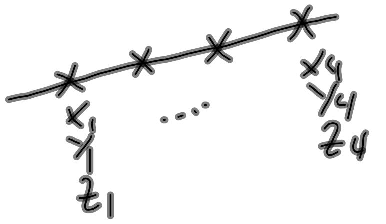
Parameter Constraints Examples:

33-1

1. $X_1 = 1000$, $Y_1 = 500$ } enforce by substitution

2.



$$\frac{Y_3 - Y_1}{X_3 - X_1} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$\frac{Z_3 - Z_1}{X_3 - X_1} = \frac{Z_2 - Z_1}{X_2 - X_1}$$

$$\frac{Y_4 - Y_1}{X_4 - X_1} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$\frac{Z_4 - Z_1}{X_4 - X_1} = \frac{Z_2 - Z_1}{X_2 - X_1}$$

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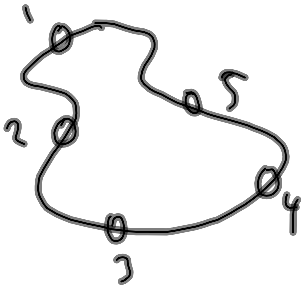
3.



enforce @ shoreline all points have same elevation

33-2

$Z_2 = Z_1$		$Z_2 = Z_1$
$Z_3 = Z_1$	OR	$Z_3 = Z_2$
$Z_4 = Z_1$		$Z_4 = Z_3$
$Z_5 = Z_1$		$Z_5 = Z_4$

4.

$$\left[(X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2\right]^{1/2} = 100$$

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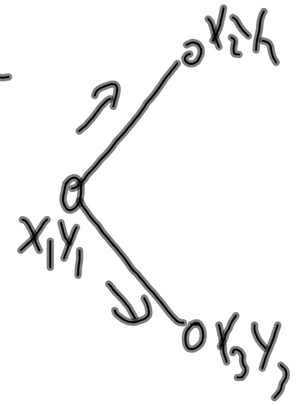
5. 2D network 33-3



$\measuredangle_{312} = 90°$

Dot Product = 0

$$\begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} \cdot \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \end{pmatrix} = 0$$

$$(x_2 - x_1)(x_3 - x_1) + (y_2 - y_1)(y_3 - y_1) = 0$$

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LS implementation of constraints 33-4

general LS : $c = r + u$ ✓

u' : independent

$u - u'$: dependent

$s : u - u'$, $u' = u - s$

$c = r + u'$

$r + u - s$

$$\boxed{c + s = r + u}$$

c: # cond eqn.; s: # constr. eq.; r: redundancy; u: total # params

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33-5

$$\underset{c,n}{A}\ \underset{n,1}{v} + \underset{c,m}{B}\ \underset{m,1}{\delta} = \underset{c,1}{f} \quad \text{constr. eqn's}$$

$$\underset{s,m}{C}\ \underset{m,1}{\delta} = \underset{s,1}{g}$$

<u>elimination</u>: after elim., no constraints, reduces # params

disadvantage: may reorder parameter vector
difficult to standardize

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33-6

$$A v + B_1 \delta_1 + B_2 \delta_2 = f$$

$$C_1 \delta_1 + C_2 \delta_2 = g$$

arrange so C_1: square & nonsingular ✓

✓ $\delta_1 = C_1^{-1}(g - C_2 \delta_2)$

$$A v + B_1 C_1^{-1}(g - C_2\delta_2) + B_2 \delta_2 = f$$

$$A v + -B_1 C_1^{-1} C_2 \delta_2 + B_2 \delta_2 = f - B_1 C_1^{-1} g$$

$$A v + (B_2 - B_1 C_1^{-1} C_2)\delta_2 = f - B_1 C_1^{-1} g$$

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33-7

$$Av + \underbrace{(B_2 - B_1 C_1^{-1} C_2)}_{\bar{B}} \Delta_2 = \underbrace{(f - B_1 C_1^{-1} g)}_{\bar{f}}$$

$$Av + \bar{B}\,\underline{\Delta_2} = \bar{f}$$

now use GLS to solve unconstrained problem

after solve LS problem for Δ_2,

$$\Delta_1 = C_1^{-1}(g - C_2 \Delta_2)$$

$\uparrow$ $\uparrow$

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more general approach for constrained LS 33-8

$$Av + B\Delta = f$$

$$C\Delta = g$$

if Ind. Obs. then $A = I$

$\phi = v^T W v$ minimize

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33-9

$$\phi' = v^T W v - 2k^T(Av + B\delta - f) - 2k_c^T(C\delta - g)$$ ✓scalar

$$\frac{\partial \phi'}{\partial v} = 2v^T W - 2k^T A = 0$$ (row vector of 0's)

$$\frac{\partial \phi'}{\partial \delta} = -2k^T B - 2k_c^T C = 0$$

$$\frac{\partial \phi'}{\partial k} = -2(Av + B\delta - f)^T = 0$$

$$-2(Av + B\delta - f)^T k$$

$$\frac{\partial \phi'}{\partial k_c} = -2(C\delta - g)^T = 0$$

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33-10

$$Wv - A^T k = 0 \qquad -Wv + A^T k = 0$$

$$-B^T k - C^T k_c = 0 \qquad B^T k + C^T k_c = 0$$

$$-(Av + B\delta - f) = 0 \qquad Av + B\delta = f$$

$$-(C\delta - g) = 0 \qquad C\delta = g$$

full N.E., constrained LS prob.

$$\begin{bmatrix} -W & 0 & A^T & 0 \\ 0 & 0 & B^T & C^T \\ A & B & 0 & 0 \\ 0 & C & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ \Delta \\ k \\ k_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ f \\ g \end{bmatrix}$$

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block gauss elimination

$-Wv + A^T k = 0$ ① 33-11

$B^T k + C^T k_c = 0$ ② ✓

$Av + B\delta = f$ ③

$C\delta = g$ ④

$Wv = A^T k$, $\boxed{v = QA^T k}$ sub into ③

$AQA^T k + B\delta = f$

$k = (AQA^T)^{-1}(f - B\delta)$

$\boxed{K = W_e(f - B\delta)}$ – sub into 2

$B^T W_e (f - B\delta) + C^T k_c = 0$

$-\underbrace{B^T W_e B}_{N}\delta + C^T k_c = -\underbrace{B^T W_e f}_{t}$

$\Big\} -N\delta + C^T k_c = -t$

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if N is full rank (nonsingular = invertible) ✓ 33-12

then proceed with elim :

if N is not full rank

$-N\delta + C^T k_c = -t$ ...

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