Lecture 23

23-1

Exam Friday This week:

1 page notes

prior exams posted online

mathematical < funct. model (equations)
stochastic model σ_0^2, { unk, obs, constr

counting n, n_0, r

LS: 2 methods: Ind Obs.
Obs. only.

concepts/vocabulary: residual, weight, σ^2, σ_0^2,
observation (meas.), adj. obs., obj. function,
lagrange mult., linear vs. non linear,
linearization via Taylor series,
∇ & Δl, rotations + rot. matrices
2D & 3D, euler angles, orthogonal,
minimal constraints

Oct 19-4:21 PM

Statistics, pdf, cdf, Σ, μ, σ^2, E.P. 23-2

problems & setting up problems

L: line fit, curve fit, surface fit,
coordinate transf., level networks, angle fig.
collinear length, ...

NL: coord. transf (7 par), range obs, 2D/3D,
azimuth obs, angle obs., dual coord. eqs.,
(quadrant problem)

counting, write cond. eqns, linearize, E.P.

Oct 19-4:22 PM

E.P. $y = a_1 x_1 + a_2 x_2$, $\Sigma_{\binom{x_1}{x_2}} = \begin{bmatrix}\sigma_{x_1}^2 & 0\\ 0 & \sigma_{x_2}^2\end{bmatrix}$ 23-3

$$y = \begin{bmatrix}a_1 & a_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\Sigma_{yy} = \sigma_y^2 = A\Sigma_{xx}A^T$$

$$= \begin{bmatrix}a_1 & a_2\end{bmatrix}\begin{bmatrix}\sigma_{x_1}^2 & 0\\ 0 & \sigma_{x_2}^2\end{bmatrix}\begin{bmatrix}a_1\\ a_2\end{bmatrix}$$

$$\sigma_y^2 = \begin{bmatrix}a_1\sigma_{x_1}^2 & a_2\sigma_{x_2}^2\end{bmatrix}\begin{bmatrix}a_1\\ a_2\end{bmatrix} = a_1^2\sigma_{x_1}^2 + a_2^2\sigma_{x_2}^2$$

ASSUMPTION : x's uncorrelated

$y = \begin{bmatrix}a_1 & a_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, $\Sigma_{xx} = \begin{bmatrix}\sigma_{x_1}^2 & \sigma_{x_1x_2}\\ \sigma_{x_1x_2} & \sigma_{x_2}^2\end{bmatrix}$ 23-4

$$\sigma_y^2 = \begin{bmatrix}a_1 & a_2\end{bmatrix}\begin{bmatrix}\sigma_{x_1}^2 & \sigma_{x_1x_2}\\ \sigma_{x_1x_2} & \sigma_{x_2}^2\end{bmatrix}\begin{bmatrix}a_1\\ a_2\end{bmatrix}$$

$$\begin{bmatrix}a_1\sigma_{x_1}^2 + a_2\sigma_{x_1x_2} & a_1\sigma_{x_1x_2} + a_2\sigma_{x_2}^2\end{bmatrix}\begin{bmatrix}a_1\\ a_2\end{bmatrix}$$

$$a_1^2\sigma_{x_1}^2 + \underbrace{a_1a_2\sigma_{x_1x_2} + a_1a_2\sigma_{x_1x_2}} + a_2^2\sigma_{x_2}^2$$

$$\sigma_y^2 = a_1^2\sigma_{x_1}^2 + \underline{2a_1a_2\sigma_{x_1x_2}} + a_2^2\sigma_{x_2}^2$$

general result

$\Sigma = \begin{bmatrix}\sigma_1^2 & \sigma_{12}\\ \sigma_{21} & \sigma_2^2\end{bmatrix}$ normalize covariance

$$\frac{\sigma_{12}}{\sigma_1\sigma_2} = \rho_{12}$$

$\Sigma = \begin{bmatrix}\sigma_1^2 & \sigma_1\sigma_2\rho_{12}\\ \sigma_1\sigma_2\rho_{12} & \sigma_2^2\end{bmatrix}$

$$\sigma_{12} = \sigma_1\sigma_2\rho_{12}$$

$\rho : -1 \to +1$ 23-5

$y_1 = x_1 + 2x_2$
$y_2 = 2x_1 + x_2$ $\Sigma_{xx} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ $z = y_1 + 2y_2$

$\Sigma_{zz} = \sigma_z^2 = ?$

$\boxed{1\ \text{step}}$ $z = x_1 + 2x_2 + 2(2x_1 + x_2)$

$z = 5x_1 + 4x_2$

$z = \begin{bmatrix} 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\sigma_z^2 = \begin{bmatrix} 5 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = 204$

$A \Sigma_{xx} A^T$

$\sigma_z = \sqrt{204}$

$\boxed{2\ \text{step}}$ next time