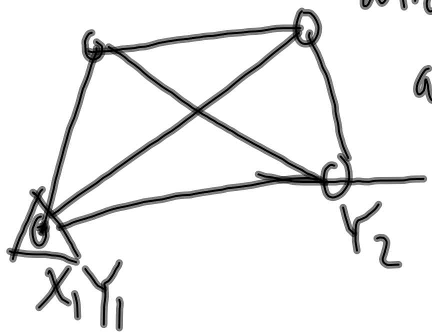
Minimal Constraints how many are needed? 19-1

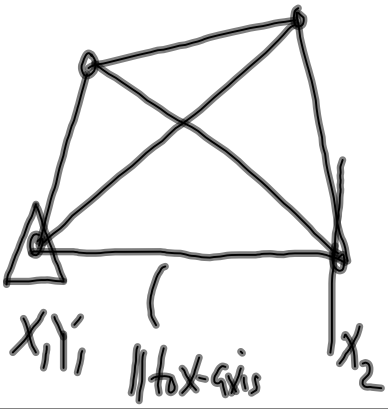
which coord. components to fix?

ang + dist obs



#MC = 3

Y_1, Y_2, Y_3 X



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want to look at point uncertainties under a MC adjustment 19-2

INNER CONSTRAINTS (Free Network Adj.)

no net shift in x –

no net shift in y –

no net rotation –

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Statistics to support E.P. + hypothesis testing 19-3

→ confidence interval
confidence region
$\sigma_{x_1}, \sigma_{x_2}^2, \sigma_{xy}$

distributions (density functions)

<u>Normal Distr.</u> $$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right)$$

$$z = \frac{x-\mu_x}{\sigma_x} : \mu = 0,\ \sigma = 1$$

standard normal $$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

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MVN multivariate Normal Distr. 19-4

$$f(\vec{x}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \exp\left[-\frac{1}{2}(\vec{x}-\vec{\mu_x})^T \Sigma^{-1} (\vec{x}-\vec{\mu_x})\right]$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \mu_x = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_n} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots & \sigma_{x_1x_n} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \sigma_{x_nx_1} & \cdots & & \sigma_{x_n}^2 \end{bmatrix}$$

variance/covariance matrix

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normal distr. 19-5

justified by Central Limit Theorem

χ^2 : chi-squared distr.

$x_1, x_2, \dots, x_n$ independent $N(0,1)$

$x_1^2 + x_2^2 + \dots + x_n^2 = \chi_n^2$ chi-squared distr., n deg. of freedom

$f(\chi_n^2) = f(u) = C_n\, u^{(n-2)/2}\, e^{-u/2}$ for $u > 0$

$C_n = \dfrac{1}{2^{n/2}\,\Gamma(n/2)}$ Γ : gamma function

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19-6

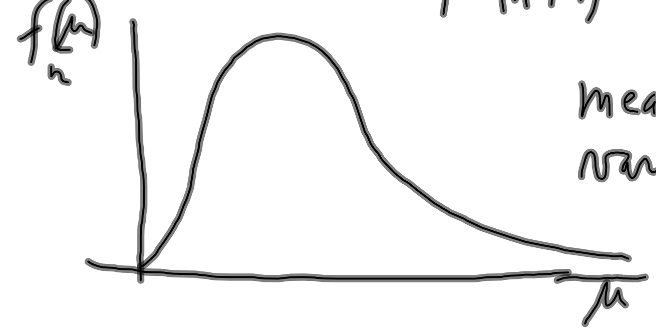
$$\Gamma(n) = \int_0^\infty e^{-t}\, t^{n-1}\, dt$$

$\Gamma(n+1) = n\,\Gamma(n)$

$\Gamma(1) = 1$

whole numbers $\Gamma(n) = (n-1)!$

$\Gamma(n+1) = n!$



mean = n
var = 2n

"shape" of typical χ^2 density function

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Theorem: $\bar{X}$, S^2 from sample size n 19-7

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\boxed{\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_r}$$

t-distr arises sampling when you compute variance from data

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19-8

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(see revision below)

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \quad \left(\frac{\text{estimate} - \text{mean}}{\text{std. dev. from data}} \right)$$

$$f_n(t) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}}$$

[note: formula for $f(t)$ was incorrect in lecture.]

$\Gamma(\cdot)$: gamma function prev. defined, n: deg. of freedom

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as $n \to \infty$ t approaches normal distr. 19-9

$n > 0$, replace t by z $N(0,1)$

F distribution

f: density
F: cumulative distr. function

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