CE 597 HW1 Solution

1.

(const) x	(obs) y
1	3.50
4	4.50
7	5.70
9	6.80

(equal weights)

$n = 4$, $n_0 = 2$, $r = 2$

Indirect Observations, $u = n_0 = 2$ parameters m, b slope and intercept

$\hat{l} = mx + b$

$l + v = mx + b$

$v = mx + b - l$

$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2$

$$\Phi = (m\cdot 1 + b - 3.5)^2 + (m\cdot 4 + b - 4.5)^2 + (m\cdot 7 + b - 5.7)^2 + (m\cdot 9 + b - 6.8)^2$$

$$\frac{\partial \Phi}{\partial m} = \tfrac{1}{2}(m + b - 3.5) + \tfrac{1}{2}(m\cdot 4 + b - 4.5)4 + \tfrac{1}{2}(m\cdot 7 + b - 5.7)7 + \tfrac{1}{2}(m\cdot 9 + b - 6.8)9 = 0$$

$$\frac{\partial \Phi}{\partial b} = \tfrac{1}{2}(m\cdot 1 + b - 3.5) + \tfrac{1}{2}(m\cdot 4 + b - 4.5) + \tfrac{1}{2}(m\cdot 7 + b - 5.7) + \tfrac{1}{2}(m\cdot 9 + b - 6.8) = 0$$

$$m + 16m + 49m + 81m + b + 4b + 7b + 9b = 122.6$$

$$m + 4m + 7m + 9m + b + b + b + b = 20.5$$

$$\left.\begin{aligned} 147m + 21b &= 122.6 \\ 21m + 4b &= 20.5 \end{aligned}\right\} \quad \begin{bmatrix} 147 & 21 \\ 21 & 4 \end{bmatrix}\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 122.6 \\ 20.5 \end{bmatrix}$$ solve by matlab

$$\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 0.4075 \\ 2.9857 \end{bmatrix}$$

$v_1 = .4075\cdot 1 + 2.9857 - 3.50 = -.1068$

$v_2 = .4075\cdot 4 + 2.9857 - 4.50 = .1157$

$v_3 = .4075\cdot 7 + 2.9857 - 5.70 = .1382$

$v_4 = .4075\cdot 9 + 2.9857 - 6.80 = -.1468$

$\hat{l}_1 = 3.3932$

$\hat{l}_2 = 4.6157$

$\hat{l}_3 = 5.8382$

$\hat{l}_4 = 6.6532$

no prior info on uncertainty of y, so we cannot really evaluate the v's.

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for 2 & 3: build a grid of Φ for $a = 25 \ldots 75$, $b = 50 \ldots 100$

for each a_i & b_j, $v_k = \frac{a_i}{125}x_k + \frac{b_j}{25} - y_k$, $k = 1 \ldots 4$

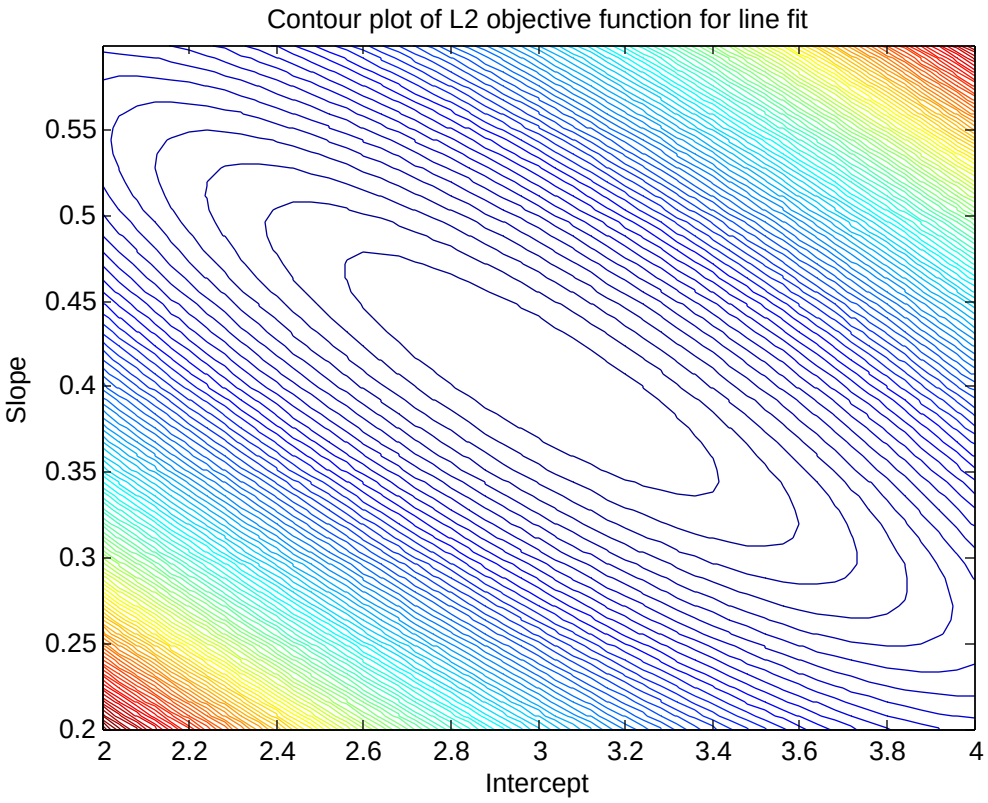
for (2) $\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2$

for (3) $\Phi = |v_1| + |v_2| + |v_3| + |v_4|$

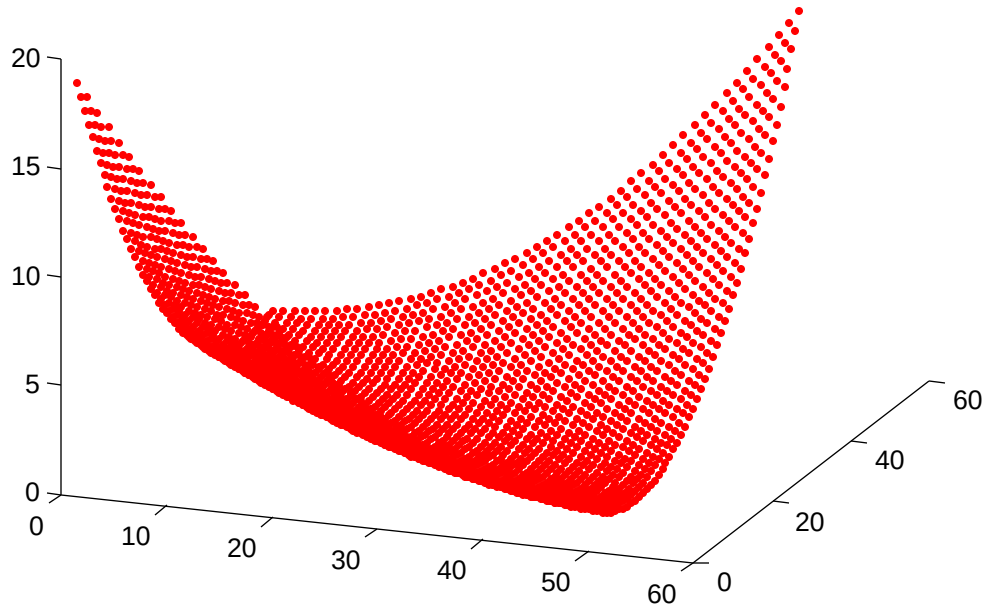
If you use search to solve <u>conventional LS problem</u>, you should repeatedly search and subdivide for greater accuracy.

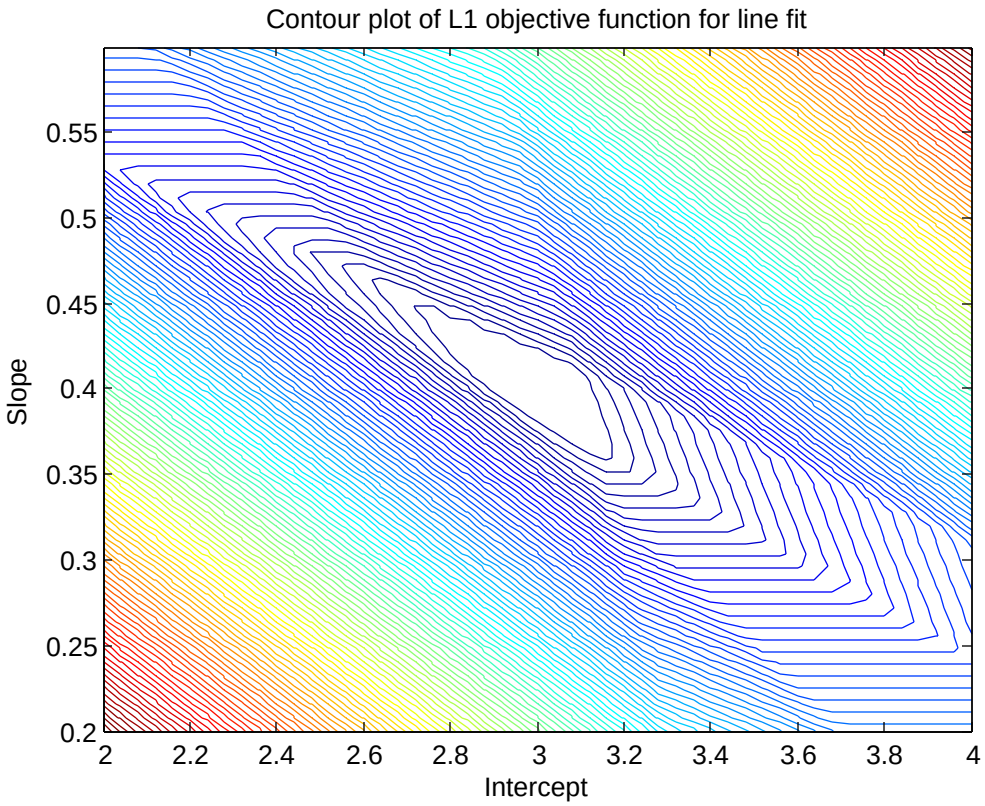
If you are solving an <u>integer LS problem</u>, then stop at integer solution

```
hw1_sol
line fit solution by conventional least squares
del (slope & intercept) =
    0.4075
    2.9857
v =
   -0.1068
    0.1156
    0.1381
   -0.1469
PHI =
    0.0654

L2 solution by integer search
msrch =
    0.4080
bsrch =
     3
svv =
   -0.0920
    0.1320
    0.1560
   -0.1280
minobj =
    0.0666
a(imin) =
    51
b(jmin) =
    75

L1 solution by integer search
msrch =
    0.3760
bsrch =
    3.0800
svv =
   -0.0440
    0.0840
    0.0120
   -0.3360
minobj =
    0.4760
a(imin) =
    47
b(jmin) =
    77
diary off
```

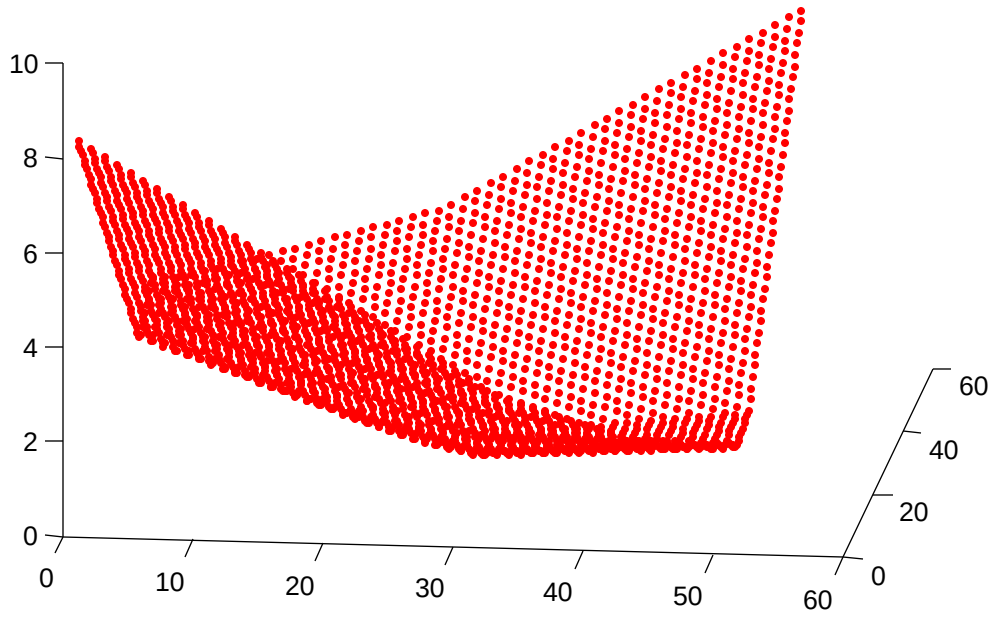

Contour plot of L2 objective function for line fit
Slope
Intercept
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4

3D plot L2 objective function




Contour plot of L1 objective function for line fit
Slope
Intercept
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4

3D plot L1 objective function



```
% hw1_sol.m  27-aug-2013
% solve line fit 3 ways
% yhat = m*x + b
% v - m*x - b = -y

x=[1;4;7;9];
y=[3.50;4.50;5.70;6.80];
[n,ndum]=size(y);

B=[-x(1) -1;
   -x(2) -1;
   -x(3) -1;
   -x(4) -1];
f=[-y(1);-y(2);-y(3);-y(4)];
N=B'*B;
t=B'*f;
del=inv(N)*t;
del
v=f-B*del
PHI=v'*v

% now search for the integer solution to same problem
% with scaled parameters

a=25:75;
b=50:100;
[mm,nn]=size(a);
n1=nn;
[mm,nn]=size(b);
n2=nn;

minobj=1.0e+12;
grd=zeros(n1,n2);
for i=1:n1
  for j=1:n2
    mm=a(i)/125;
    bb=b(j)/25;
    yhat=mm*x + bb;
    v=yhat - y;
    objf=v'*v;
    grd(i,j)=objf;
    if(objf < minobj)
      mni=i;
      mnj=j;
      minobj=objf;
      svv=v;
      end
    end
  end

disp('L2 solution by integer search');
msrch=a(mni)/125
bsrch=b(mnj)/25
svv
minobj
a(mni)
b(mnj)

ap=a/125;
bp=b/25;
contour(bp,ap,grd,100);
% contour(b,a,grd,100);
xlabel('Intercept');
ylabel('Slope');
title('Contour plot of L2 objective function for line fit');

% make 3D plot of L2 objective function
nn=n1*n2;
xx=zeros(nn,1);
yy=zeros(nn,1);
zz=zeros(nn,1);
k=0;
for i=1:n1
  for j=1:n2
    k=k+1;
    xx(k)=j;
    yy(k)=i;
    zz(k)=grd(i,j);
    end
```

```
  end
figure
plot3(xx,yy,zz,'.r');
title('3D plot L2 objective function');

% now search for the integer solution to the L1 problem
% with scaled parameters

a=25:75;
b=50:100;
[mm,nn]=size(a);
n1=nn;
[mm,nn]=size(b);
n2=nn;

minobj=1.0e+12;
grd=zeros(n1,n2);
for i=1:n1
  for j=1:n2
    mm=a(i)/125;
    bb=b(j)/25;
    yhat=mm*x + bb;
    v=yhat - y;
    % make L1 objective function
    objf=0;
    for k=1:n
      objf=objf + abs(v(k));
      end
    grd(i,j)=objf;
    if(objf < minobj)
      mni=i;
      mnj=j;
      minobj=objf;
      svv=v;
      end
    end
  end

disp('L1 solution by integer search');
msrch=a(mni)/125
bsrch=b(mnj)/25
svv
minobj
a(mni)
b(mnj)

ap=a/125;
bp=b/25;
% make a new figure
figure
contour(bp,ap,grd,100);
% contour(b,a,grd,100);
xlabel('Intercept');
ylabel('Slope');
title('Contour plot of L1 objective function for line fit');

% make 3D plot of L1 objective function
nn=n1*n2;
xx=zeros(nn,1);
yy=zeros(nn,1);
zz=zeros(nn,1);
k=0;
for i=1:n1
  for j=1:n2
    k=k+1;
    xx(k)=j;
    yy(k)=i;
    zz(k)=grd(i,j);
    end
  end
figure
plot3(xx,yy,zz,'.r');
title('3D plot L1 objective function');
```