34-1

$N\Delta = t + C^T k_c$ assuming N fullrank ✓ invertible

$\Delta = N^{-1}t + N^{-1}C^T k_c$ sub into eq ④ $\underline{C}\Delta = \underline{g}$

$C[N^{-1}t + N^{-1}C^T k_c] = g$

$\underline{CN^{-1}C^T} k_c = g - \underline{CN^{-1}t}$

$\boxed{k_c = (CN^{-1}C^T)^{-1}(g - CN^{-1}t)}$ solve this numerically

back subs.

$\Delta = N^{-1}t + N^{-1}C^T k_c$

$k = W_e(f - B\Delta)$

$v = QA^T k$

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34-2

what if N singular?

$$\left.\begin{aligned} -N\Delta + C^T k_c &= -t \\ C\Delta &= g \end{aligned}\right\} \text{ must solve simultaneously}$$

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta \\ k_c \end{bmatrix} = \begin{bmatrix} -t \\ g \end{bmatrix}$$ ✓ partially reduced NE

(1) if N: fullrank $Q_{\Delta\Delta} = N^{-1}(I - C^T(CN^{-1}C^T)^{-1}CN^{-1})$

(2) N NOT fullrank

$Q_{\Delta\Delta} = -\alpha$, $\begin{bmatrix} \alpha & \beta^T \\ \beta & \gamma \end{bmatrix} = \begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix}^{-1}$

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34-3


$$\left[(x_1-x_0)^2+(y_1-y_0)^2\right]^{1/2}-R=0$$
$$\left[(x_2-x_0)^2+(y_2-y_0)^2\right]^{1/2}-R=0$$
$$\vdots$$
$$\left[(x_n-x_0)^2+(y_n-y_0)^2\right]^{1/2}-R=0$$

regular parameters — $x_1, y_1, x_2, y_2, \dots x_n, y_n$

constraint equations

$1^{st}$ approach just append $x_0, y_0, R$ to existing param. vector

$2^{nd}$ separate 2 groups

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34-4

$$Av + B\Delta = f \qquad \Delta\text{: regular params}$$

$$D_1\Delta + D_2\Delta' = h$$

$$\Downarrow$$

$$\begin{bmatrix} -W & A^T & 0 & 0 & 0 \\ A & 0 & B & 0 & 0 \\ 0 & B^T & 0 & D_1^T & 0 \\ 0 & 0 & D_1 & 0 & D_2 \\ 0 & 0 & 0 & D_2^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \\ k_c \\ \Delta' \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ h \\ 0 \end{bmatrix}$$

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$\sigma = 0$   $0 < \sigma < \infty$   $\sigma = \infty$   unified LS   34-5

constant   observations   unknown

how to handle ?

(1) group the "parameter" with observations
solve using observation only method

(2) Keep obs separate from parameters

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Counting $l$, $n \times 1$, $\Sigma, W, Q$ correction $v$   34-6

$X$, $u \times 1$   $\Sigma_{xx}, W_{xx}, Q_{xx}$   $v_x$, $\Delta$

original LS problem

$n$
$n_0$
$r$   $v$

$C = r + u$

new problem   $v_x = \Delta$

# obs. $n + u$
min # $n_0$
redundancy $n + u - n_0$
$(n - n_0) + u$
$r + u$
# cond. eqn $r + u + u$

$n, u, r, c$ refer to original problem

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Linear Case

34-7

$$A(\ell+v)+B(x+\delta)=d$$

$$Av+B\delta=\underbrace{d-A\ell-Bx}_{\bar{f}}$$

$$\hat{x}=x+V_x=x+\delta$$

$$V_x=\delta$$

$$V_x-\delta=0$$

$$Av+B\delta=\bar{f}$$
$$V_x-\delta=0$$

$$;\ \underbrace{\begin{bmatrix}A & 0\\ 0 & I\end{bmatrix}}\underbrace{\begin{bmatrix}v\\ V_x\end{bmatrix}}+\begin{bmatrix}B\\ -I\end{bmatrix}\delta=\begin{bmatrix}\bar{f}\\ 0\end{bmatrix}$$

$$\rightarrow \boxed{\acute{A}\ \acute{v}+\acute{B}\delta=\acute{f}}$$ solve this as G.L.S.

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34-8

$$\begin{bmatrix}v\\ V_x\end{bmatrix}:\begin{bmatrix}W & 0\\ 0 & W_{xx}\end{bmatrix},\ \underbrace{\begin{pmatrix}Q & 0\\ 0 & Q_{xx}\end{pmatrix}=\acute{Q}},\ \begin{pmatrix}\Sigma & 0\\ 0 & \Sigma_{xx}\end{pmatrix}$$

$$\acute{Q}_e=\acute{A}\acute{Q}\acute{A}^T=\begin{bmatrix}A & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}Q & 0\\ 0 & Q_{xx}\end{bmatrix}\begin{pmatrix}A^T & 0\\ 0 & I\end{pmatrix}$$

$$\acute{Q}_e=\begin{bmatrix}AQA^T & 0\\ 0 & Q_{xx}\end{bmatrix},\quad \acute{W}_e=\acute{Q}_e^{-1}$$

$$\acute{N}=\acute{B}^T\acute{W}_e\acute{B}=\begin{bmatrix}B^T & -I\end{bmatrix}\begin{bmatrix}W_e & 0\\ 0 & W_{xx}\end{bmatrix}\begin{bmatrix}B\\ -I\end{bmatrix}$$

$$\acute{N}=\underbrace{B^TW_eB}_{N}+W_{xx}=\boxed{N+W_{xx}}$$

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34-9

$$\dot{t} = \dot{B}^T \dot{W}_\ell \dot{f} = \begin{bmatrix} B^T & -I \end{bmatrix} \begin{pmatrix} W_\ell & 0 \\ 0 & W_{xx} \end{pmatrix} \begin{pmatrix} \bar{f} \\ 0 \end{pmatrix}$$

$$\dot{t} = B^T W_\ell \bar{f}$$

$$\dot{N} \Delta = \dot{t}, \quad \Delta = \dot{N}^{-1} \dot{t}$$

$$\Delta = \left[N + \underline{\underline{W_{xx}}}\right]^{-1} \left(B^T W_\ell \underline{\underline{\bar{f}}}\right)$$

$$\rightarrow \dot{v} = \dot{Q} \dot{A}^T \dot{k} \qquad \begin{pmatrix} v \\ v_x \end{pmatrix} \qquad \Delta = v_x$$

$$\dot{k} = \dot{W}_\ell (\dot{f} - \dot{B} \Delta)$$

$$v = Q A^T W_\ell (\bar{f} - B \Delta)$$

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still linear, now $\Delta$ total parameter value 34-10

$$x + v_x = \Delta \qquad A(\ell + v) + B\Delta = d$$

$$\underline{v_x - \delta = -x} \qquad Av + B\Delta = d - A\ell$$

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