41-1

1. $X_i = \underline{\Phi}_{i-1} X_{i-1} + w_{i-1}$ cov: Q

(prediction)

$$X_i = \begin{bmatrix} \underline{\Phi}_{i-1} & I \end{bmatrix} \begin{bmatrix} x_{i-1} \\ w_{i-1} \end{bmatrix} \rightsquigarrow \Sigma: \begin{bmatrix} P_{i-1} & 0 \\ 0 & Q \end{bmatrix}$$

$$\Sigma_{x_i} = P_i^- = \begin{bmatrix} \underline{\Phi}_{i-1} & I \end{bmatrix} \begin{bmatrix} P_{i-1} & 0 \\ 0 & Q \end{bmatrix} \begin{bmatrix} \underline{\Phi}_{i-1} \\ I \end{bmatrix}$$

$$\boxed{P_i^- = \underline{\Phi}_{i-1} P_{i-1} \underline{\Phi}_{i-1}^T + Q}$$

$$\boxed{X_i^- = \underline{\Phi}_{i-1} X_{i-1}}$$

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2. observations $z_i = H x_i + v_i$ cov: R

$Hx = z$ (R)

$x = x^-$ (Q)

$$\rightarrow \begin{bmatrix} H \\ I \end{bmatrix} (x) \approx \begin{bmatrix} z \\ x^- \end{bmatrix}, \text{weight} \begin{bmatrix} R^{-1} & 0 \\ 0 & (P^-)^{-1} \end{bmatrix}$$

dropped subscripts $\underline{B}\, 0 \approx \underline{f}$ W

X, x^-, P, P^-

$B^T W B 0 = B^T W f$

$$\begin{bmatrix} H^T & I \end{bmatrix} \begin{bmatrix} R^{-1} & 0 \\ 0 & (P^-)^{-1} \end{bmatrix} \begin{bmatrix} H \\ I \end{bmatrix} (x) = \begin{bmatrix} H^T & I \end{bmatrix} \begin{bmatrix} R^{-1} & 0 \\ 0 & (P^-)^{-1} \end{bmatrix} \begin{bmatrix} z \\ x^- \end{bmatrix}$$

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$$\rightarrow X = \left[H^T R^{-1} H + (P^-)^{-1}\right]^{-1} \left(H^T R^{-1} z + (P^-)^{-1} x^-\right)$$

$$\left\{ (B^T w B + w_{xx}) \quad \left(B^T w f + \overset{(P_m)}{w_{xx}} x^0 \right) \right\}$$

$$\rightarrow \underline{P} = \left[H^T R^{-1} H + (P^-)^{-1} \right]^{-1} \quad v$$

$$K = P^- H^T (R + H P^- H^T)^{-1}$$

Kalman Gain Matrix

$$P = (I - KH) P^-$$

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41-4

$$X = X^- + K(z - H x^-)$$

$z_1, z_2, z_3, \ldots$

$$K = P^{-1} H^T (H P^- H^T + R)^{-1}$$

$$X = X^- + K(z - H x^-)$$

$$P = (\underline{I}_4 - KH) P^-$$

$$x^- = \underline{\Phi} x$$

$$P^- = \underline{\Phi} P \underline{\Phi}^T + Q$$

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41-5

Φ : $X_i = \Phi_{i-1} X_{i-1}$

simple example : assume constant accel.

- $\ddot{x}_{i+1} = \ddot{x}_i$

integrate (integration constant)

- $\dot{x}_{i+1} = \ddot{x}_i t + \dot{x}_i$

integrate

- $x_{i+1} = \ddot{x}_i \frac{t^2}{2} + \dot{x}_i t + x_i$

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41-6

$$\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}_{i+1} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}_i + \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

noise

assume constant velocity

$$\begin{pmatrix} x \\ \dot{x} \end{pmatrix}_{i+1} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix}_i$$

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