```
sph02
n =
        1023
n0 =
    686
r =
    337
keep_going =
     1
iter =
     1
ans =
  Columns 1 through 3
       0.00375729684859144       -0.0159687429771287        0.0101736827247065
  Column 4
       -0.0276730388451996
iter =
     2
ans =
  Columns 1 through 3
     -0.000365746146370238      8.72845926982572e-05      -9.4165549102458e-05
  Column 4
      0.000314721675637143
iter =
     3
ans =
  Columns 1 through 3
     -6.80321876721359e-07     -2.69856687137055e-06      8.28905520375208e-07
  Column 4
     -1.28339115602296e-06
iter =
     4
ans =
  Columns 1 through 3
     -5.67214489018159e-08      1.51086894036624e-08      -4.8672242017811e-09
  Column 4
     -2.72682488122806e-08
iter =
     5
ans =
  Columns 1 through 3
     -1.80250233595711e-10     -1.64591037117247e-09      3.15178651750384e-10
  Column 4
     -7.42040268437066e-10
we have converged
p =
          3.05339081347865
          1.91411585651148
         -0.539919658470921
          0.272640371428992
vTv =
       0.00063126853449787
sig0sqr_hat =
      1.87320039910347e-06
TS =
          25.2507413799148
cv1 =
          273.885718342905
cv2 =
          407.622706245951
diary off
```

```
% sph02.m  1-nov-12

% update the observations
% LS fit of sphere to point cloud
% use sequential formation of N.E.
% code from sphere.m, data1, 2009

% initial approximations

degrad=180/pi;
xc=3.05;
yc=1.93;
zc=-0.55;
R=0.30;
p=[xc;yc;zc;R];
% summary stats
vTv=0.0;
sumvxsqr=0;
sumvysqr=0;
sumvzsqr=0;
maxvx=0;
maxvy=0;
maxvz=0;
load sphere_m.xyz
% XYZ original observations
X=sphere_m(:,1);
Y=sphere_m(:,2);
Z=sphere_m(:,3);
[np,nn]=size(X);
% X0Y0Z0 updated observations
X0=X;
Y0=Y;
Z0=Z;
n=np*3
n0=4 + np*2
r=n-n0
sigi=0.005; % 1 mm
sig0=0.005;
Wi=eye(3);
Qi=eye(3);
% so we can leave these out of expressions

keep_going=1
iter=0;
while(keep_going == 1)
  iter=iter+1
  Ai=zeros(1,3);
  Bi=zeros(1,4);
  N=zeros(4,4);
  Ni=zeros(4,4);
  t=zeros(4,1);
  ti=zeros(4,1);
  % get current parameters from the parameter vector
  xc=p(1);
  yc=p(2);
  zc=p(3);
  R=p(4);
  for i=1:np
    XX=X0(i);
    YY=Y0(i);
    ZZ=Z0(i);
    Fi=R - sqrt((XX-xc)^2 + (YY-yc)^2 + (ZZ-zc)^2);
    d=sqrt((XX-xc)^2 + (YY-yc)^2 + (ZZ-zc)^2);
    dFdX=-(XX-xc)/d;
    dFdY=-(YY-yc)/d;
    dFdZ=-(ZZ-zc)/d;
    dFdxc=-dFdX;
    dFdyc=-dFdY;
    dFdzc=-dFdZ;
    dFdR=1;
    Ai=[dFdX dFdY dFdZ];
    Bi=[dFdxc dFdyc dFdzc dFdR];
    fi=-Fi - Ai*[X(i)-XX;Y(i)-YY;Z(i)-ZZ];
    Qei=Ai*Ai';
    Wei=1/Qei;
    Ni=Bi'*Wei*Bi;
    ti=Bi'*Wei*fi;
    N=N + Ni;
    t=t + ti;
    end
  del=inv(N)*t;
  del'
  % don't update p yet

  % now go through points again to compute residuals to update obs
  % use same values as in prior loop except delta !
  vTv=0;
  for i=1:np
    XX=X0(i);
    YY=Y0(i);
    ZZ=Z0(i);
```

```
    Fi=R - sqrt((XX-xc)^2 + (YY-yc)^2 + (ZZ-zc)^2);
    d=sqrt((XX-xc)^2 + (YY-yc)^2 + (ZZ-zc)^2);
    dFdX=-(XX-xc)/d;
    dFdY=-(YY-yc)/d;
    dFdZ=-(ZZ-zc)/d;
    dFdxc=-dFdX;
    dFdyc=-dFdY;
    dFdzc=-dFdZ;
    dFdR=1;
    Ai=[dFdX dFdY dFdZ];
    Bi=[dFdxc dFdyc dFdzc dFdR];
    fi=-Fi - Ai*[X(i)-XX;Y(i)-YY;Z(i)-ZZ];
    Qei=Ai*Ai';
    Wei=1/Qei;
    % v=Q*A'*We*(f - B*del)
    v=Ai'*Wei*(fi - Bi*del);
    vTv=vTv + v'*v;
    X0(i)=X(i)+v(1);
    Y0(i)=Y(i)+v(2);
    Z0(i)=Z(i)+v(3);
    end

  p=p+del;
  if((all(abs(del) < 1e-08)) && (iter < 10))
    disp('we have converged');
    keep_going=0;
  else
    if(iter >= 10)
      keep_going=0;
      end
    end
  end % while(keep_going==1)

p
vTv
sig0sqr_hat=vTv/r
TS=vTv/sig0^2
alpha=0.01;
cv1=icdf('chi2',alpha/2,r)
cv2=icdf('chi2',1-alpha/2,r)
```

```
lin2
keep_going =
     1
iter =
     1
ans =
  Columns 1 through 3
        0.0116162495371994         -108.294266182525          -0.1149587067839
  Column 4
         115.466273954279
iter =
     2
ans =
  Columns 1 through 3
       0.000262657061540296        -0.014647508798513       -0.00393419484380924
  Column 4
        0.0914044602046753
iter =
     3
ans =
  Columns 1 through 3
       1.68458956879531e-05     -0.000939439449530636       0.00010477061123434
  Column 4
      -0.00243417053433734
iter =
     4
ans =
  Columns 1 through 3
      -1.9760545224039e-08      1.10197973783886e-06     -1.34267086103146e-07
  Column 4
       3.11947197213341e-06
iter =
     5
ans =
  Columns 1 through 3
       4.33699299126954e-09     -2.41859642777737e-07      2.85293351612974e-08
  Column 4
      -6.6283155444346e-07
iter =
     6
ans =
  Columns 1 through 3
      -5.26214544053154e-12      2.93453176620865e-10     -3.46158858671048e-11
  Column 4
       8.04236475998675e-10
we have converged
parameters
p =
          0.389895737065613
           1.69014772964097
          -2.56478823678884
           114.555246701394
residuals
v =
         0.0395946441482597
         -0.101551877549926
          0.213295843841192
         -0.547058671232455
         -0.252890487989451
           0.64861054878238
          0.483848175306389
          0.188650340941483
         0.0395804917603532
         0.0154322650085306
         -0.523428667066743
         -0.204082605950014
diary off
```

```
% lin2.m  4-dec-2013
% two lines with perp. constraint (hw6)

m1=0.378;
b1=110;
m2=-2.446;
b2=-1;
p=[m1;b1;m2;b2];

x1=[20.5;51.5;95.3];
y1=[9.8;22.4;38.1];
x2=[11.2;22.2;36.3];
y2=[84.4;57.5;23.0];
x10=x1;
y10=y1;
x20=x2;
y20=y2;

n=12;
n0=9; % 3+3+3
r=n-n0; % 3
u=4;
c=6;
s=1;
% c+s = 7
% r+u = 7  check

keep_going=1
iter=0;
while(keep_going == 1)
  iter=iter+1
  A=zeros(c,n);
  B=zeros(c,u);
  C=zeros(s,u);
  f=zeros(c,1);
  % get current parameters from the parameter vector
  m1=p(1);
  b1=p(2);
  m2=p(3);
  b2=p(4);

  % line 1
  rowc=1;
  colc=1;
  for i=1:3
    dFdx=-m1;
    dFdy=1;
    dFdm1=-x10(i);
    dFdb1=-1;
    Ai=[dFdx dFdy];
    Bi=[dFdm1 dFdb1 0 0];
    F=y10(i) - m1*x10(i) - b1;
    fi=-F - Ai*[x1(i)-x10(i);y1(i)-y10(i)];
    A(rowc,colc:colc+1)=Ai;
    B(rowc,:)=Bi;
    f(rowc)=fi;
    rowc=rowc+1;
    colc=colc+2;
    end
  % line 2
  for i=1:3
    dFdx=-m2;
    dFdy=1;
    dFdm2=-x20(i);
    dFdb2=-1;
    Ai=[dFdx dFdy];
    Bi=[0 0 dFdm2 dFdb2];
    F=y20(i) - m2*x20(i) - b2;
    fi=-F - Ai*[x2(i)-x20(i);y2(i)-y20(i)];
    A(rowc,colc:colc+1)=Ai;
    B(rowc,:)=Bi;
    f(rowc)=fi;
    rowc=rowc+1;
    colc=colc+2;
    end

  % constraint
  dFcdm1=1;
  dFcdm2=-1/m2^2;
  C=[dFcdm1 0 dFcdm2 0];
  Fc=m1 + 1/m2;
  g=-Fc;

  Qe=A*A';
  We=inv(Qe);
  N=B'*We*B;
  t=B'*We*f;

  M=[-N C';C 0];
  tc=[-t;g];
  edel=inv(M)*tc;
```

```
  del=edel(1:4);
  del'
  k=We*(f-B*del);
  v=A'*k;
  % line 1 obs
  for i=1:3
    idx=(i-1)*2 + 1;
    x10(i)=x1(i) + v(idx);
    y10(i)=y1(i) + v(idx+1);
    end
  % line 2 obs
  for i=1:3
    idx=6 + (i-1)*2 + 1;
    x20(i)=x2(i) + v(idx);
    y20(i)=y2(i) + v(idx+1);
    end

  p=p+del;
  if((all(abs(del) < 1e-08)) && (iter < 10))
    disp('we have converged');
    keep_going=0;
  else
    if(iter >= 10)
      keep_going=0;
      end
    end
  end % while(keep_going==1)

disp('parameters');
p
disp('residuals');
v
```