13-1

linear: $5x + 3y = 7$ linear in x, y
(constant coefficients)

non linear: $\sqrt{x} + xy = 14$

LS: unknowns: parameters & observations

$$\boxed{f(x) = f(x_0) + \frac{f'(x_0)}{1!}\underbrace{(x - x_0)}_{\Delta x}} + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots$$

$$f(x,y) = f(x_0, y_0) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y \quad \vdots$$

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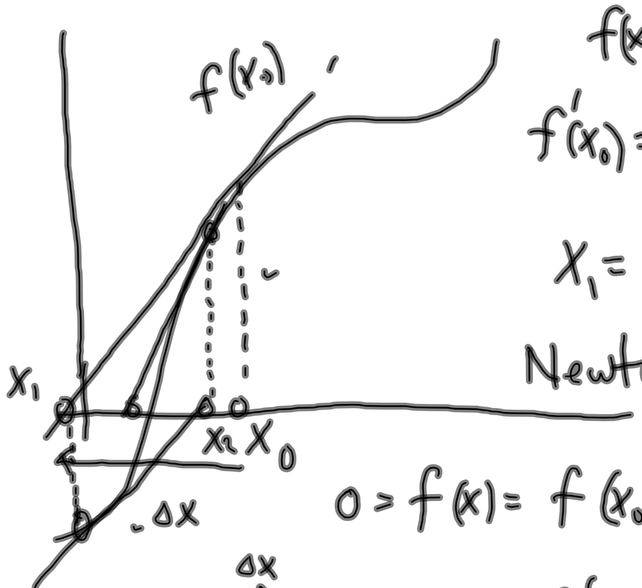
13-2



$$f(x) = 0$$

$$f'(x_0) = \frac{f(x_0)}{-\Delta x}, \quad \boxed{\Delta x = \frac{-f(x_0)}{f'(x_0)}}$$

$$x_1 = x_0 + \Delta x$$

Newton Iteration

$$0 = f(x) = f(x_0) + f'(x_0)\Delta x$$

$$\Delta x \cdot f'(x_0) = -f(x_0)$$

$$\boxed{\Delta x = \frac{-f(x_0)}{f'(x_0)}}$$

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

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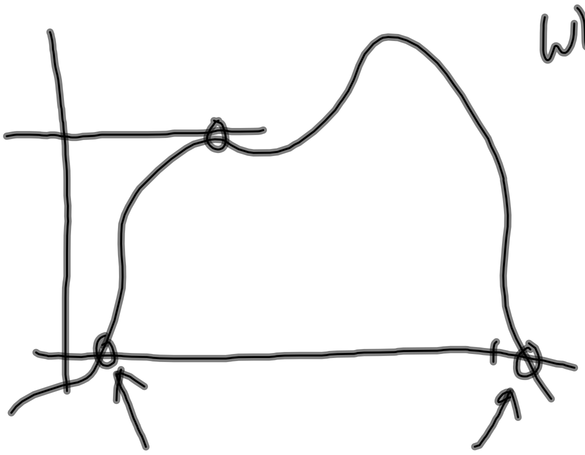

What can go wrong? 13-3

1. divergence
   non-convergence
2. converge to another solution
3. incorrect equation
   wrong derivatives ✓

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13-4

$$F_1(x_1, x_2, \ldots) = 0 = F_1(x_1^0, x_2^0, \ldots) + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 + \ldots$$

$$F_2(x_1, x_2, \ldots) = 0 = F_2(x_1^0, x_2^0, \ldots) + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 + \ldots$$

$$\vdots$$

$$F_n(x_1, x_2, \ldots) = 0 = F_n(x_1^0, x_2^0, \ldots) + \frac{\partial F_n}{\partial x_1}\Delta x_1 + \frac{\partial F_n}{\partial x_2}\Delta x_2 + \ldots$$

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} F_1^0 \\ F_2^0 \\ \vdots \\ F_n^0 \end{bmatrix} + \begin{bmatrix} \partial F_1/\partial x_1 & \partial F_1/\partial x_2 & \cdots \\ \partial F_2/\partial x_1 & \partial F_2/\partial x_2 & \cdots \\ \vdots & & \\ \partial F_n/\partial x_1 & \partial F_n/\partial x_2 & \cdots \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

Jacobian

$$0 = F(x_0) + J\Delta x$$

$$J\Delta x = -F(x_0)$$

$$\boxed{\Delta x = -J^{-1}F(x_0)}$$

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13-5

$$\vec{\Delta x} = -J^{-1} F(x_0)$$

$$\boxed{\vec{X}_1 = \vec{X}_0 + \vec{\Delta x}}$$

$$\boxed{\vec{X}_1 = \vec{X}_0 - J^{-1} F(x_0)}$$ multivariate

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ ← single function/unknown

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2 eqn / 2 unknown

$\begin{pmatrix} x \\ y \end{pmatrix}$

$x^2 + y = 2$ | $f_1(x,y) = x^2 + y - 2 = 0$ ← 13-6

$x - 3y^2 = -2$ | $f_2(x,y) = x - 3y^2 + 2 = 0$

$$\vec{X}_1 = \vec{X}_0 - \underline{J}^{-1} \underline{F(x_0)}$$

$$J = \begin{bmatrix} 2x & 1 \\ 1 & -6y \end{bmatrix}$$

$(1,1)$: $(x^0, y^0) = (1.05, 1.05)$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 1.05 \\ 1.05 \end{pmatrix} - \begin{pmatrix} 2.10 & 1 \\ 1 & -6.30 \end{pmatrix}^{-1} \begin{pmatrix} .1525 \\ -.2575 \end{pmatrix} = \begin{pmatrix} 1.05 \\ 1.05 \end{pmatrix} - \underbrace{\begin{pmatrix} .0494 \\ .0487 \end{pmatrix}}$$

$\begin{matrix} 1.0006 \\ 1.0013 \end{matrix}$ $\begin{matrix} -.0006 \\ -.0013 \end{matrix}$ → $\begin{pmatrix} 1.0000 \\ 1.0000 \end{pmatrix}$

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newton process : converge 13-7

converges quadratically

$1, 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}$ } stop

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Big Picture 13-8

analyze problem n, n_0, r
initial approx

$r=0$ unique solution	$r>0$ LS choose I/0, 0/0

linearize J, B, A + Eval Funct.

solve uniquely $\Delta = -J^{-1}F(x_0)$	$\Delta = (B^TWB)^{-1}B^TWf$

$x_{i+1} = x_i + \Delta$

Newton Iteration Loop

...... repeat until Δ small

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