10-1

$$\underbrace{\begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_1 \\ | \\ v_5 \end{bmatrix}}_{v} = d - Al = f$$

$$v = \begin{bmatrix} .375 \\ -.375 \\ -.250 \\ -.125 \\ .125 \end{bmatrix}$$

A, f, W

$Q = W^{-1}$

$Q_e = AQA^T$

$W_e = Q_e^{-1}$

$K = (AQA^T)^{-1} f$

$v = QA^T k$

$K = \begin{bmatrix} -.375 \\ -.125 \end{bmatrix}$

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10-2

I/O $v + Bo = f$ ($d - l$) B, f, W $o = (B^T W B)^{-1} B^T W f$ o

$v = f - Bo$ v

l

O/O $Av = f$ ($d - Al$) A, f, W $k = (AQA^T)^{-1} f$ v

$v = QA^T k$ $\hat{l}$

SUMMARY OF EQUATIONS for LS by I/O & O/O

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Curve fitting $y = a_0 + a_1 x = mx + b$ line 10-3

x: constant

$y = a_0 + a_1 x + a_2 x^2$ parabola

general polynomial

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots + a_n x^n$

high degree polynomials : dangerous

(1) low redundancy : between data points unexpected curve

(2) condition number large (numerical stability)

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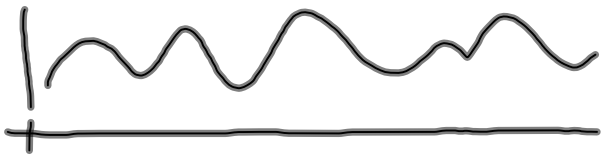
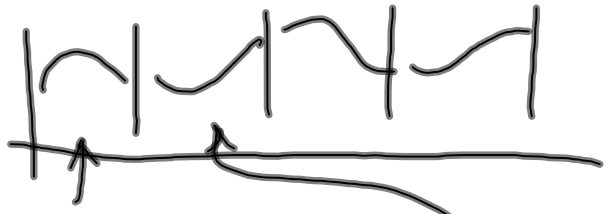


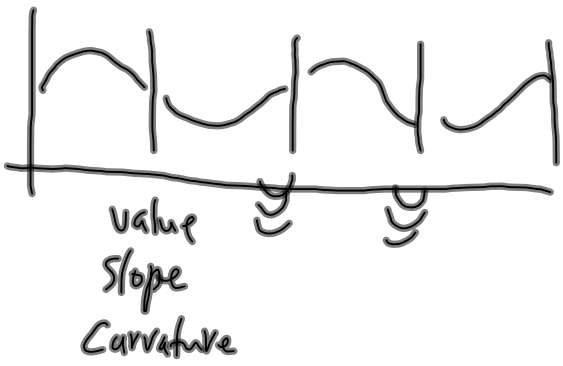
high degree polynomial 10-4

$n_0 = n+1$



piecewise polynomials

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ $\quad y = b_0 + b_1 x + b_2 x^2 + b_3 x^3$



$n_0 = n.seg \times 4$

value

slope

curvature

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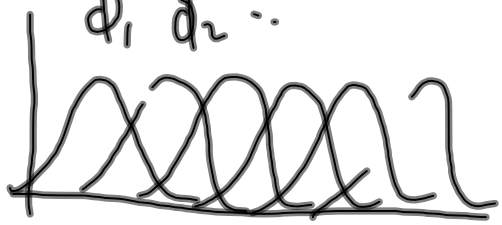
10-5

$x = f_x(s)$
$y = f_y(s)$
} splines



$y = a_1\phi_1 + a_2\phi_2 + \cdots + a_n\phi_n$

Spline, no constraint

trigonometric $y = a_0 + a_1 \cos x + a_2 \sin x + a_3 \cos 2x \ldots$

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10-6

$z = a_0 + a_1 x + a_2 y$

$z = a_0 + a_1 x + a_2 y + a_3 xy$, $n_0 = 4$
bilinear model

$X = a_0 + a_1 X + a_2 Y$
$y = b_0 + b_1 X + b_2 Y$
$\begin{pmatrix} X \\ Y \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix}$ 2D

coordinate transformation

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ 3D, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix}$ 3D → 2D projection

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components of coord. transf. rotation ✓ 10-7
shift
scale
skew

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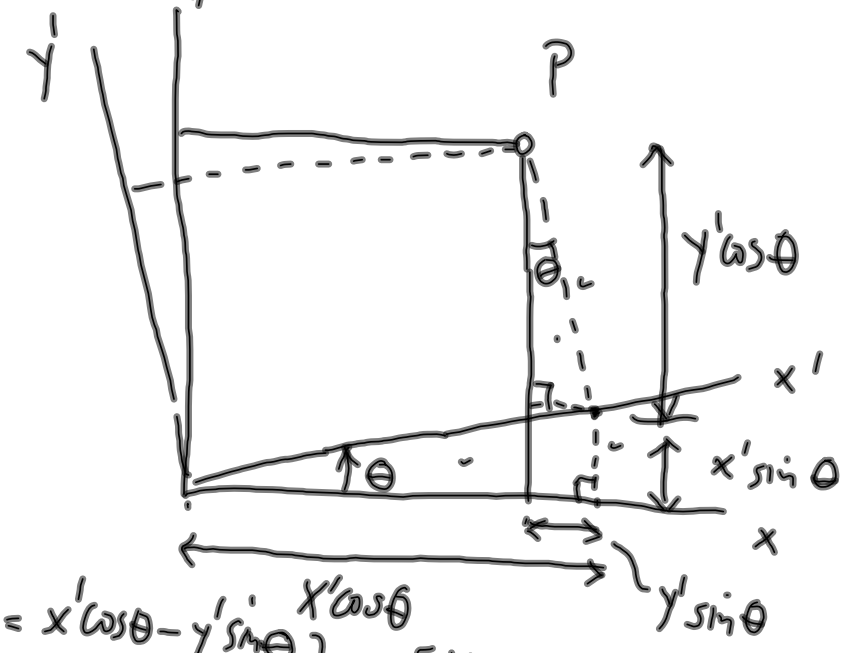
2D rotation matrix 10-8



$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

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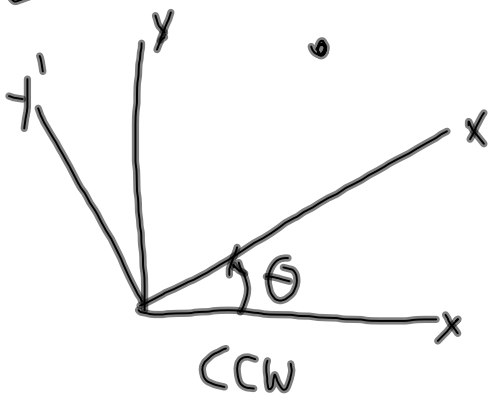
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

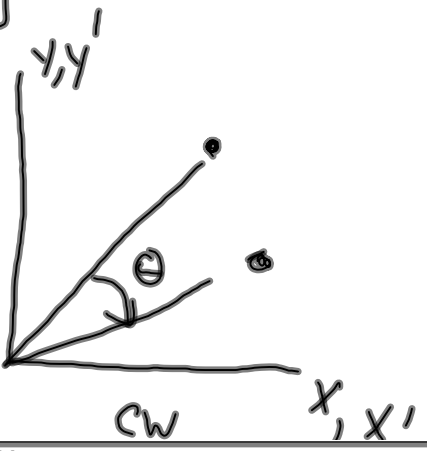
10-9

↑ orthogonal $\cos^2\theta + \sin^2\theta = 1$

$-\cos\theta \cdot \sin\theta + \cos\theta \cdot \sin\theta = 0$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$





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10-10

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_z & \sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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