

$$\begin{bmatrix} -W & A^T & 0 \\ A & 0 & B \\ 0 & B^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}$$

$$\begin{aligned} -Wv + A^T k &= 0 \\ Au + B\Delta &= f \\ B^T k &= 0 \end{aligned}$$

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$$v = QA^T k$$

$$\begin{aligned} (AQA^T k + B\Delta) &= f \\ k &= (AQA^T)^{-1} (f - B\Delta) \\ B^T W_e (f - B\Delta) &= 0 \\ \underbrace{B^T W_e B}_N \Delta &= \underbrace{B^T W_e f}_t \end{aligned}$$

$$\begin{aligned} \Delta &= N^{-1} t \\ k &= W_e (f - B\Delta) \\ v &= QA^T k \\ x &= x + \Delta \\ l^0 &= l + v \\ \Delta l &= l^0_{\text{new}} - l^0_{\text{prior}} \end{aligned}$$

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$$A: \frac{\partial F}{\partial l}$$

$$B: \frac{\partial F}{\partial x}$$

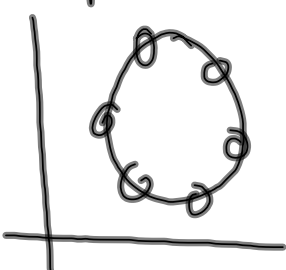
$$f: -F(l^0, x^0) - A(l - l^0)$$

$$\begin{aligned} n &= 11 \times 2 = 22 \\ n_0 &= 3 + 11 = 14 \\ \hline r &= 8 \end{aligned}$$

$$M = 3, \quad x_0, y_0, R$$

$$C = r + M, \quad 8 + 3 = 11$$

Example GLS ²⁹⁻²



x, y observed
#points = 11

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o/o

gls
 $0 < \mu < n_0$

i/o

r

r + \mu

n

$C = 11$, also # of points

$$(x_i - x_0)^2 + (y_i - y_0)^2 = R^2$$

$$F: (x_i - x_0)^2 + (y_i - y_0)^2 - R^2 = 0$$

$$F: \left[\underbrace{(x_i - x_0)^2}_{=} + \underbrace{(y_i - y_0)^2}_{=} \right]^{1/2} - \underbrace{R}_{=} = 0 \quad \checkmark$$

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$$F = \left[(x_i - x_0)^2 + (y_i - y_0)^2 \right]^{1/2} - R = 0$$

$$\frac{\partial F}{\partial x_i} = \frac{1}{2} [\cdot]^{-1/2} \cdot 2(x_i - x_0) = \frac{x_i - x_0}{[\cdot]^{1/2}}$$

$$\frac{\partial F}{\partial y_i} = \frac{1}{2} [\cdot]^{-1/2} \cdot 2(y_i - y_0) = \frac{y_i - y_0}{[\cdot]^{1/2}}$$

$x_1 \ y_1 \ x_2 \ y_2 \ x_3 \ y_3 \ \dots \ x_n \ y_n$

A

(C, n)

$(11, 22)$

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B
 $c_{1,2}$
 $11,3$

$$\frac{\partial F}{\partial x_0} = -\frac{(x_i - x_0)}{(\cdot)^{3/2}}, \quad \frac{\partial F}{\partial y_0} = -\frac{(y_i - y_0)}{(\cdot)^{3/2}}, \quad \frac{\partial F}{\partial R} = -1 \quad 29-5$$

	x_0	y_0	R
point 1	$-\frac{(x_1 - x_0)}{(\cdot)^{3/2}}$	$-\frac{(y_1 - y_0)}{(\cdot)^{3/2}}$	-1
point 2	$-\frac{(x_2 - x_0)}{(\cdot)^{3/2}}$	$-\frac{(y_2 - y_0)}{(\cdot)^{3/2}}$	-1
⋮		⋮	

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$f = -F^0(x^0, y^0) - A \underline{(u-l^0)}$
 $c_{1,1}$
 $11,1$

29-6

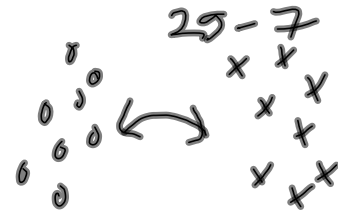
$$f_i = -F_i(x_i^0, y_i^0, x_0^0, y_0^0, R^0) - A_i \begin{pmatrix} x_i - x_0^0 \\ y_i - y_0^0 \end{pmatrix}$$

$1,2$ $2,1$
non zero

3D conformal coordinate transformations
 7-parameter transf. (rigid body transf.)

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$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \checkmark$$



$$F = \begin{matrix} F_1 \\ F_2 \\ F_3 \end{matrix} : \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

observed SAT systems

$$n = 8 \times 6 = 48$$

$$n_0 = 7 + 3 \times 8 = 31$$

$$\underline{r = 17}$$

$$\mu = 7 \quad \lambda, \omega, \phi, k, t_x, t_y, t_z$$

$$C = r + \mu = 17 + 7 = \underline{24}$$

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