

# Adj. Geospa. Obs. HW1 Solution

1,  $n=7$  Indirect Observations, parameters  $a_0, a_1, a_2$   $\hat{y}_i = a_0 + a_1 X_i + a_2 X_i^2$

$n_0=3$   
 $r=4$

$v_i - a_0 - a_1 X_i - a_2 X_i^2 = -y_i$

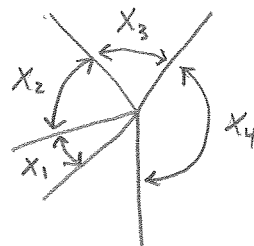
$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & -1 \\ -1 & -2 & -4 \\ -1 & -3 & -9 \\ -1 & -4 & -16 \\ -1 & -5 & -25 \\ -1 & -6 & -36 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -3.244 \\ -4.142 \\ -4.746 \\ -4.959 \\ -4.652 \\ -4.271 \\ -3.216 \end{bmatrix}$$

$V + B \Delta = f$

$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3.2308 \\ 1.1264 \\ -0.1873 \end{bmatrix}$   $\Rightarrow V = \begin{bmatrix} .0132 \\ .0280 \\ -.0014 \\ -.0342 \\ .0884 \\ -.0896 \\ .0320 \end{bmatrix}$

2(a),  $n=8$  Indirect Observations, parameters  $x_1, x_2, x_3, x_4$

$n_0=4$   
 $r=4$



$\hat{\alpha}_1 = x_3$   $v_1 - x_3 = -\alpha_1$   
 $\hat{\alpha}_2 = x_2$   $v_2 - x_2 = -\alpha_2$   
 $\hat{\alpha}_3 = x_1$   $v_3 - x_1 = -\alpha_3$   
 $\hat{\alpha}_4 = x_4$   $v_4 - x_4 = -\alpha_4$   
 $\hat{\alpha}_5 = 360 - x_2 - x_3 - x_4$   $v_5 + x_2 + x_3 + x_4 = 360 - \alpha_5$   
 $\hat{\alpha}_6 = x_2 + x_3$   $v_6 - x_2 - x_3 = -\alpha_6$   
 $\hat{\alpha}_7 = x_4$   $v_7 - x_4 = -\alpha_7$   
 $\hat{\alpha}_8 = x_1 + x_2$   $v_8 - x_1 - x_2 = -\alpha_8$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -64.92 \\ -50.13 \\ -24.82 \\ -160.02 \\ -85.02 \\ -114.93 \\ -159.98 \\ -74.97 \end{bmatrix}$$

$V + B \Delta = f$

$V = \begin{bmatrix} -.0558 \\ -.0305 \\ .0253 \\ -.0095 \\ .0058 \\ .0337 \\ .0305 \\ -.0253 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 24.8453 \\ 50.0995 \\ 64.8642 \\ 160.0105 \end{bmatrix}$

$W = I_8$

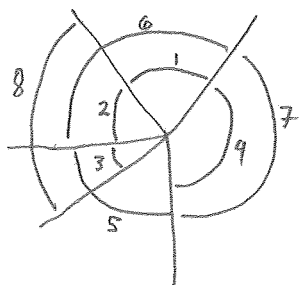
2(b),  $n=8$  Observations Only

$n_0=4$   
 $r=4$

$C=r=4$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 360 \end{bmatrix} - A \begin{bmatrix} 64.92 \\ 50.13 \\ 24.82 \\ 160.02 \\ 85.02 \\ 114.93 \\ 159.98 \\ 74.97 \end{bmatrix}$$

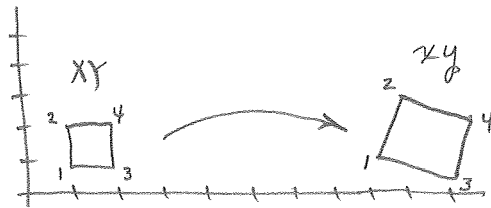
$V = d - A \Delta = f$



$\hat{\alpha}_4 = \hat{\alpha}_2$   
 $\hat{\alpha}_6 = \hat{\alpha}_1 + \hat{\alpha}_2$   
 $\hat{\alpha}_8 = \hat{\alpha}_2 + \hat{\alpha}_3$   
 $\hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_4 + \hat{\alpha}_5 = 360$

$V =$  same as previous problem

$$3. \quad \begin{aligned} n &= 8 \\ n_0 &= 4 \\ \hline r &= 4 \end{aligned}$$



$$\begin{aligned} x &= aX + bY + c \\ y &= -bX + aY + d \end{aligned}$$

parameter  $a, b, c, d$

$$\begin{aligned} V_x - aX - bY - c &= -x \\ V_y + bX - aY - d &= -y \end{aligned}$$

$$\begin{bmatrix} V_{x1} \\ V_{x1} \\ V_{x2} \\ V_{y2} \\ V_{x3} \\ V_{y3} \\ V_{x4} \\ V_{y4} \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & -2 & -1 & 0 \\ -2 & 1 & 0 & -1 \\ -2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -2 & -2 & -1 & 0 \\ -2 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -7.2990 \\ -0.6960 \\ -7.6120 \\ -2.7210 \\ -9.3000 \\ -0.3870 \\ -9.6220 \\ -2.3820 \end{bmatrix} \quad \text{?} \quad V = \begin{bmatrix} .0041 \\ .0090 \\ .0028 \\ -.0102 \\ .0007 \\ -.0019 \\ -.0014 \\ .0089 \end{bmatrix}$$

$$V + B \Delta = f$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2.0058 \\ .3199 \\ 4.9692 \\ -.9810 \end{bmatrix}$$

$$\begin{aligned} \sigma_0^2 &= (.02)^2 \\ W_i &= \frac{(.02)^2}{\sigma_i^2} \end{aligned}$$

$$W = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ & & & 4 & 4 \\ 0 & & & & 1 \end{bmatrix}$$

See accompanying matlab code

```
% data1_14_hw1_sol.m 16-sep-2014
```

```
% #1
```

```
clear
disp('problem 1');
n=7;
n0=3;
r=4;
x=[0;1;2;3;4;5;6];
y=[3.244;4.142;4.746;4.959;4.652;4.271;3.216];
B=zeros(7,3);
f=zeros(7,1);
for i=1:7
    B(i,:)=[-1 -x(i) -x(i)^2];
    f(i)=-y(i);
end
W=eye(7);
B
W
f
del=inv(B'*W*B)*B'*W*f;
a0=del(1)
a1=del(2)
a2=del(3)
v=f-B*del
yhat=y+v
```

```
% #2a
```

```
clear
disp('problem 2a');
n=8;
n0=4;
r=4;
l=[64.92;50.13;24.82;160.02;85.02;114.93;159.98;74.97];
B=[0 0 -1 0;
   0 -1 0 0;
   -1 0 0 0;
   0 0 0 -1;
   0 1 1 1;
   0 -1 -1 0;
   0 0 0 -1;
   -1 -1 0 0];
W=diag([1 1 1 2 2 2 1 1]);
f=[-1(1);
   -1(2);
   -1(3);
   -1(4);
   360-1(5);
   -1(6);
   -1(7);
   -1(8)];
del=inv(B'*W*B)*B'*W*f;
v=f-B*del;
lhat=l+v
```

```
% #2b
```

```
clear
disp('problem 2b');
l=[64.92;50.13;24.82;160.02;85.02;114.93;159.98;74.97];
W=diag([1 1 1 2 2 2 1 1]);
A=[0 0 0 1 0 0 -1 0;
   -1 -1 0 0 0 1 0 0;
   0 -1 -1 0 0 0 0 1;
   1 1 0 1 1 0 0 0];
d=[0;0;0;360];
f=d-A*l;
A
W
f
z4=zeros(4,4);
N=[-W A;
   A z4];
t=[0;0;0;0;0;0;0;f];
sol=inv(N)*t;
v=sol(1:8);
lhat=l+v
```

```
% #3
```

```
clear
disp('problem 3');
n=8;
n0=4;
r=4;
B=zeros(8,4);
f=zeros(8,1);
X=[1;1;2;2];
Y=[1;2;1;2];
x=[7.299;7.612;9.300;9.622];
y=[0.696;2.721;0.387;2.382];
sx=[.02;.02;.01;.02];
sy=[.02;.02;.01;.02];
B=zeros(8,4);
f=zeros(8,1);
W=eye(8);
l=zeros(8,1);
s0=0.02;
s0sqr=s0^2;
idx=0;
```

```
for i=1:4
    idx=idx+1;
    B(idx,:) = [-X(i) -Y(i) -1 0];
    f(idx) = -x(i);
    W(idx,idx) = s0sqr/sx(i)^2;
    l(idx) = x(i);
    idx=idx+1;
    B(idx,:) = [-Y(i) X(i) 0 -1];
    f(idx) = -y(i);
    W(idx,idx) = s0sqr/sy(i)^2;
    l(idx) = y(i);
end
B
W
f
del=inv(B'*W*B)*B'*W*f;
a=del(1)
b=del(2)
c=del(3)
d=del(4)
v=f-B*del
lhat=l+v
```

data1\_14\_hw1\_sol

problem 1

B =

```

-1  0  0
-1 -1 -1
-1 -2 -4
-1 -3 -9
-1 -4 -16
-1 -5 -25
-1 -6 -36

```

W =

```

1  0  0  0  0  0  0
0  1  0  0  0  0  0
0  0  1  0  0  0  0
0  0  0  1  0  0  0
0  0  0  0  1  0  0
0  0  0  0  0  1  0
0  0  0  0  0  0  1

```

f =

```

-3.2440
-4.1420
-4.7460
-4.9590
-4.6520
-4.2710
-3.2160

```

a0 =

3.2308

a1 =

1.1264

a2 =

-0.1873

v =

```

-0.0132
0.0280
-0.0114
-0.0342
0.0884
-0.0896
0.0320

```

yhat =

```

3.2308
4.1700
4.7346
4.9248
4.7404
4.1814
3.2480

```

problem 2a

B =

```

0  0  -1  0
0 -1  0  0
-1  0  0  0
0  0  0 -1
0  1  1  1
0 -1 -1  0
0  0  0 -1
-1 -1  0  0

```

W =

```

1  0  0  0  0  0  0  0
0  1  0  0  0  0  0  0
0  0  1  0  0  0  0  0
0  0  0  2  0  0  0  0
0  0  0  0  2  0  0  0
0  0  0  0  0  2  0  0
0  0  0  0  0  0  1  0
0  0  0  0  0  0  0  1

```

f =

```

-64.9200
-50.1300
-24.8200
-160.0200
274.9800
-114.9300
-159.9800
-74.9700

```

del =

```

24.8453
50.0995
64.8642
160.0105

```

v =

```

-0.0558
-0.0305
0.0253
-0.0095
0.0058
0.0337
0.0305
-0.0253

```

lhat =

```

64.8642
50.0995
24.8453
160.0105
85.0258
114.9637
160.0105
74.9447

```

problem 2b

A =

```

0  0  0  1  0  0  -1  0
-1 -1  0  0  0  1  0  0
0 -1 -1  0  0  0  0  1

```

data1\_14\_hw1\_sol\_lst

```
W = 1 1 0 1 1 0 0 0
    1 0 0 0 0 0 0 0
    0 1 0 0 0 0 0 0
    0 0 1 0 0 0 0 0
    0 0 0 2 0 0 0 0
    0 0 0 0 2 0 0 0
    0 0 0 0 0 2 0 0
    0 0 0 0 0 0 1 0
    0 0 0 0 0 0 0 1
```

```
f = -0.0400
     0.1200
    -0.0200
    -0.0900
```

```
sol = -0.0558
       -0.0305
        0.0253
       -0.0095
        0.0058
        0.0337
        0.0305
       -0.0253
       -0.0305
        0.0674
       -0.0253
        0.0116
```

```
v = -0.0558
     -0.0305
      0.0253
     -0.0095
      0.0058
      0.0337
      0.0305
     -0.0253
```

```
lhat = 64.8642
       50.0995
       24.8453
       160.0105
       85.0258
       114.9637
       160.0105
       74.9447
```

```
problem 3
s0sqr = 4.0000e-04
```

```
B = -1 -1 -1 0
     -1 1 0 -1
     -1 -2 -1 0
     -2 1 0 -1
     -2 -1 -1 0
     -1 2 0 -1
     -2 -2 -1 0
     -2 2 0 -1
```

```
W = 1 0 0 0 0 0 0 0
    0 1 0 0 0 0 0 0
    0 0 1 0 0 0 0 0
    0 0 0 1 0 0 0 0
    0 0 0 0 4 0 0 0
    0 0 0 0 0 4 0 0
    0 0 0 0 0 0 1 0
    0 0 0 0 0 0 0 1
```

```
f = -7.2990
     -0.6960
     -7.6120
     -2.7210
     -9.3000
     -0.3870
     -9.6220
     -2.3820
```

```
a = 2.0058
```

```
b = 0.3199
```

```
c = 4.9692
```

```
d = -0.9810
```

```
v = -0.0041
     0.0090
     0.0028
    -0.0102
     0.0007
    -0.0019
    -0.0014
     0.0089
```

```
lhat = 7.2949
       0.7050
       7.6148
       2.7108
       9.3007
       0.3851
       9.6206
       2.3909
```

```
diary off
```