16-1

$$F_1 = \hat{\ell}_1^2 + \hat{\ell}_2^2 - \hat{\ell}_3^2 = 0$$

$$F_2 = \hat{\ell}_4 + \hat{\ell}_5 - \pi/2 = 0$$

$$F_3 = \hat{\ell}_4 - \tan^{-1}(\hat{\ell}_2/\hat{\ell}_1)$$

$$\boxed{\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\cdot\frac{du}{dx}}$$

$$\frac{\partial F_1}{\partial \ell_1} = 2\ell_1 \Big|_{\ell^0}, \quad \frac{\partial F_1}{\partial \ell_2} = 2\ell_2 \Big|_{\ell^0}, \quad \frac{\partial F_1}{\partial \ell_3} = -2\ell_3$$

$$\frac{\partial F_2}{\partial \ell_4} = 1, \quad \frac{\partial F_2}{\partial \ell_5} = 1 ,$$

$$\frac{\partial F_3}{\partial \ell_1} = -\frac{1}{1+\left(\ell_2/\ell_1\right)^2}(-1)\frac{\ell_2}{\ell_1^2} = \frac{\ell_2}{\ell_1^2+\ell_2^2}\Big|_{\ell^0}$$

$$\frac{\partial F_3}{\partial \ell_2} = -\frac{1}{1+\left(\ell_2/\ell_1\right)^2}\cdot\frac{1}{\ell_1}\left[\frac{\ell_1}{\ell_1}\right] = \frac{-\ell_1}{\ell_1^2+\ell_2^2}, \quad \frac{\partial F_3}{\ell_4} = 1$$

Sep 28-9:23 AM

16-2

$$\left.\begin{aligned} \ell_1 &= 10.1 \\ \ell_2 &= 7.4 \\ \ell_3 &= 12.5 \end{aligned}\right\} \sigma = 0.1$$

$$\left.\begin{aligned} \ell_4 &= 36.22^\circ \\ \ell_5 &= 53.78^\circ \end{aligned}\right\} \sigma = 0.005\,R$$

$$A = \begin{bmatrix} 2\ell_1^0 & 2\ell_2^0 & -2\ell_3^0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ \dfrac{\ell_2^0}{\ell_1^{0\,2}+\ell_2^{0\,2}} & \dfrac{-\ell_1^0}{\ell_1^{0\,2}+\ell_2^{0\,2}} & 0 & 1 & 0 \end{bmatrix}$$

Sep 28-9:24 AM

16-3

$$f = -F - A\cdot(l - l^0)$$

$$\underset{r,1}{f} = \left[\begin{bmatrix} -(l_1^2 + l_2^2 - l_3^2) \\ -(l_4 + l_5 - \pi/2) \\ -(l_4 - \tan^{-1}(l_2/l_1)) \end{bmatrix} - A\cdot(l - l^0)\right]$$

$$\begin{array}{cccc} \Delta l & 1e\text{-}05 & 1e\text{-}09 & 1e\text{-}14 \\ \begin{bmatrix} -.0074 \\ -.0075 \\ 0.100 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} -.6 \\ .5 \\ .8 \\ -.0018 \\ .0018 \end{bmatrix} & \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} & \begin{bmatrix} \vdots \\ \vdots \end{bmatrix} \end{array}$$

Sep 28-9:24 AM

16-4

dist $\sigma = 0.1$ $\quad \sigma_0^2 = .01$ $\quad W_d = \frac{.01}{.01} = 1$

avg $\sigma = .005$ $\quad W_a = \frac{.01}{.000025} = 400$

$$W = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 400 & \\ & & & & 400 \end{bmatrix}$$

$A, f, W \Rightarrow$ LS solution

$$k = (AQA^T)^{-1} f$$

$$Q = W^{-1}$$

$$V = QA^T k$$

$$l + v = l^0 + \Delta l$$

$$\Delta = l - l^0 + v$$

Sep 28-9:24 AM

16-5

$$v = \begin{bmatrix} -.0074 \\ -.0075 \\ .0104 \\ 0 \\ 0 \end{bmatrix} \quad \sigma = 0.1 \quad \hat{l} = \begin{bmatrix} 10.0926 \\ 7.3925 \\ 12.5104 \\ 0.6322 \\ 0.9386 \end{bmatrix}$$

computing derivatives

1. analytically
2. numerical approximation
3. symbolic processing

Sep 28-9:24 AM

16-6

Symbolic (matlab)

S = 'atan(y/x)'

$$\text{diff}(S, \text{'y'}) \rightarrow \left(\frac{x}{x}\right) \underbrace{\frac{1}{x\left(\frac{y^2}{x^2}+1\right)}}_{\text{result}} = \frac{x}{x^2+y^2}$$

$$\text{diff}(S, \text{'x'}) \rightarrow \frac{-y}{x^2 \cdot \left(\frac{y^2}{x^2}+1\right)} = \frac{-y}{x^2+y^2}$$

Sep 28-9:24 AM

<u>numerical approximation</u> $\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ 16-7

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} \; ; \; \Delta x \text{ : small number}$$

$$\frac{\partial f}{\partial x_i} = \lim_{\Delta x \to 0} \frac{f(x_1, x_2, \dots, x_i + \Delta x, \dots, x_n) - f(x_1, \dots x_n)}{\Delta x}$$

$$\frac{\partial f}{\partial x_i} = \frac{f(x_1 \dots, x_i + \Delta x, \dots x_n) - f(x_1, \dots x_n)}{\Delta x}$$

Sep 28-9:24 AM

implement in matlab : put f into function m-file 16-8

dx : array (vector) of perturbation (Δx's)

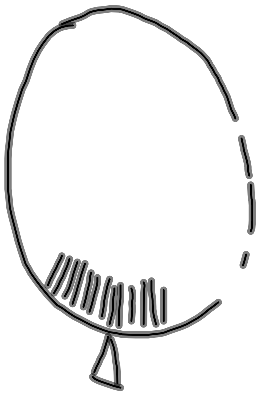
x, f

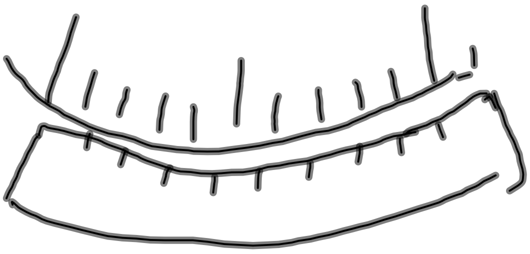
```
for i=1:n
    xx = x ;
    xx(i) = xx(i)+ dx(i)
    df_dx(i) = [f(xx) - f(x)]/ dx(i)
end
```

Sep 28-9:24 AM

angle

16-9





Vernier
subdivide graduations
by 10x

Sep 28-9:24 AM