23-1

$$Q_{vv} = Q + \underbrace{BN^{-1}B^T - BN^{-1}B^T}_{0} - BN^{-1}B^T$$

$$\boxed{Q_{vv} = Q - BN^{-1}B^T}$$

$Q_{\hat{\ell}\hat{\ell}}$

$Q_{vv} = Q - Q_{\hat{\ell}\hat{\ell}}$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}$

$Q = Q_{\hat{\ell}\hat{\ell}} + Q_{vv}$

↑ Ind. Obs.

↓ Obs. Only $Q_{vv}, Q_{\hat{\ell}\hat{\ell}}$ $v = [\,] + [\,]\ell$ ← ✓

$Q_{vv} = [\,] Q_{\ell\ell} [\,]^T$

$Av = f$

$Av = d - A\ell$

$v = QA^Tk, \; k = W_e f$, $\underline{v = QA^T W_e (d - A\ell)}$

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23-2

$$v = \underbrace{QA^TW_ed} - \underbrace{QA^TW_eA}\underbrace{\ell} \qquad Q_{\ell\ell} = Q$$

$$Q_{vv} = QA^TW_eA \cdot Q \cdot [QA^TW_eA]^T$$

A^TW_eAQ

$$\boxed{Q_{vv} = QA^TW_eAQ}$$

$\hat{\ell}: \; \hat{\ell} = \ell + v, = \ell + QA^TW_ed - QA^TW_eA\ell$

$$\hat{\ell} = \underline{QA^TW_ed} + \underline{(I - QA^TW_eA)}\underline{\ell} \leftarrow$$

$$Q_{\hat{\ell}\hat{\ell}} = (I - QA^TW_eA) \cdot Q \cdot (I - QA^TW_eA)^T$$

$(I - A^TW_eAQ)$

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23-3

$$(Q - QA^TW_eAQ)(I - A^TW_eAQ)$$

$$Q + \underbrace{QA^TW_e\underbrace{AQA^T}\underbrace{W_e}AQ - QA^TW_eAQ}_{W_e} - QA^TW_eAQ$$

$$\boxed{Q_{\hat{l}\hat{l}} = Q - \underbrace{QA^TW_eAQ}_{Q_W}} \qquad Q_{\hat{l}\hat{l}} = Q - Q_W$$

$\Sigma = \sigma_0^2 Q$ $\quad v^TWv = \sigma_0^2 v^T\Sigma^{-1}v$ $\quad E\left(\frac{v^TWv}{r}\right) = \sigma_0^2$

$Q = \frac{1}{\sigma_0^2}\Sigma$ $\quad E(v^TWv) = \sigma_0^2 \underbrace{E(v^T\Sigma^{-1}v)}_{"r}$

$\underline{W = \sigma_0^2\Sigma^{-1}}$

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23-4

σ_0^2 a priori ref. var

$\hat{\sigma}_0^2$ a posteriori ref. var. $= \frac{v^TWv}{r}$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}, \qquad \frac{r\cdot\hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_r$$

$$\frac{r\cdot\frac{v^TWv}{r}}{\sigma_0^2} = \boxed{\frac{v^TWv}{\sigma_0^2} \sim \chi^2_r}$$ global test

---

choice of σ_0^2 has no impact on LS adjustment

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estimation 23-5

$$\Delta = (B^T W B)^{-1} B^T W f \qquad \underline{W = \sigma_0^2 \Sigma^{-1}}$$

$$\frac{1}{\underline{\sigma_0^2}}(B^T \Sigma^{-1} B)^{-1} \cdot B^T \Sigma^{-1} f \cdot \underline{\sigma_0^2}$$

$$\Delta = (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} f$$

σ_0^2 disappeared

error prop $\quad \Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} = \sigma_0^2 N^{-1}$

$$= \sigma_0^2 (B^T W B)^{-1}, \quad \sigma_0^2 (B^T \Sigma^{-1} B)^{-1} \frac{1}{\sigma_0^2}$$

$$\Sigma_{\Delta\Delta} = (B^T \Sigma^{-1} B)^{-1}$$

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hypothesis test $\quad \boxed{\frac{v^T W v}{\sigma_0^2} \sim \chi_r^2}$ 23-6

$$\frac{\sigma_0^2 v^T \Sigma^{-1} v}{\sigma_0^2} : \; v^T \Sigma^{-1} v \sim \chi_r^2$$

post adjustment statistics

Global Test on reference variance

Hypothesis Test : $H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 > \sigma_0^2$

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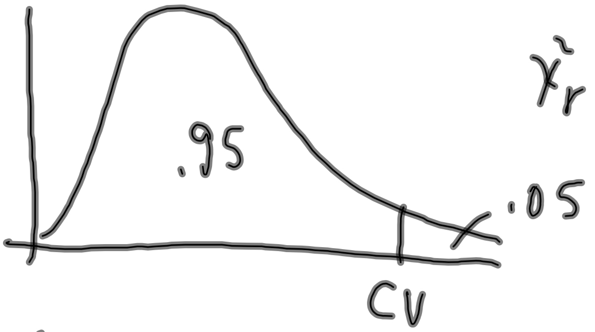
$$\frac{v^T W v}{\sigma_0^2} = \chi^{2*} = \text{test statistic} \sim \chi^2_r$$ 23-7



@ significance level of .05

Decision rule

if $\chi^* < cv$, accept H_0 @ .05 L.O.S.

if $\chi^* > cv$, reject H_0 " "

accept H_1

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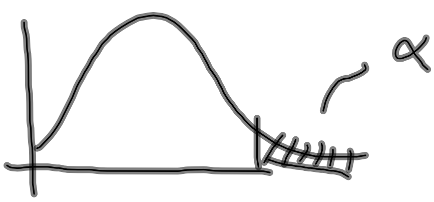
23-8



Probability (reject H_0 when true)

Prob (type 1 error)

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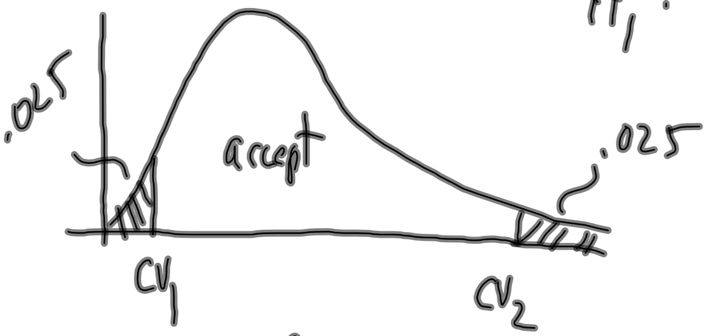
two sided global test $H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 \neq \sigma_0^2$



decision rule: if $cv_1 < \chi^* < cv_2$ accept H_0, otherwise reject H_0

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23-9

$$\left.\begin{array}{l} CV_1 = icdf('chi2', .025, r) \\ CV_2 = icdf('chi2', \underbrace{1-.025}_{1-\alpha/2}, r) \end{array}\right\} \text{two sided test}$$

$$CV = icdf('chi2', \underbrace{.95}_{1-\alpha}, r) \qquad \text{one sided test}$$

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