std normal

26-1

$\text{Prob}(-z < RV < +z) = F(z) - F(-z)$

$F(-z) = 1 - F(z)$, $F(z) + F(z) - 1$

$\text{Prob}(-z < RV < +z) = \underbrace{P = 2F(z) - 1}$

$\boxed{\frac{P+1}{2} = F(z)}$ given P, calculate z

$z = \text{icdf}\left('\text{norm}', 0, 1, \frac{P+1}{2}\right)$

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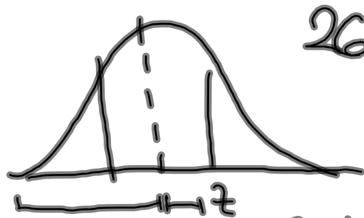
$\hat{x}$, $\frac{\hat{x} - \mu_x}{\sigma_x}$, pass global test $\Sigma_{xx} = \sigma_0^2 \underline{Q_{xx}}$

26-2



$\frac{1}{2} + \frac{P}{2} = \frac{P+1}{2}$

$\text{Prob}\left(-z < \frac{\hat{x} - \mu_x}{\sigma_x} < +z\right) = P$

$(-z\sigma_x < \hat{x} - \mu_x < +z\sigma_x)$

$(-\hat{x} - z\sigma_x < -\mu_x < -\hat{x} + z\sigma_x)$

$(\underline{\hat{x} + z\sigma_x} > \mu_x > \underline{\hat{x} - z\sigma_x})$

$\text{Prob}(\underline{\hat{x} - z\sigma_x} < \mu_x < \underline{\hat{x} + z\sigma_x}) = P$

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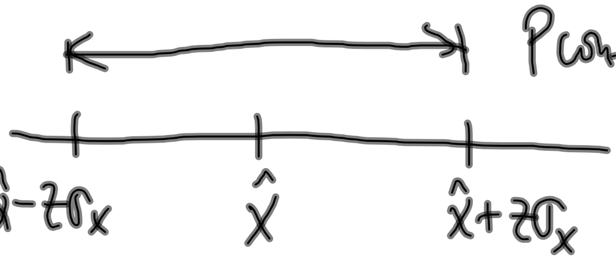

P conf. interval 26-3

assumes prior σ_0^2 : pass global test

1. select P
2. solve for z
3. get σ_x, $\hat{x}$
4. construct interval

x_i : parameter Σ_{xx}

v_i : residual Σ_{vv}

$\hat{l}_i$: adj. obs. $\Sigma_{\hat{l}\hat{l}}$

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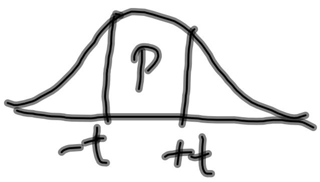
what if don't pass global test 26-4

have no info about prior σ's

$$\boxed{\frac{\hat{x}-\mu_x}{\hat{\sigma}_x}} \sim t_r \qquad \hat{\sigma}_x : \underline{\hat{\sigma}_0^2} Q_{xx}, \quad \hat{\sigma}_0^2 = \frac{v^T W v}{r}$$

r: redundancy in adj.

$$\text{Prob}\left(-t < \frac{\hat{x}-\mu_x}{\hat{\sigma}_x} < +t\right) = P \qquad P = 2F(t)-1$$



$$, \quad P\left(\hat{x}+t\hat{\sigma}_x > \mu_x > \hat{x}-t\hat{\sigma}_x\right)$$

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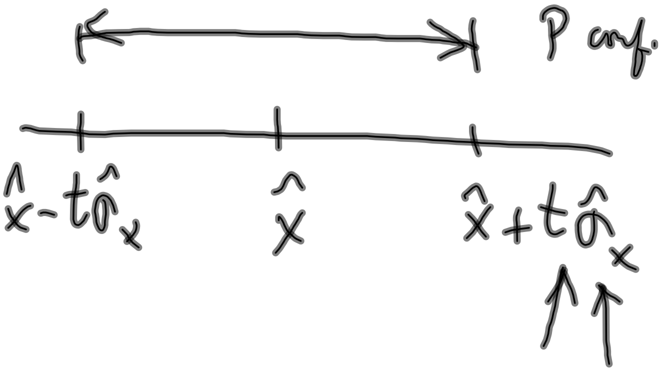
26-5

$F(t) = \frac{P+1}{2}$, $t = \text{icdf}('t', r, \frac{P+1}{2})$



$\underline{\underline{\hat{x} \pm t\hat{\sigma}_x}}$

theorem if $x \sim N(0, \Sigma)$, if $A\Sigma$ idempotent rank r $\Big\}$ $x^T A x \sim \chi^2_r$

$(A\Sigma)^2 = A\Sigma$

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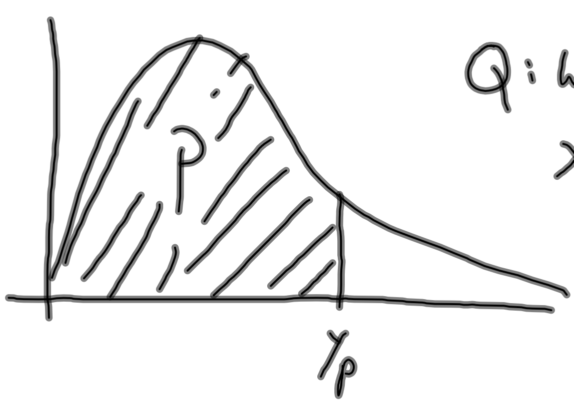
26-6

$\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r$

$y = \underline{\underline{(x-\mu_x)^T \Sigma_{xx}^{-1} (x-\mu_x)}} \sim \chi^2_n$ n: #elements in x



Q: what is the locus of x for which $(^T \Sigma^{-1} () < y_p$

$x : \begin{bmatrix} x \\ y \end{bmatrix}$

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eigenvalues 26-7

$Av = \lambda v$

$Av - \lambda v = 0$

$Av - \lambda I v = 0$

$(A - \lambda I) v = 0$

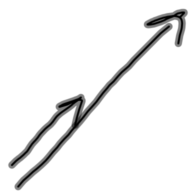
if system of eqn's has non-trivial solution then matrix singular

$\underline{\det(A - \lambda I) = 0}$

for small matrix solve for λ, then solve for v's up to scale factor

λ : eigenvalue

v : eigenvector



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(2×2) 26-8

$$A[v_1 v_2] = [v_1 v_2]\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$AV = V\Lambda$

$A = V\Lambda V^{-1}$ eigenvalue decomposition

A symmetric : λ's real

V orthogonal

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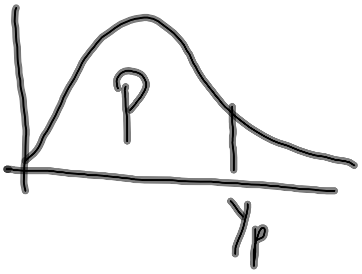

26-9

$(\ )^T \Sigma^{-1} (\ ) \sim \chi^2_n$

what is form of x ?

$(x-\mu_x)^T \underline{\Sigma}^{-1} (x-\mu_x)$

$\Sigma = V \Lambda V^{-1}$

$\Sigma = R^T D R \qquad D = R \Sigma R^T$

$D^{-1} = R \Sigma^{-1} R^T$

$D = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}, \quad D^{-1} = \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix}$

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26-10

$\text{Prob}\left[(x-\mu_x)^T \Sigma_{xx}^{-1} (x-\mu_x) < \chi^2_{p,n} \right] = p$

$+ \ w = R(x-\mu) \qquad R^T R$

$\text{Prob}\left[\underbrace{(x-\mu_x)^T R^T}_{w^T} \underbrace{R \Sigma_{xx}^{-1} R^T}_{D^{-1}} \underbrace{R (x-\mu_x)}_{w} < \chi^2_{p,n} \right]$

$\text{Prob}\left[w^T D^{-1} w < \chi^2_{p,n} \right] = p$

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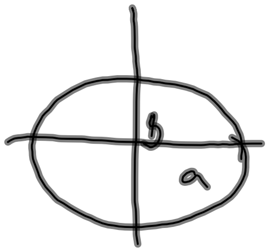
26-11

$$P\left[[w_1\ w_2] \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} < \chi^2_{p,n} \right] = p$$

$$\left[\frac{w_1^2}{d_1} + \frac{w_2^2}{d_2} < \underline{\chi^2_{p,n}} \right]$$

$$\text{Prob}\left[\frac{w_1^2}{d_1 \chi^2_{p,n}} + \frac{w_2^2}{d_2 \chi^2_{p,n}} < 1 \right] = p$$



$$\boxed{\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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