

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -2.0 \\ -2.6 \\ -2.9 \end{bmatrix} \quad W = I_3 \quad 38-1$$

$$v + B \theta = f$$

$$v_1 + \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = (-2.0) \quad v_1 + b_1 \theta = f_1$$

$$v_2 + \begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = (-2.6) \quad v_2 + b_2 \theta = f_2$$

$$v_3 + \begin{bmatrix} -3 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = (-2.9) \quad v_3 + b_3 \theta = f_3$$

$$N_1 = b_1^T b_1 \quad t_1 = b_1^T f_1$$

$$N_2 = b_2^T b_2 \quad t_2 = b_2^T f_2$$

$$N_3 = b_3^T b_3 \quad t_3 = b_3^T f_3$$

$$N = N_1 + N_2 + N_3$$

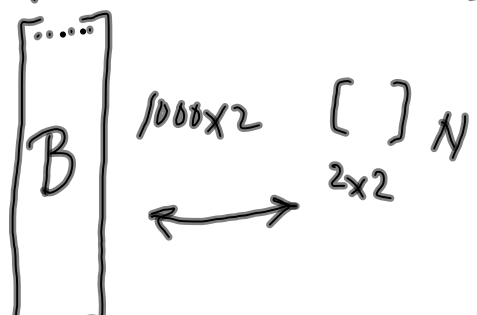
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$$t_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad t_2 = \begin{bmatrix} 5.2 \\ 2.6 \end{bmatrix}, \quad t_3 = \begin{bmatrix} 8.7 \\ 2.9 \end{bmatrix} \quad 38-2$$

$$\left(\sum_{i=1}^n N_i \right) \theta = \left(\sum_{i=1}^n t_i \right)$$

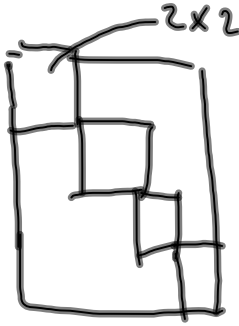
$$\left(\sum_{i=1}^n b_i^T b_i \right) \theta = \sum_{i=1}^n (b_i^T f_i)$$

Suppose 1000 obs. $N = N_1 + N_2 + \dots + N_{1000}$
 $t = t_1 + t_2 + \dots + t_{1000}$



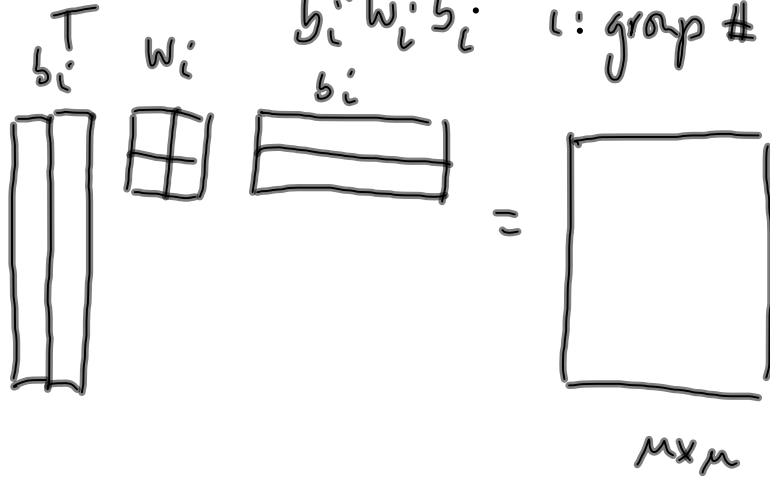
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$W =$ 

block diagonal: from N_i
 i^{th} equation group, from N_i
 2 equations at a time

$b_i^T W_i b_i$ i : group #
 b_i



$m \times m$

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up to now Indirect Obs. model
 $Av + B\delta = f$ Gen. LS
 $Q_e = AQA^T$ diagonal or block diagonal
 $W_e = Q_e^{-1}$ " " "
 $N_i = b_i^T W_e b_i$ i : point number
 $t_i = b_i^T W_e f_i$

LINEAR: solutions same identical
 NONLINEAR: slightly different

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$$N_i = N_{i-1} + B_i^T W_i B_i, \quad N_i: \text{N up to } i^{\text{th}} \text{ iteration}$$

$$t_i = t_{i-1} + B_i^T W_i f_i \quad t_i: "$$

Sherman Morrison Woodbury Schur
Matrix Inversion Lemma

$$(Y \pm UZV)^{-1} = Y^{-1} \mp Y^{-1}U(\bar{Z} \pm VY^{-1}U)^{-1}VY^{-1}$$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ 0 & I_n \end{bmatrix} \quad \text{Solve for } B_{ij} \text{ in terms of } A_{ij}$$

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$$N_i = (N_{i-1} + B_i^T W_i B_i) \quad 38-8$$

$$N_i^{-1} = N_{i-1}^{-1} - N_{i-1}^{-1} B_i^T (Q_i + B_i N_{i-1}^{-1} B_i^T)^{-1} B_i N_{i-1}^{-1}$$

$$t_i = t_{i-1} + B_i^T W_i f_i$$

$$\Delta_i = N_i^{-1} t_i$$

$$\Delta_i = \left[N_{i-1}^{-1} - N_{i-1}^{-1} B_i^T (Q_i + B_i N_{i-1}^{-1} B_i^T)^{-1} B_i N_{i-1}^{-1} \right] (t_{i-1} + B_i^T W_i f_i)$$

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Multiply prior expression

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$$\Delta_i = N_{i-1}^{-1} \cdot t_{i-1}$$

$$+ N_{i-1}^{-1} B_i^T W_i f_i$$

$$- N_{i-1}^{-1} B_i^T (Q_i + \boxed{B_i N_{i-1}^{-1} B_i^T})^{-1} B_i N_{i-1}^{-1} t_{i-1}$$

$$- N_{i-1}^{-1} B_i^T (Q_i + J)^{-1} J W_i f_i$$

$$(A+B)^{-1} = A^{-1} (A^{-1} + B^{-1})^{-1} B^{-1} \quad A, B \text{ matrices exist}$$

$$- N_{i-1}^{-1} B_i^T W_i (W_i + J^{-1})^{-1} J^{-1} J W_i f_i$$

$$(u, v = \mathbb{I}) \quad \text{Apply matrix inversion form.}$$

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$$- N_{i-1}^{-1} B_i^T W_i [Q_i - Q_i (J + Q_i)^{-1} Q_i] W_i f_i$$

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$$- N_{i-1}^{-1} B_i^T W_i f_i + N_{i-1}^{-1} B_i^T (J + Q_i)^{-1} f_i$$

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