11-1

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

R_3, R_z, M_z, M_k

elementary rotation matrix
sequential rotation

next R_x, M_x, M_w :

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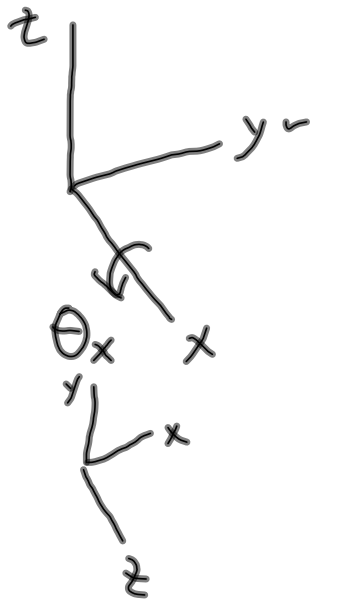
11-2



$$\begin{pmatrix} y' \\ z' \\ x' \end{pmatrix} = \begin{pmatrix} \cos\theta_x & \sin\theta_x & 0 \\ -\sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} Y \\ Z \\ X \end{pmatrix}$$

$$\begin{pmatrix} y' \\ z' \\ x' \end{pmatrix} = \begin{pmatrix} 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_y \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

R_1, R_x, M_x, M_w

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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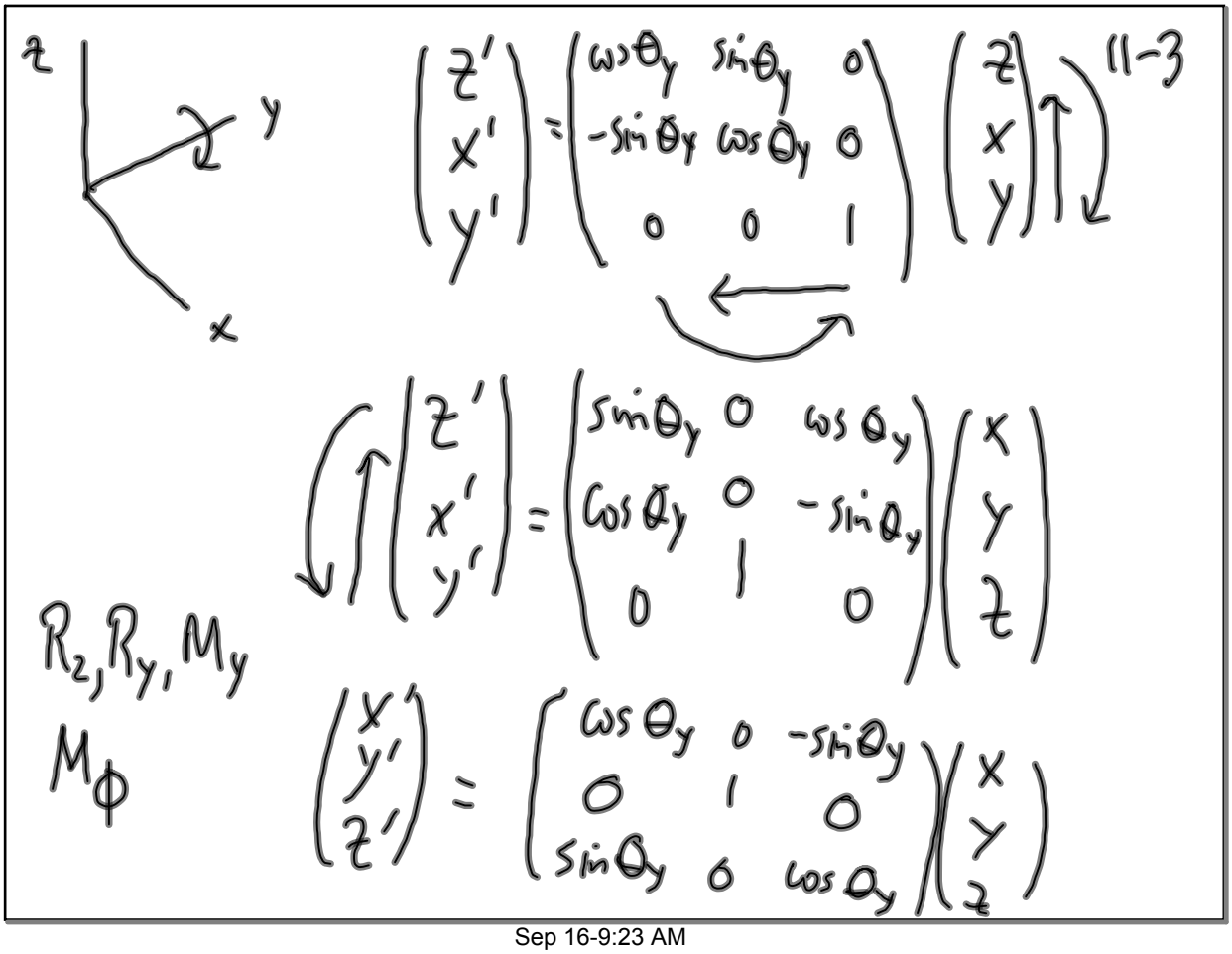

$$\begin{pmatrix} z' \\ x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta_y & \sin\theta_y & 0 \\ -\sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ x \\ y \end{pmatrix}$$

$$\begin{pmatrix} z' \\ x' \\ y' \end{pmatrix} = \begin{pmatrix} \sin\theta_y & 0 & \cos\theta_y \\ \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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11-4

$R = R_3 R_2 R_1$, $M = M_k M_\phi M_\omega$

$R_3 R_2 R_1 \neq R_1 R_2 R_3$ $\quad Y = R_3 R_2 R_1 X$, $Y = RX$

$M = M_k M_\phi M_\omega$ tertiary ↑ ↑ primary, secondary

$$\begin{bmatrix} \cos\phi\cos k & \cos\omega\sin k + \sin\omega\sin\phi\cos k & \sin\omega\sin k - \cos\omega\sin\phi\cos k \\ -\cos\phi\sin k & \cos\omega\cos k - \sin\omega\sin\phi\sin k & \sin\omega\cos k + \cos\omega\sin\phi\sin k \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix}$$

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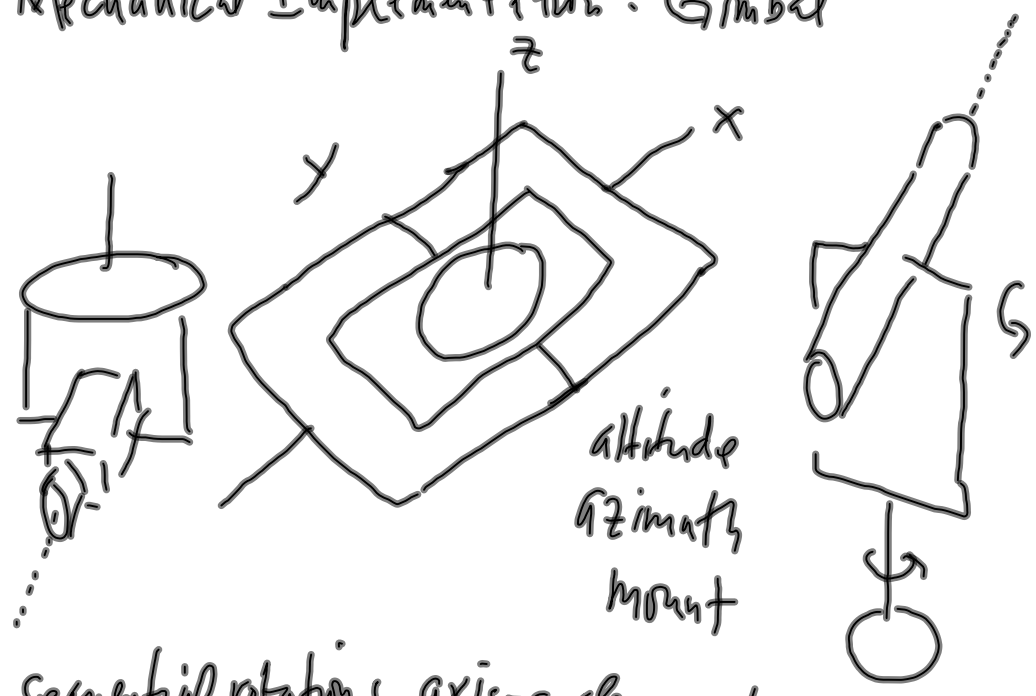
Mechanical Implementation: Gimbal

11-5



sequential rotations, axis-angle, quaternions, direction cosines

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Coordinate Transformation

11-6

2D conformal Coord Transf.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

4 parameter

scale — rotation — shift x, y

non linear: x, y

$$x' = \lambda\cos\theta\, x + \lambda\sin\theta\, y + t_x$$
$$y' = -\lambda\sin\theta\, x + \lambda\cos\theta\, y + t_y$$

constant

$$\lambda\cos\theta = a, \quad \lambda\sin\theta = b$$

x', y' obs.

$$x' = ax + by + c$$
$$y' = -bx + ay + d$$

Linear Model

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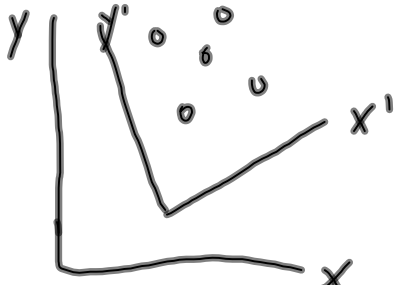

$x' = ax + by + c$
$y' = -bx + ay + d$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$ 4 parameter 11-7

$n = 5 \times 2 = 10$ ±10

$n_0 = 4$ n cond eqn

$r = 6$

$x_1' = ax_1 + by_1 + c$
$y_1' = -bx_1 + ay_1 + d$
$\vdots$
$x_5' = ax_5 + by_5 + c$
$y_5' = -bx_5 + ay_5 + d$
} 10

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