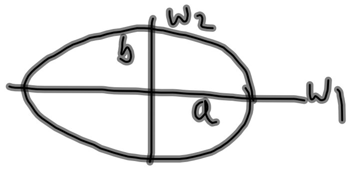
27-1

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$w = P(x-\mu)$

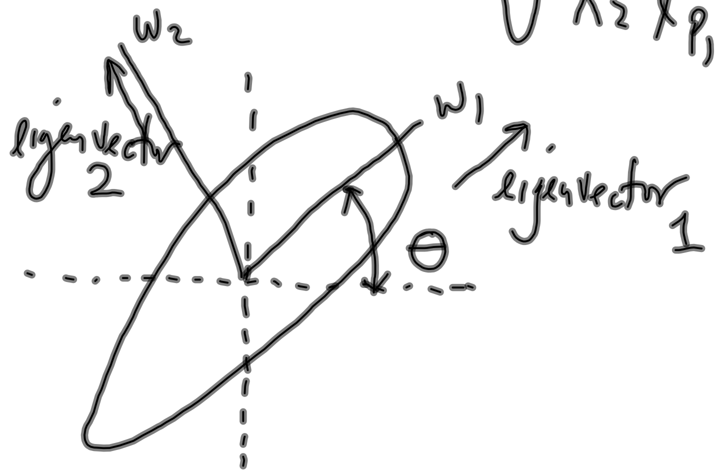
semi major axis $\sqrt{\lambda_1 \chi^2_{p,2}}$

semi minor axis $\sqrt{\lambda_2 \chi^2_{p,2}}$

ordered λ_1, λ_2 and v_1, v_2 by mag.

λ_1 larger

λ_2 smaller



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Specific λ for Σ 2x2

27-2

$\det(A-\lambda I)=0$ A: sym.

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{12} & a_{22}-\lambda \end{vmatrix} = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$(a_{11}-\lambda)(a_{22}-\lambda) - a_{12}^2 = 0$$

$$a_{11}a_{22} + \lambda^2 - a_{11}\lambda - a_{22}\lambda - a_{12}^2 = 0$$

$$\underbrace{1}\lambda^2 \underbrace{-(a_{11}+a_{22})}\lambda + \underbrace{a_{11}a_{22}-a_{12}^2} = 0$$

$$\frac{a_{11}+a_{22}}{2} \pm \frac{\sqrt{(a_{11}+a_{22})^2 - 4(a_{11}a_{22}-a_{12}^2)}}{2}$$

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$$\frac{a_{11}+a_{22}}{2} + \sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22} - 4a_{11}a_{22} + 4a_{12}^2}\;/2$$

27-3

$$\sqrt{\frac{a_{11}^2 + a_{22}^2 - 2a_{11}a_{22} + 4a_{12}^2}{4}}$$

$A : \Sigma$

$$\lambda = \frac{a_{11}+a_{22}}{2} \pm \sqrt{\frac{(a_{11}-a_{22})^2 + 4a_{12}^2}{4}}$$

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\boxed{\lambda = \frac{\sigma_x^2+\sigma_y^2}{2} \pm \left[\frac{(\sigma_x^2-\sigma_y^2)^2}{4} + \sigma_{xy}^2\right]^{1/2}}$$

$=$

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orientation, $\theta = ?$ $\quad \Sigma = R^T D R$ $\quad R: \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

27-4

$\underline{R \Sigma R^T = D}$

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}\begin{bmatrix} c & -s \\ s & c \end{bmatrix} = \begin{pmatrix} \lambda_1 & \boxed{0} \\ 0 & \lambda_0 \end{pmatrix}$$

$$\begin{bmatrix} c\sigma_x^2 + s\sigma_{xy} & c\sigma_{xy} + s\sigma_y^2 \end{bmatrix}\begin{bmatrix} -s \\ c \end{bmatrix} = 0$$

$$-sc\sigma_x^2 - s^2\sigma_{xy} + c^2\sigma_{xy} + sc\sigma_y^2 = 0$$

$$\sigma_{xy}(c^2 - s^2) + \underline{sc(\sigma_y^2 - \sigma_x^2)} = 0$$

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27-5

$$\sigma_{xy}(c^2 - s^2) = sc(\sigma_x^2 - \sigma_y^2)$$

$$\frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} = \frac{2sc}{c^2 - s^2} \quad \begin{matrix} \div c^2 \\ \div c^2 \end{matrix}$$

obscure trig identity

$$\frac{2\, s/c}{1 - s^2/c^2} = \frac{2\tan\theta}{1 - \tan^2\theta} = \tan 2\theta$$

$$\boxed{\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}}, \quad \boxed{2\theta = \tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}\right)}$$

quadrants

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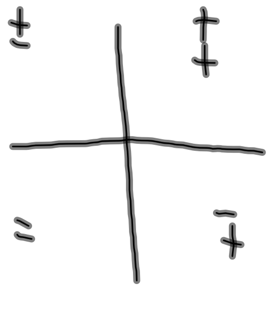
27-6

use $2\theta = \mathrm{atan2}(2\sigma_{xy}, \sigma_x^2 - \sigma_y^2)$



$$\Sigma_{xx} \Rightarrow \lambda_1, \lambda_2, \theta$$

$$\chi^2_{p,2}$$

assumed $\Sigma = \sigma_0^2 Q$, passed global test

if fail global test OR don't do it

$$y = (x - \mu_x)^T \Sigma^{-1} (x - \mu_x) \sim n \cdot F_{n,r}$$

n: 2

r: redundancy d.o.f.

$$\Sigma = \hat{\sigma}_0^2 Q \qquad \hat{\sigma}_0^2 = v^T W v / r$$

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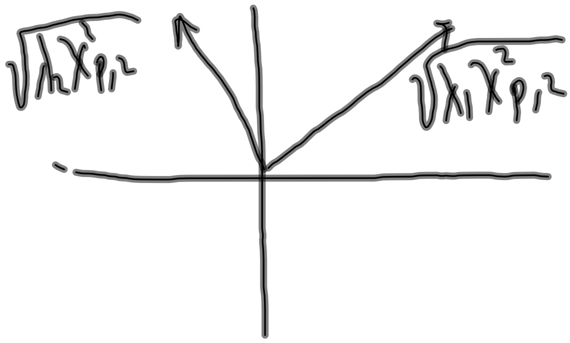
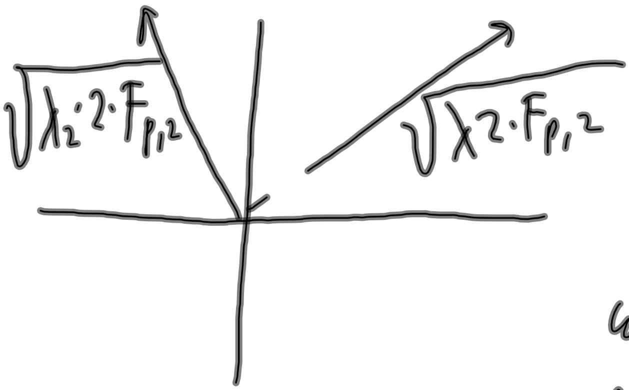
ellipse centered at $\hat{x}, \hat{y}$ 27-7



pass



do not pass

conf. interval z, t

conf region χ^2, F

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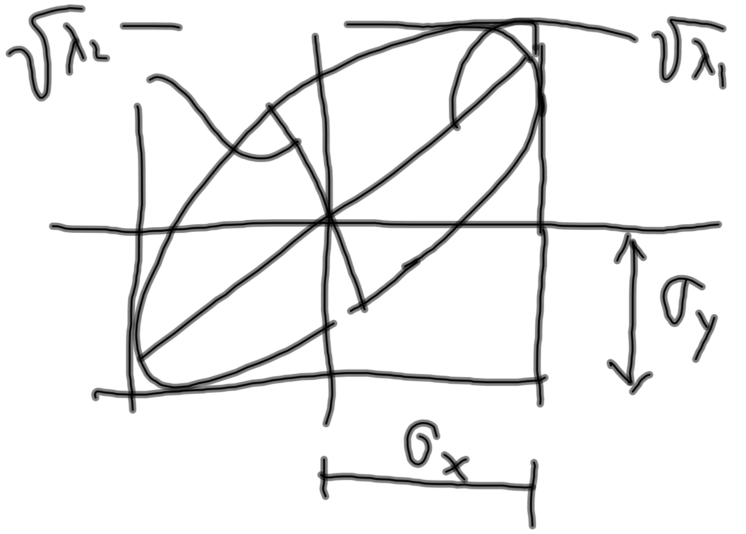
matlab: $[V, D] = \mathrm{eig}(\Sigma)$ 27-8

recommend reorder V & D $d_1 = \lambda_1 =$ larger of two



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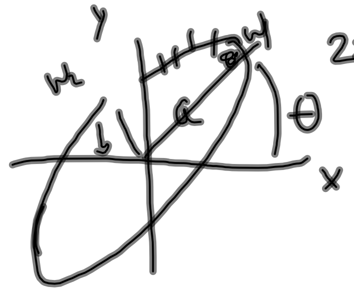
want to plot rotated ellipse :



27-9

th = θ
a = semi major
b = semi minor

$x_0 = a$
$y_0 = 0$
nseg = 50
dalpha = 2*pi/nseg
for i = 1:nseg
    alpha = i*dalpha

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27-10

```
x1 = a * cos(alpha)
y1 = b * sin(alpha)
px0 = cos(th) * x0 - sin(th) * y0
py0 = sin(th) * x0 + cos(th) * y0
px1 = cos(th) * x1 - sin(th) * y1
py1 = sin(th) * x1 + cos(th) * y1
plot([px0 px1], [py0 py1], '-r')
if (i == 1)
    hold on
end
x0 = x1, y0 = y1 end
```

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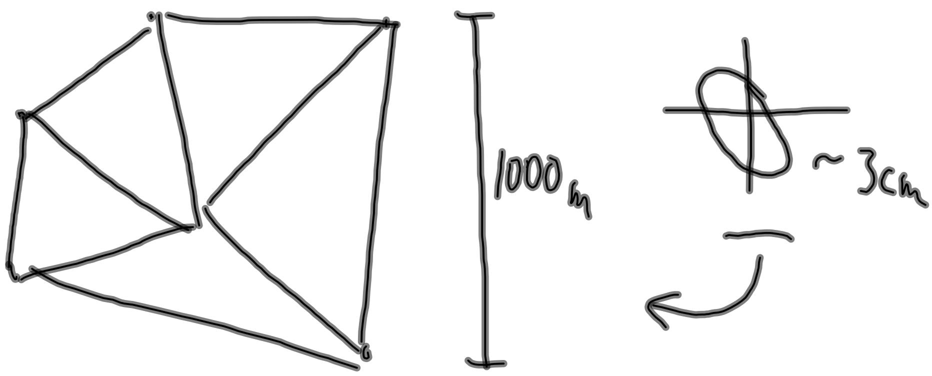
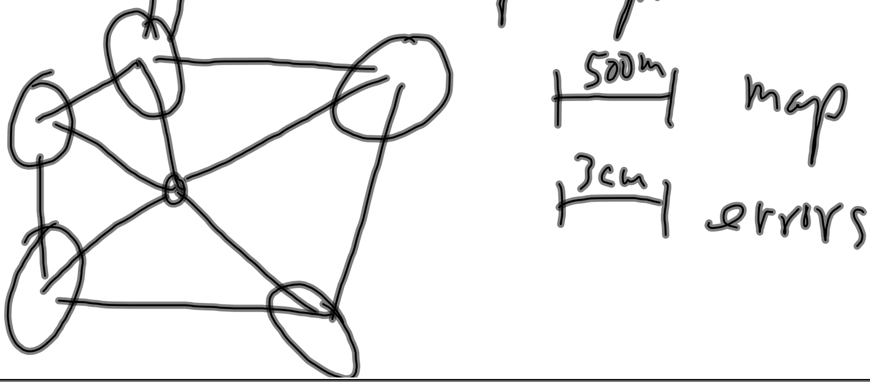
27-11



exaggerate scale of ellipses



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