Lect. 11

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$$f(x,y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\cdot \Delta y$$

$$f(x_1,x_2,\dots,x_n) \approx f(x_1^0, x_2^0, \dots, x_n^0) + \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \dots \frac{\partial f}{\partial x_n}\Delta x_n$$

$$F_1(x_1,x_2,\dots x_n) = 0 \approx F_1(x_1^0 x_2^0 \dots x_n^0) + \frac{\partial F_1}{\partial x_1}\Delta x_1 + \frac{\partial F_1}{\partial x_2}\Delta x_2 + \dots \frac{\partial F_1}{\partial x_n}\Delta x_n$$

$$F_2(x_1,x_2,\dots x_n) = 0 \approx F_2(x_1^0 x_2^0 \dots x_n^0) + \frac{\partial F_2}{\partial x_1}\Delta x_1 + \frac{\partial F_2}{\partial x_2}\Delta x_2 + \dots \frac{\partial F_2}{\partial x_n}\Delta x_n$$

$$\vdots$$

$$F_n(x_1,x_2,\dots x_n) = 0 \approx \underbrace{F_n(x_1^0 x_2^0 \dots x_n^0)} + \underbrace{\frac{\partial F_n}{\partial x_1}\Delta x_1 + \frac{\partial F_n}{\partial x_2}\Delta x_2 + \dots \frac{\partial F_n}{\partial x_n}\Delta x_n}$$

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \approx \begin{bmatrix} F_1^0 \\ F_2^0 \\ \vdots \\ F_n^0 \end{bmatrix} + \begin{bmatrix} \partial F_1/\partial x_1 & \partial F_1/\partial x_2 & \cdots & \partial F_1/\partial x_n \\ \partial F_2/\partial x_1 & \partial F_2/\partial x_2 & \cdots & \partial F_2/\partial x_n \\ \vdots & & & \\ \partial F_n/\partial x_1 & \partial F_n/\partial x_2 & \cdots & \partial F_n/\partial x_n \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

$$0 = F(x_0) + J\,\Delta x$$

$$\boxed{\Delta x = -J^{-1} F(x_0)}$$

multivariate version of newton iteration

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$$X_1 = X_0 + \Delta X$$

$X_{i+1} = X_i + \Delta X_i$ keep doing until $\|\Delta x\|$ small

numerical example $x, y = ?$

$x^2 + y = 2 \qquad f_1(x,y) = x^2 + y - 2 = 0$

$x - 3y^2 = -2 \qquad f_2(x,y) = x - 3y^2 + 2 = 0$

$$\boxed{\begin{matrix} x^0 = 1.05 \\ y^0 = 1.05 \end{matrix}}$$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{bmatrix} 2x & 1 \\ 1 & -6y \end{bmatrix} \qquad F = \begin{bmatrix} f_1^0 \\ f_2^0 \end{bmatrix}$$

$$J = \begin{bmatrix} 2.10 & 1 \\ 1 & -6.3 \end{bmatrix}, \quad F = \begin{bmatrix} 0.15 25 \\ -0.2575 \end{bmatrix}$$

$$\Delta X = -J^{-1}F(x_0) = \begin{matrix} -0.0494 \\ -0.0487 \end{matrix} \qquad \begin{bmatrix} 1.05 \\ 1.05 \end{bmatrix} + \begin{bmatrix} -.0494 \\ -.0487 \end{bmatrix} = \begin{bmatrix} 1.0006 \\ 1.0013 \end{bmatrix}$$

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after 2^{nd} iteration

$$\begin{bmatrix} 1.0000 \\ 1.0000 \end{bmatrix}$$

Convergence $1, 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, \ldots$

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Classical Newton Iteration

Solving NL equations which are UNIQUELY DETERMINED, n eqn, n unk $(r=0)$

NL LS

Solving NL equations which are OVER-DETERMINED, m eqn, n unk $m>n$ $(r>0)$

Big Picture for NL solutions

analyze problem n, n_0, r

$r=0$	$r>0$	
unique sol.	LS problem Chone I/O	G/O
linearize $J, F(x_0)$	B, f, W	A, f, W
$\Delta = -J^{-1}F(x_0)$	$\Delta = (B^TWB)^{-1}B^TWf$	$k = W_ef$ $v = QA^Tk$
$x_{i+1} = x_i + \Delta$	$x_{i+1} = x_i + \Delta$	$\mathring{l}_{i+1} = l + v$

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linearize I/O problem

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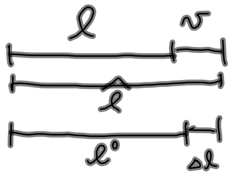
$\hat{\ell} = G(x)$

$F(\hat{\ell}, x) = \hat{\ell} - G(x) = 0$

$F(\hat{\ell}, x) \approx F(\ell^0, x^0) + \frac{\partial F}{\partial \ell}\Delta \ell + \frac{\partial F}{\partial x}\Delta x = 0$



$v \rightarrow v$

$\Delta \ell \rightarrow$ zero

$\ell + v = \ell^0 + \Delta \ell$

$\Delta \ell = (\ell - \ell^0) + v$

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