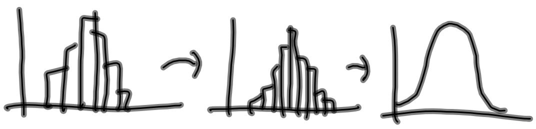
Lecture 21

21-1

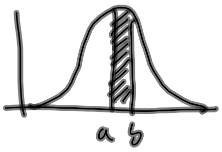


Scale so area = 1

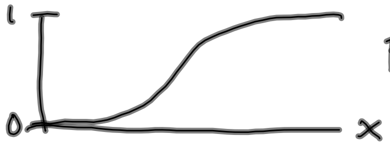
probability density function

$p(a < X < b) : \int_a^b f(x)\,dx$

$f(x)$



cumulative distribution functions



$F(x) : P(X < x) = \int_0^x f(u)\,du$

$$\left.\begin{array}{l} \text{pdf} : f = \text{pdf}(x) \\ \text{cdf} : P = \text{cdf}(x) \\ \text{icdf} : x = \text{icdf}(P) \end{array}\right\}$$

in matlab call need arg : to specify densityfunct.

$f(x) = \frac{d}{dx} F(x)$, $\int_a^b f(x)\,dx = F(b) - F(a)$

Oct 14-4:22 PM

Expectation

21-2

discrete case $E(x) = \sum_i x_i \cdot p(x_i)$

continuous case $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

moment (statistical)

r^{th} moment @ $\left\{\begin{array}{c} 0 \\ \mu \end{array}\right\}$ $\quad \int_{-\infty}^{\infty} (x-0)^r f(x)\,dx$

$$\int_{-\infty}^{+\infty} (x-\mu_x)^r f(x)\,dx$$

Mean $\mu_x = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

1^{st} moment @ origin

Variance

$\sigma_x^2 = E\{(x-\mu_x)^2\} = \int_{-\infty}^{+\infty} (x-\mu_x)^2 f(x)\,dx$

2^{nd} moment @ mean

Oct 14-4:23 PM

21-3

$$\sigma_x^2 = E\{(x-\mu_x)(x-\mu_x)\} \leftarrow \sigma_{xx}$$

$$\sigma_x = \sqrt{\sigma_x^2}$$

$$\sigma_{xy} = E\{(x-\mu_x)(y-\mu_y)\}$$

$$\rho_{xy} = \text{correlation coefficient} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} : -1 \rightarrow +1$$

pop stat.	Sample statistics
μ_x	$\bar{x}$
σ_x^2	s_x^2
σ_x	s_x
σ_{xy}	s_{xy}
ρ_{xy}	r_{xy}

Oct 14-4:23 PM

21-4

Expectation Operator : Linear

$$E(x+y) = E(x) + E(y)$$

$$E(ax) = a\,E(x)$$

random vector

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad E(\vec{x}) = \begin{bmatrix} E(x_1) \\ E(x_2) \\ \vdots \\ E(x_n) \end{bmatrix} = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_n} \end{bmatrix} = \vec{\mu}_x$$

$$\Sigma_{xx} = E\{(\vec{x}-\vec{\mu})(\vec{x}-\vec{\mu}_x)^T\}$$

$$E\{ [\;] \cdot [\;] \} = E\{\square\}$$

Oct 14-4:23 PM

21-5

$$\Sigma_{xx} = E\left\{ \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \\ \vdots \\ x_n-\mu_n \end{bmatrix} \cdot \begin{bmatrix} x_1-\mu_1 & x_2-\mu_2 & \cdots & x_n-\mu_n \end{bmatrix} \right\}$$

$$\Sigma_{xx} = E\left\{ \begin{bmatrix} (x_1-\mu_1)(x_1-\mu_1) & (x_1-\mu_1)(x_2-\mu_2) & \cdots & (x_1-\mu_1)(x_n-\mu_n) \\ (x_2-\mu_2)(x_1-\mu_1) & (x_2-\mu_2)(x_2-\mu_2) & \cdots & (x_2-\mu_2)(x_n-\mu_n) \\ \vdots & & & \\ (x_n-\mu_n)(x_1-\mu_1) & (x_n-\mu_n)(x_2-\mu_2) & \cdots & (x_n-\mu_n)(x_n-\mu_n) \end{bmatrix} \right\}$$

$$\Sigma_{xx} = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots & \sigma_{x_1x_n} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 & & \vdots \\ \vdots & & \ddots & \\ \sigma_{x_nx_1} & \cdots & & \sigma_{x_n}^2 \end{bmatrix}$$

variance/covariance matrix

covariance matrix

Oct 14-4:23 PM

21-6

Covariance Matrix

Symmetric

positive semi-definite $x^T\Sigma_{xx}x \geq 0$ any x

Σ_{xx} describes precision of random vector $\vec{x}$

Error Propagation

$$\vec{l},\ \Sigma_{ll} \longrightarrow \boxed{LS} \longrightarrow \begin{matrix} x,\ \Sigma_{xx} \\ \Delta,\ \Sigma_{\Delta\Delta} \\ v,\ \Sigma_{vv} \\ \hat{l},\ \Sigma_{\hat{l}\hat{l}} \end{matrix}$$

Oct 14-4:23 PM