F distribution 2 χ^2: χ^2_m, χ^2_n 20-1

$$\rightarrow u = \frac{\chi^2_m/m}{\chi^2_n/n} \sim F \text{ with } m,n \text{ d.o.f.}$$

$$f_{m,n}(u) = \frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}\left(\frac{m}{n}\right)^{m/2} \frac{u^{(m-2)/2}}{\left[1+\left(\frac{m}{n}\right)u\right]^{(m+n)/2}}$$

compare 2 var. estimates →

$$\frac{(n_1-1)\,S_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1}$$

$$\frac{(n_2-1)\,S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{n_1-1,\,n_2-1}$$

Oct 7-9:27 AM

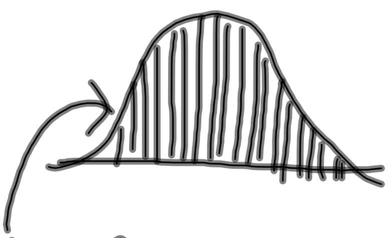
PDF prob. density function R.V. X 20-2

sample many times

samples → ∞

reduce bin width

normalize so that area under curve = 1



envelope function

P.D.F.

R discrete

continuous ✓

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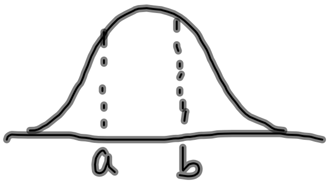
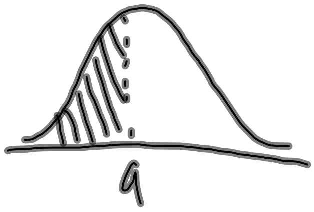


areas : probabilities 20-3

$P(a < X < b)$ = area under curve between a, b

$$\int_a^b f(x)\,dx$$



$$P(X < a) = \int_{-\infty}^{a} f(x)\,dx$$

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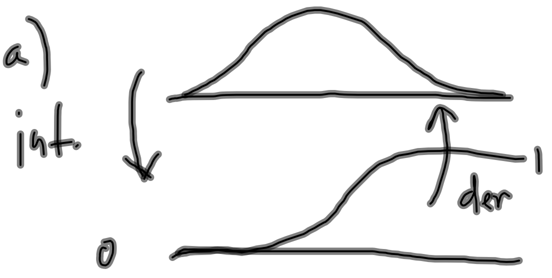
$F(a) = P(X < a)$



$f(x)$ 20-4

$F(x)$

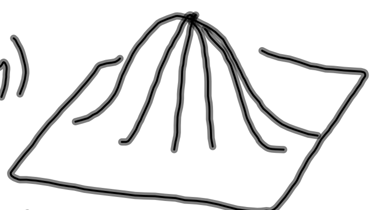
$$F(a) = \int_{-\infty}^{a} f(x)\,dx$$

$$F(x) \int_{-\infty}^{x} f(u)\,du$$

$$f(x) = \frac{d}{dx} F(x)$$

2 RV's

$f(x,y)$



vol under surface = 1

$F(a,b) = P\{(X<a) \,\&\, (Y<b)\}$

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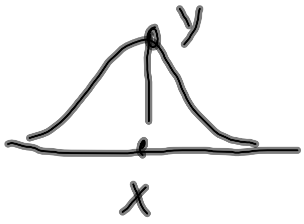
normal, MVN, χ^2, t, F MATLAB functions 20-5

tables
calculators
matlab

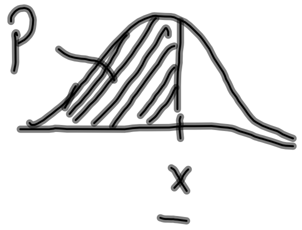
$y = \mathrm{pdf}(\text{'norm'}, x, \mu, \sigma)$



$P = \mathrm{cdf}(\text{'norm'}, x, \mu, \sigma)$

$x = \mathrm{icdf}(\text{'norm'}, P, \mu, \sigma)$

↳ critical values



'chi2', 't', 'f',

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<u>expectation</u> $E(x) = \mu_x$ 20-6

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

centroid of density function

$$E\{(x-\mu_x)^2\} = \sigma_x^2 \text{ variance } \sigma_{xx}$$

$$E\{(x-\mu_x)(y-\mu_y)\} = \sigma_{xy} \leftarrow$$

$$E\{(x-\mu_x)^2\} = \int_{-\infty}^{\infty} (x-\mu_x)^2 \cdot f(x)\, dx$$

Oct 7-9:27 AM

20-7

$$\frac{\sigma_{xy}}{\sigma_x \sigma_y} = r_{xy}$$ correlation coefficient

$-1 \rightarrow +1$

x,y move together , r +

x,y move opposite , r −

x,y independent , r 0

$$E\{(x-\mu_x)(y-\mu_y)\} = \sigma_{xy} \quad \checkmark\checkmark$$

$$E\{(y-\mu_y)(x-\mu_x)\} = \sigma_{yx}$$

Oct 7-9:27 AM

20-8

$$E(x+y) = E(x) + E(y)$$

$$E(aX) = a\,E(X)$$

$$\vec{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \quad E(\vec{X}) = \begin{bmatrix} E(X_1) \\ E(X_2) \\ \vdots \\ E(X_n) \end{bmatrix} = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_n} \end{bmatrix}$$

$$\Sigma_{XX} = E\{\underbrace{(\vec{X}-\vec{\mu}_x)}\underbrace{(\vec{X}-\vec{\mu}_x)^T}\} =$$

Oct 7-9:27 AM

20-9

$$E\left\{\begin{bmatrix} x_1-\mu_{x_1} \\ x_2-\mu_{x_2} \\ \vdots \\ x_n-\mu_{x_n}\end{bmatrix}\begin{bmatrix} x_1-\mu_{x_1} & x_2-\mu_{x_2} & \cdots & x_n-\mu_{x_n}\end{bmatrix}\right\}$$

$$E\left\{\begin{bmatrix} (x_1-\mu_{x_1})(x_1-\mu_{x_1}) & (x_1-\mu_{x_1})(x_2-\mu_{x_2}) & \cdots \\ (x_2-\mu_{x_2})(x_1-\mu_{x_1}) & & \\ \vdots & & \\ (x_n-\mu_{x_n})(x_1-\mu_{x_1}) & \cdots & (x_n-\mu_{x_n})(x_n-\mu_{x_n}) \end{bmatrix}\right\}$$

n, n

Oct 7-9:27 AM

20-10

$$\begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots & \sigma_{x_1x_n} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 & & \vdots \\ \vdots & & \ddots & \\ \sigma_{x_nx_1} & \cdots & & \sigma_{x_n}^2 \end{bmatrix} = \Sigma$$ variance/covariance matrix

Σ : symmetric
pos. definite $\quad x^T \Sigma_{xx} x > 0$
any x

Oct 7-9:27 AM

20-11

Error Propagation

$$\begin{matrix} \ell \\ \Sigma_{\ell\ell} \end{matrix} \longrightarrow \boxed{LS} \longrightarrow \begin{matrix} x & \Sigma_{xx} \\ \Delta & \Sigma_{\Delta\Delta} \\ v & \Sigma_{vv} \\ \hat{\ell} & \Sigma_{\hat{\ell}\hat{\ell}} \end{matrix}$$

Σ: precision, dispersion

does not describe biases, syst. errors

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20-12

$$\Sigma_{xx} = \begin{bmatrix} \sigma_{x_1}^2 & & & 0 \\ & \sigma_{x_2}^2 & & \\ & & \ddots & \\ 0 & & & \sigma_{x_n}^2 \end{bmatrix}$$

$$\Sigma_{xx}^{-1} = \begin{bmatrix} 1/\sigma_{x_1}^2 & & & \\ & 1/\sigma_{x_2}^2 & & \\ & & \ddots & \\ & & & 1/\sigma_{x_n}^2 \end{bmatrix}$$

$$\sigma_0^2 \Sigma_{xx}^{-1} = \begin{bmatrix} \sigma_0^2/\sigma_{x_1}^2 & & & \\ & \sigma_0^2/\sigma_{x_2}^2 & & \\ & & \ddots & \\ & & & \sigma_0^2/\sigma_{x_n}^2 \end{bmatrix} = W_{xx}$$

$$\sigma_0^2 \Sigma_{xx}^{-1} = W_{xx}$$

$$\frac{1}{\sigma_0^2} \Sigma_{xx} = Q_{xx}$$

$$\boxed{\underbrace{\Sigma_{xx}}_{\text{abs. cov}} = \sigma_0^2 \underbrace{Q_{xx}}_{\text{scaled cov. matrix}}}$$

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Derive Error Prop. Law 20-13

$\underline{Y = AX}$, have Σ_{xx}, $\Sigma_{yy} = ?$

A : matrix of constants

$$\underbrace{E(Y)}_{\mu_y} = E(AX) = \underbrace{A\,E(X)}_{A\mu_x}$$

Oct 7-9:27 AM