Circular Errors (compared vs. ell. errors) 28-1

MVN: $f(x) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \cdot \exp\left\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right\}$

n: dim of x

n=2

$\sigma_x = \sigma_y$

$\sigma_{xy} = 0$

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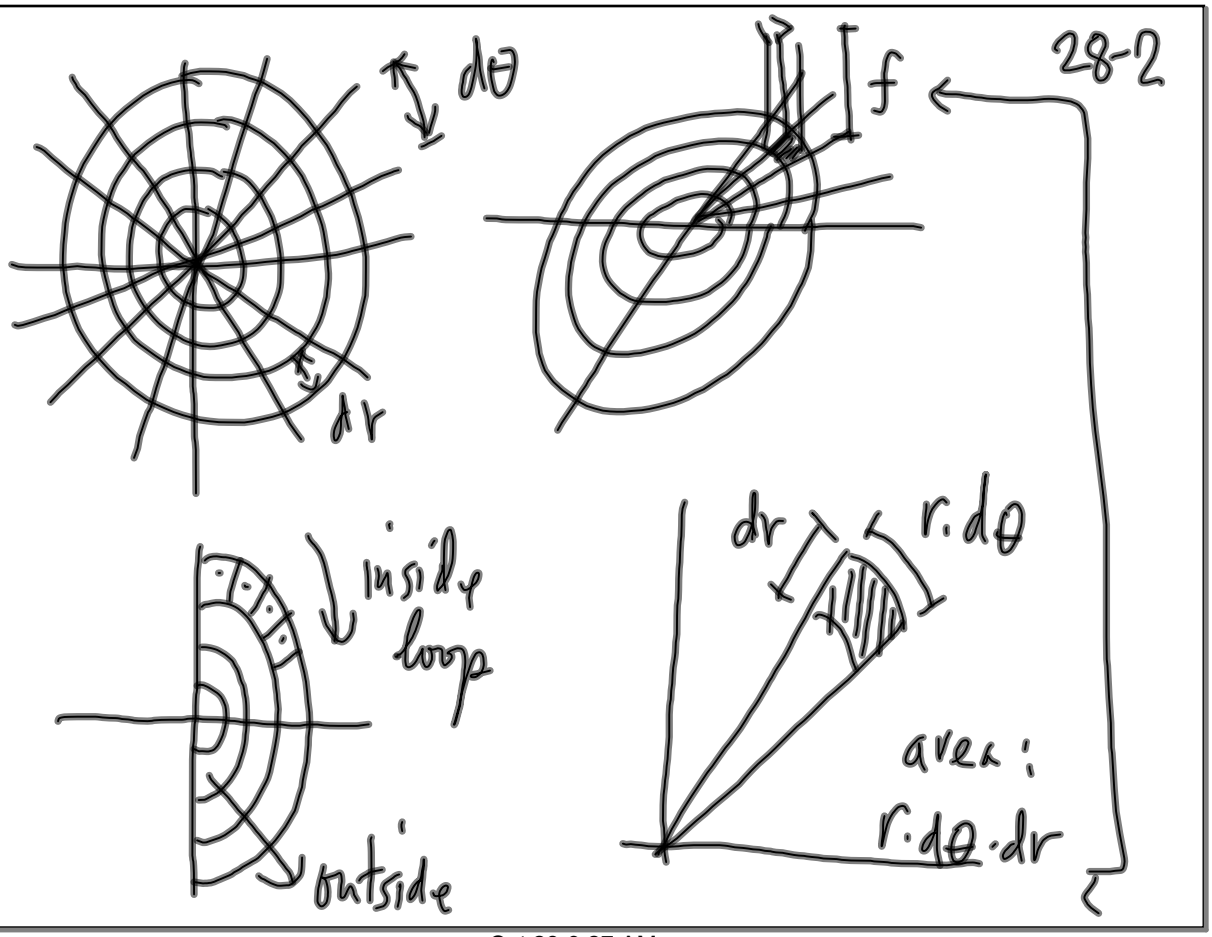
28-2



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28-3

$$\iint_{\text{circle}} f(x,y)\,dx\,dy$$

$x = r\cos\theta$
$y = r\sin\theta$

$$J = \begin{bmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{bmatrix} = \begin{bmatrix} \cos & -r\sin \\ \sin & r\cos \end{bmatrix}$$

$$\int_0^{2\pi}\int_0^R f(r\cos\theta, r\sin\theta)\,|J|\,dr\,d\theta$$

$|J| = r$

$r\cos^2\theta + r\sin^2\theta = r(1) = r$

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28-4

General LS, Mixed Model

observed in both X & Y

5 points, 2 obs per point

$n = 10$

$n_0 = 2 + 5 = 7$

$r = 3$

$n = 5$
$n_0 = 2$
$r = 3$

old $y = mx + b$ linear, $y = \underline{m}\underline{x} + b$ nonlinear

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28-5

$F_1(\hat{\ell}, x) = 0$
$F_2(\hat{\ell}, x) = 0$
$\vdots$
$F_c(\hat{\ell}, x) = 0$

Linearize

$\ell + v = \ell^0 + \delta$
$\delta\ell = \ell - \ell^0 + v$

$$F(\hat{\ell}, x) \approx \underline{F(\ell^0, x^0)} + \underbrace{\frac{\partial F}{\partial \ell}}_{A} \cdot \delta\ell + \underbrace{\frac{\partial F}{\partial x}}_{B} \delta x = 0$$

$A(\ell - \ell^0 + v)$

$$Av + B\delta = \underbrace{-F(\ell^0, x^0) - A(\ell - \ell^0)}_{f}$$

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$\boxed{Av + B\delta = f}$ GLS / mixed model 286

$f = -F(\ell^0, x^0) - \underline{A(\ell - \ell^0)}$

maintain 2 obs vectors:

ℓ : original

ℓ^0 : current

params: obs only 0
ind. obs n_0
GLS $0 < m < n_0$

$\underline{C = r + u}$

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28-7

Minimize $\phi' = \underline{v^T W v} - 2k^T(Av + Bo - f)$

$\frac{\partial \phi'}{\partial v} = 2v^T W - 2k^T A = 0$ $\quad -2(Av+Bo-f)^T k$

$\frac{\partial \phi'}{\partial o} = -2k^T B = 0$

$\frac{\partial \phi'}{\partial k} = -2(Av + Bo - f)^T = 0$

$v^T W - k^T A = 0 \quad \Big| \quad Wv - A^T k = 0 \quad \Big| \quad -Wv + A^T k = 0$

$-k^T B = 0 \quad \Big| \quad -B^T k = 0 \quad \Big| \quad Av + Bo = f$

$-(Av + Bo - f)^T = 0 \quad \Big| \quad -(Av + Bo - f) = 0 \quad \Big| \quad B^T k = 0$

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28-8

Matrix form:

$$\begin{bmatrix} -W & A^T & 0 \\ A & 0 & B \\ 0 & B^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ o \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}$$

full normal equations for GLS

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