} \begin{bmatrix} l_1 + v_1 \\ l_2 + v_2 \\ \vdots \\ l_5 + v_5 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$Av = \underbrace{d - Al}_{f}$$

$$\underset{r,n}{A}\,\underset{n,1}{v} = \underset{r,1}{f}$$

$\hat{l} = d$

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9-5

$\Phi = v^T W v$, use LM

$\Phi' = v^T W v - 2k_1(\cdots) - 2k_2(\cdots) \cdots$

$\Phi' = v^T W v - \underbrace{2k^T(Av - f)}_{2(Av-f)^T k}$ $\quad \underline{\underline{\min}}$ v,k

$\frac{\partial \Phi'}{\partial v} = \cancel{2} v^T W - \cancel{2} k^T A = 0$ (row vector)

$\frac{\partial \Phi'}{\partial k} = -\cancel{2}(Av - f)^T = 0$ " "

$Wv - A^T k = 0$ col. vector

transpose: $-Av = -f$

9-6

$$\begin{bmatrix} W & -A^T \\ -A & 0 \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ -f \end{bmatrix}$$ full normal eqns L.M.

n+r x n+r

$Wv - A^T k = 0$ $\quad (W^{-1} = Q)$

$v - QA^T k = 0$, $\boxed{v = QA^T k}$ $\quad W_e = Q_e^{-1}$

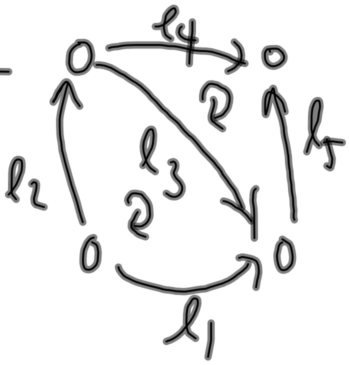
$-Av = -f$

$Av = f \leftarrow$

$\underbrace{AQA^T}_{Q_e} k = f$

$\boxed{k = W_e f} \leftarrow$

level network



$n = 5$
$n_0 = 3$
$r = 2$

0/0

9-7

$$\ell = \begin{pmatrix}10\\8\\3\\4\\1\end{pmatrix}$$

$$\hat{\ell}_3 - \hat{\ell}_1 + \hat{\ell}_2 = 0$$

$$\hat{\ell}_4 - \hat{\ell}_5 - \hat{\ell}_3 = 0$$

$$\underset{A}{\begin{array}{c}\begin{array}{ccccc}1 & 2 & 3 & 4 & 5\end{array}\\ \begin{bmatrix}-1 & 1 & 1 & 0 & 0\\ 0 & 0 & -1 & 1 & -1\end{bmatrix}\end{array}} \begin{pmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{pmatrix} = \underbrace{\begin{pmatrix}0\\0\end{pmatrix} - A\cdot\ell}_{f}$$

$W = I$

V =

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9-8

$K = W_\ell f$

$K = (AQA^T)^{-1} f$

$V = QA^T K$

$$K = \begin{pmatrix}-.375\\-.125\end{pmatrix}$$

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