5-1

$$\frac{\hat{y}_2-\hat{y}_1}{x_2-x_1}=\frac{\hat{y}_3-\hat{y}_1}{x_3-x_1}$$

$x_2-x_1=1$, $x_3-x_1=2$

$$2\hat{y}_2-2\hat{y}_1=\hat{y}_3-\hat{y}_1$$

$$-\hat{y}_1+2\hat{y}_2-\hat{y}_3=0$$

$$\boxed{\hat{y}_1-2\hat{y}_2+\hat{y}_3=0}$$

$$y_1+v_1-2(y_2+v_2)+y_3+v_3=0$$

$$v_1-2v_2+v_3=-(y_1-2y_2+y_3)$$

1 2 3

$$v_1-2v_2+v_3=0.4$$

select v_1, v_2 $\quad v_3=0.4-v_1+2v_2$

$$\phi=\sum v_i^2=v_1^2+v_2^2+v_3^2$$

$w_i=1$, $\phi=v_1^2+v_2^2+(0.4-v_1+2v_2)^2$

$$\frac{\partial\phi}{\partial v_1}=\tfrac{1}{2}v_1+\tfrac{1}{2}(0.4-v_1+2v_2)(-1)=0$$

$$\frac{\partial\phi}{\partial v_2}=\tfrac{1}{2}v_2+\tfrac{1}{2}(0.4-v_1+2v_2)2=0$$

$$\left.\begin{aligned}v_1+v_1-2v_2&=0.4\\ v_2+4v_2-2v_1&=-0.8\end{aligned}\right\}\times 2$$

$$\begin{bmatrix}2&-2\\-2&5\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}0.4\\-0.8\end{bmatrix}$$

$N \quad v = t$

$v=N^{-1}t$

$v_1=0.0667$

$v_2=-0.1333$

$v_3=0.4-v_1+2v_2=0.0667$

Same as I/O solution

$\hat{y}=y+v$, $\hat{t}=t+v$

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5-2

Solve O/O problem again with Lagrange Mult.

$\phi=v_1^2+v_2^2+v_3^2$ subject to $v_1-2v_2+v_3=0.4$

$$\phi'=v_1^2+v_2^2+v_3^2-2k(v_1-2v_2+v_3-0.4)$$

$$\frac{\partial\phi'}{\partial v_1}=\tfrac{1}{2}v_1-\tfrac{1}{2}k=0$$

$$\frac{\partial\phi'}{\partial v_2}=\tfrac{1}{2}v_2+\tfrac{2}{2}k=0$$

$$\frac{\partial\phi'}{\partial v_3}=\tfrac{1}{2}v_3-\tfrac{1}{2}k=0$$

$$\frac{\partial\phi'}{\partial k}=-(v_1-2v_2+v_3-0.4)=0$$

$$v_1-k=0$$
$$v_2+2k=0$$
$$v_3-k=0$$
$$-(v_1-2v_2+v_3-0.4)=0$$

$\times(-1)$

$$-v_1+k=0$$
$$-v_2-2k=0$$
$$-v_3+k=0$$
$$v_1-2v_2+v_3=0.4$$

$$\left[\begin{array}{ccc|c}-1&0&0&1\\0&-1&0&-2\\0&0&-1&1\\\hline 1&-2&1&0\end{array}\right]\left[\begin{array}{c}v_1\\v_2\\v_3\\\hline k\end{array}\right]=\left[\begin{array}{c}0\\0\\0\\\hline 0.4\end{array}\right]$$

Solve by matrix or ...

$$\begin{bmatrix}v_1\\v_2\\v_3\\k\end{bmatrix}=\begin{matrix}0.0667\\-0.1333\\0.0667\\-0.0667\end{matrix}$$

k : nuisance parameter

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5-3

Derive MATRIX LS solution
1. I/O
2. O/O

I/O : analyze prob. $n_y n_o r$
choose $u = n_o$ parameters
write $c = n$ cond. eqns.
in form $V + B\delta = f$

resid. Coeff. mx. (design) param. RHS

$y_1 + v_1 = m x_1 + b$
$y_2 + v_2 = m x_2 + b$
$y_3 + v_3 = m x_3 + b$

$v_1 - m x_1 - b = -y_1$
$v_2 - m x_2 - b = -y_2$
$v_3 - m x_3 - b = -y_3$

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