Kalman Filter Discrete version, continuous (K-Bucy Filter) 40-1

parameter vector = state vector

$$\begin{bmatrix} X \\ \dot{X} \\ Y \\ \dot{Y} \\ \omega \\ \dot{\omega} \end{bmatrix}_i \quad \begin{bmatrix} X \\ \dot{X} \\ Y \\ \dot{Y} \\ \omega \\ \dot{\omega} \end{bmatrix}_{i-1}$$

condition equations

@ epoch i

$$B_i x_i \approx f_i \ , \ W_0, \ Q_0$$

$\sigma_0^2 = 1$

$$x_{i+1} = \underset{\sim}{\Phi} x_i$$

$\underset{\sim}{\Phi}$ state transition matrix, $-\underset{\sim}{\Phi} x_i + x_{i+1} = 0$

Nov 30-9:24 AM

$$-\Phi_i x_i + x_{i+1} = 0$$

$\uparrow$ pseudo observation 40-2

$$x_{i+1} = \underset{\sim}{\Phi}_i x_i \qquad W_t, \ Q_t$$

"prediction" equation

assume all data is available

Nov 30-9:24 AM

$$\begin{bmatrix} B_1 & 0 & 0 & 0 \\ -\Phi_1 & I & 0 & 0 \\ & B_2 & 0 & 0 \\ & -\Phi_2 & I & 0 \\ & & B_3 & 0 \\ \phi & & -\Phi_3 & I \\ & & & B_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \\ f_2 \\ 0 \\ f_3 \\ 0 \\ f_4 \end{bmatrix}, \quad \begin{bmatrix} W_0 & & & & & & \\ & W_t & & & & & \\ & & W_0 & & & & \\ & & & W_t & & & \\ & & & & W_0 & & \\ & & & & & W_t & \\ & & & & & & W_0 \end{bmatrix}$$

$B^T W B o = B^T W f$

$$B \quad o \quad = \quad f \quad , \quad W$$

40-3

Nov 30-9:24 AM

40-4

$$\left[\begin{array}{c|c|c} B_1^T W_0 B_1 + \Phi_1^T W_t \phi & -\Phi_1^T W_t & 0 \\ \hline -W_t \Phi_1 & W_t + B_2^T W_0 B_2 + \Phi_2^T W_t \Phi_2 & -\Phi_2^T W_t \\ \hline 0 & -W_t \Phi_2 & \\ \hline 0 & 0 & \end{array}\right]$$

Nov 30-9:24 AM

40-5

$$\begin{bmatrix} \ddots & & \phi \\ & \ddots & \\ \phi & & \ddots \end{bmatrix}$$

not diagonal

tridiagonal

<u>block tridiagonal</u>

$$\rightarrow \begin{bmatrix} n_{11} & n_{12} & & & \\ n_{21} & n_{22} & n_{23} & & \phi \\ & n_{32} & n_{33} & n_{34} & \\ & & n_{43} & n_{44} & n_{45} \\ \phi & & & n_{54} & n_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$$

Nov 30-9:24 AM

40-6

$$n_{11} x_1 + \underline{n_{12} x_2} = t_1$$

$$x_1 = n_{11}^{-1}(t_1 - n_{12} x_2) = \underline{n_{11}^{-1} t_1 - n_{11}^{-1} n_{12} x_2}$$

$$n_{21} \underset{\uparrow}{x_1} + n_{22} x_2 + n_{23} x_3 = t_2$$

$$\underline{n_{21}}\left[n_{11}^{-1} t_1 - \underline{n_{11}^{-1} n_{12} x_2}\right] + \underline{n_{22}} x_2 + \underline{n_{23}} x_3 = t_2$$

$$\underline{-n_{21} n_{11}^{-1} n_{12} x_2 + n_{22} x_2} + n_{23} x_3 = t_2 - n_{21} n_{11}^{-1} t_1$$

$$(n_{22} - n_{21} n_{11}^{-1} n_{12}) x_2 + n_{23} x_3 = t_2 - n_{21} n_{11}^{-1} t_1$$

$$n_{22}' x_2 + n_{23} x_3 = t'$$

Nov 30-9:24 AM

40-7

$$\begin{bmatrix} \eta'_{22} & \eta_{23} & 0 & 0 \\ \eta_{32} & \eta_{33} & \eta_{34} & 0 \\ 0 & \eta_{43} & \eta_{44} & \eta_{45} \\ 0 & 0 & \eta_{54} & \eta_{55} \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t'_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$$

tridiagonal, elimination step

<u>still tridiagonal</u>

do this forever, solving numerically for current state vector

Nov 30-9:24 AM

Change notation, <u>Brown + Hwang</u>

40-8

1. prediction step $x_i = \Phi_{i-1} x_{i-1}$

cov matrix Q

implicit $\underbrace{x_i = \Phi_{i-1} x_{i-1} + w_{i-1}}$

$$x_i = \begin{bmatrix} \Phi_{i-1} & I \end{bmatrix} \begin{bmatrix} x_{i-1} \\ w_{i-1} \end{bmatrix} \qquad \begin{matrix} P_{i-1} \\ Q \end{matrix}$$

$P_i^- = \begin{bmatrix} \Phi_{i-1} & I \end{bmatrix} \ldots$ (P is cov. matrix of x)

Nov 30-9:24 AM