Small angle assumption for 3D rotations

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$\cos\theta \approx 1$

$\sin\theta \approx \theta$

$\sin\theta_1 \cdot \sin\theta_2 \approx 0$

$M_z \; M_y \; M_x$

$$\begin{pmatrix} ck & sk & 0 \\ -sk & ck & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\phi & 0 & -s\phi \\ 0 & 1 & 0 \\ s\phi & 0 & c\phi \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & cw & sw \\ 0 & -sw & cw \end{pmatrix}$$

$$\downarrow \qquad \begin{pmatrix} 1 & 0 & -\phi \\ 0 & 1 & 0 \\ \phi & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \omega \\ 0 & -\omega & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & k & 0 \\ -k & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -\phi \\ 0 & 1 & \omega \\ \phi & -\omega & 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & K & -\phi \\ -K & 1 & \omega \\ \phi & -\omega & 1 \end{bmatrix}$$

rotation matrix assume all angles are small

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$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$ 1 par. transf.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \lambda\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$ 2 par. transf.

$\lambda\cos\theta = a$

$\lambda\sin\theta = b$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

if x,y constant, then

$x' = ax + by$

$y' = -bx + ay$

<u>add shift or translation parameters</u>

4 parameter transformation

helmert transf.

2D <u>conformal</u> coord. transf.

└ shape preserved

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1. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$ 9-3

2. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x+e \\ y+f \end{pmatrix}$

estimate using model 1 a, b, c, d

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix}}_{\begin{pmatrix} c \\ d \end{pmatrix}}$$

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix}$$

$$\begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}^{-1} \begin{pmatrix} c \\ d \end{pmatrix}$$

a, b, c, d : convenient for calculations

$\theta, \lambda, t_x, t_y$: physical parameters

$a = \lambda \cos\theta$, $b = \lambda \sin\theta$

$a^2 + b^2 = \lambda^2 \cos^2\theta + \lambda^2 \sin^2\theta = \lambda^2(\cos^2\theta + \sin^2\theta)$

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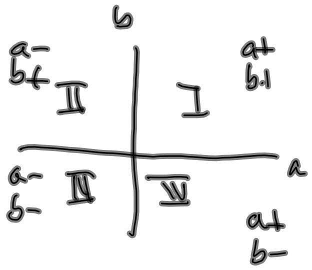
$a^2 + b^2 = \lambda^2$, $\lambda = \sqrt{a^2+b^2}$ 9-4

$\frac{b}{a} = \frac{\lambda \sin\theta}{\lambda \cos\theta} = \tan\theta$

$\theta = \arctan(b/a)$,

to get θ in correct quadrant



$\theta = \text{atan2}(b, a)$ 4 quadrant version

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$$\boxed{\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}}$$ apply transf

without scale : rigid body transf

$$x' = ax + by + c$$
$$y' = -bx + ay + d$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{pmatrix} x & y & 1 & 0 \\ y & -x & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$V + B\Delta = f$, if x', y' observations
x, y constant

$$\hat{x}' = x' + v_{x'}$$
$$\hat{y}' = y' + v_{y'}$$

$$\begin{pmatrix} \hat{x}' \\ \hat{y}' \end{pmatrix} = \begin{pmatrix} x & y & 1 & 0 \\ y & -x & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \Rightarrow \begin{pmatrix} v_{x'} \\ v_{y'} \end{pmatrix} + \begin{pmatrix} -x & -y & -1 & 0 \\ -y & x & 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -x' \\ -y' \end{pmatrix}$$

$$V + B\Delta = f$$

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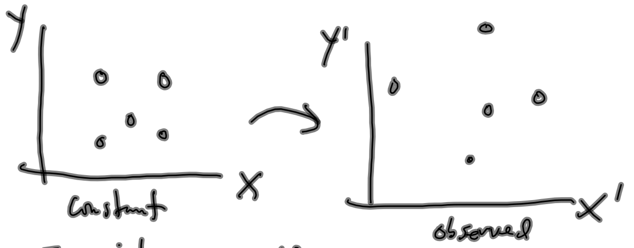

5 points $n = 10$
$n_0 = 4$
$r = 6$

$$\begin{bmatrix} v_{x_1'} \\ v_{y_1'} \\ v_{x_2'} \\ v_{y_2'} \\ \vdots \\ v_{x_n'} \\ v_{y_n'} \end{bmatrix} + \begin{bmatrix} -x_1 & -y_1 & -1 & 0 \\ -y_1 & x_1 & 0 & -1 \\ -x_2 & -y_2 & -1 & 0 \\ -y_2 & x_2 & 0 & -1 \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

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