

$$\begin{aligned}
 \hat{x}_1 &= ax_1 + by_1 + c \\
 \hat{y}_1 &= -bx_1 + ay_1 + d \\
 &\vdots \\
 \hat{x}_n &= ax_n + by_n + c \\
 \hat{y}_n &= -bx_n + ay_n + d
 \end{aligned}$$

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ \vdots \\ v_{x_n} \\ v_{y_n} \end{bmatrix} - \begin{bmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & 1 & 0 \\ y_n & -x_n & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x_1 \\ -y_1 \\ \vdots \\ -x_n \\ -y_n \end{bmatrix}$$

I/O

$$V + B \Delta = f$$

$$\Delta = (B^T W B)^{-1} B^T W f$$

$$v = f - B \Delta$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

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$$\begin{aligned}
 a &= \lambda \cos \theta \\
 b &= \lambda \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 a^2 + b^2 &= \lambda^2 \cos^2 \theta + \lambda^2 \sin^2 \theta \\
 &= \lambda^2 (\sin^2 \theta + \cos^2 \theta) \\
 &= \lambda^2 \cdot 1
 \end{aligned}$$

12-2

$$\lambda = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right) \rightarrow \text{atan2}(b, a)$$

$$t_x : c, \quad t_y : d$$

4 parameter transform

| | | |
|-----------|---|---|
| 2D affine | $ \begin{aligned} x' &= a_0 + a_1 x + a_2 y \\ y' &= b_0 + b_1 x + b_2 y \end{aligned} $ | 2 shifts rotation 2 scales 1 non-orth. |
|-----------|---|---|

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3D: Conformal transf.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \lambda \cdot M \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

scale

3 rotations

3 shifts

Non linear

7 parameter
transf. 12-3

Rigid Body

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$$5x + 3y = 7 \quad \text{linear in } x, y \quad 12-4$$

$$x^2 + \sin y = 10 \quad \text{nonlinear}$$

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