Steps to solve I/O problem :

7-1

1. analyze problem n, n_0, r
2. choose $u = n_0$ unknowns (parameters)
3. write $c = n$ condition eqn's with one obs. per equation, expressed as function of param + constants $v + B\Delta = f$
4. select weights populate W, either from knowledge of σ_i's, + choice of σ_0^2 OR, assumption about weights
5. have B, f, W
6. $\Delta = (B^T W B)^{-1} B^T W f$
7. finish $v = f - B\Delta$
   $\hat{l} = l + v$
8. examine/evaluate magnitude of v's reconcile with assumptions

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Derivation of Matrix approach to Observation Only problem

7-2

analyzing problem n, n_0, r
write $c = r$ condition equations, use only observations + constants of the form
$Av = f$

example from line fit

$\hat{y}_1 - 2\hat{y}_2 + \hat{y}_3 = 0$

$y_1 + v_1 - 2(y_2 + v_2) + y_3 + v_3 = 0$

$v_1 - 2v_2 + v_3 = \underbrace{-(y_1 - 2y_2 + y_3)}_{\text{obs I have numbers}}$

$$\begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{matrix} 0.4 \\ 0.4 \end{matrix}$$

$$\underset{\substack{(c,n)\\(r,n)}}{A} \quad \underset{(n,1)}{v} = \underset{\substack{(c,1)\\(r,1)}}{f}$$

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minimize our obj. function & satisfy cond. eqn's 7-3

$$\phi' = v^T W v - 2k^T(Av - f)$$
$$- 2(Av-f)^T k$$

$$\frac{\partial \phi'}{\partial v} = \cancel{2} v^T W - \cancel{2} k^T A = 0 \text{ (row vector of 0's)}$$

$$\frac{\partial \phi'}{\partial k} = -\cancel{2}(Av-f)^T = 0 \quad ''$$

$$Wv - A^T k = 0 \text{ (col vect. of 0's)}$$

(transp) $-(Av - f) = 0$

$$-Av = -f$$

(x-1)

$$-Wv + A^T k = 0$$
$$Av = f$$

$$\begin{bmatrix} -W & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$$ normal equations for obs only

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7-4

$$\begin{bmatrix} -W & A^T \\ A & 0 \end{bmatrix} \begin{pmatrix} v \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix}$$

$Wv = A^T k$

$v = W^{-1} A^T k$, $v = Q A^T k$

$Av = f$

$\underbrace{AQA^T}_{Q_e} k = f$

$k = W_e f$

$\boxed{v = QA^T W_e f}$

$\hat{\ell} = \ell + v$

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Steps to solve O/O LS problem 7-5

1. analyze problem n, n_0, r
2. write $c = r$ cond. eqn's of form
   $Av = f$
3. select W matrix choosing σ_0^2, $w_i = \frac{\sigma_0^2}{\sigma_i^2}$
4. solve $\begin{pmatrix} -W & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} v \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix}$ OR

   $k = W_e f$
   $v = QA^T k$
   $\hat{l} = l + v$

5. examine/eval v's

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